Mathematica 11.3 Integration Test Results

Test results for the 913 problems in "1.1.3.4 (e x)^m (a+b x^n)^p (c+d x^n)^q.m"

Problem 30: Result more than twice size of optimal antiderivative.

$$\int x^2 \left(a+b x^3\right)^5 \left(A+B x^3\right) \, dx$$

Optimal (type 1, 42 leaves, 3 steps):

$$\frac{(A b-a B) \left(a+b x^3\right)^6}{18 b^2}+\frac{B \left(a+b x^3\right)^7}{21 b^2}$$

Result (type 1, 107 leaves):

$$\frac{1}{126} x^3 \left(42 a^5 A+21 a^4 (5 A b+a B) x^3+70 a^3 b (2 A b+a B) x^6+105 a^2 b^2 (A b+a B) x^9+42 a b^3 (A b+2 a B) x^{12}+7 b^4 (A b+5 a B) x^{15}+6 b^5 B x^{18}\right)$$

Problem 54: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(a+b x^3\right)^5 \left(A+B x^3\right)}{x^{22}} \, dx$$

Optimal (type 1, 48 leaves, 3 steps):

$$-\frac{A \left(a+b x^3\right)^6}{21 a x^{21}}+\frac{(A b-7 a B) \left(a+b x^3\right)^6}{126 a^2 x^{18}}$$

Result (type 1, 118 leaves):

$$-\frac{1}{126 x^{21}}\left(21 b^5 x^{15} \left(A+2 B x^3\right)+35 a b^4 x^{12} \left(2 A+3 B x^3\right)+35 a^2 b^3 x^9 \left(3 A+4 B x^3\right)+21 a^3 b^2 x^6 \left(4 A+5 B x^3\right)+7 a^4 b x^3 \left(5 A+6 B x^3\right)+a^5 \left(6 A+7 B x^3\right)\right)$$

Problem 155: Result more than twice size of optimal antiderivative.

$$\int \frac{x^{7/2} \left(A+B x^3\right)}{a+b x^3} \, dx$$

Optimal (type 3, 73 leaves, 5 steps):

$$\frac{2 (A b-a B) x^{3/2}}{3 b^2}+\frac{2 B x^{9/2}}{9 b}-\frac{2 \sqrt{a} (A b-a B) \text{ArcTan}\left[\frac{\sqrt{b} x^{3/2}}{\sqrt{a}}\right]}{3 b^{5/2}}$$

Result (type 3, 180 leaves):

$$\frac{2\left(A b-a B\right) x^{3/2}}{3 b^2}+\frac{2 B x^{9/2}}{9 b}+\frac{2 \sqrt{a}\left(-A b+a B\right) \text{ArcTan}\left[\frac{-\sqrt{3}\, a^{1/6}+2 b^{1/6} \sqrt{x}}{a^{1/6}}\right]}{3 b^{5/2}}+\frac{2 \sqrt{a}\left(-A b+a B\right) \text{ArcTan}\left[\frac{\sqrt{3}\, a^{1/6}+2 b^{1/6} \sqrt{x}}{a^{1/6}}\right]}{3 b^{5/2}}-\frac{2 \sqrt{a}\left(-A b+a B\right) \text{ArcTan}\left[\frac{b^{1/6} \sqrt{x}}{a^{1/6}}\right]}{3 b^{5/2}}$$

Problem 158: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{x}\left(A+B x^3\right)}{a+b x^3}\, dx$$

Optimal (type 3, 53 leaves, 4 steps):

$$\frac{2 B x^{3/2}}{3 b}+\frac{2\left(A b-a B\right) \text{ArcTan}\left[\frac{\sqrt{b}\, x^{3/2}}{\sqrt{a}}\right]}{3 \sqrt{a}\, b^{3/2}}$$

Result (type 3, 139 leaves):

$$\frac{1}{3 \sqrt{a}\, b^{3/2}} 2\left(\sqrt{a} \sqrt{b}\, B x^{3/2}+\left(-A b+a B\right) \text{ArcTan}\left[\sqrt{3}-\frac{2 b^{1/6} \sqrt{x}}{a^{1/6}}\right]+\left(A b-a B\right) \text{ArcTan}\left[\sqrt{3}+\frac{2 b^{1/6} \sqrt{x}}{a^{1/6}}\right]-A b\, \text{ArcTan}\left[\frac{b^{1/6} \sqrt{x}}{a^{1/6}}\right]+a B\, \text{ArcTan}\left[\frac{b^{1/6} \sqrt{x}}{a^{1/6}}\right]\right)$$

Problem 161: Result more than twice size of optimal antiderivative.

$$\int \frac{A+B x^3}{x^{5/2}\left(a+b x^3\right)}\, dx$$

Optimal (type 3, 53 leaves, 4 steps):

$$-\frac{2 A}{3 a x^{3/2}}-\frac{2\left(A b-a B\right) \text{ArcTan}\left[\frac{\sqrt{b}\, x^{3/2}}{\sqrt{a}}\right]}{3 a^{3/2} \sqrt{b}}$$

Result (type 3, 160 leaves):

$$-\frac{2 A}{3 a x^{3/2}}+\frac{2\left(-A b+a B\right) \text{ArcTan}\left[\frac{-\sqrt{3}\, a^{1/6}+2 b^{1/6} \sqrt{x}}{a^{1/6}}\right]}{3 a^{3/2} \sqrt{b}}+\frac{2\left(-A b+a B\right) \text{ArcTan}\left[\frac{\sqrt{3}\, a^{1/6}+2 b^{1/6} \sqrt{x}}{a^{1/6}}\right]}{3 a^{3/2} \sqrt{b}}-\frac{2\left(-A b+a B\right) \text{ArcTan}\left[\frac{b^{1/6} \sqrt{x}}{a^{1/6}}\right]}{3 a^{3/2} \sqrt{b}}$$

Problem 185: Result unnecessarily involves imaginary or complex numbers.

$$\int x^3 \sqrt{a+b x^3}\left(A+B x^3\right) dx$$

Optimal (type 4, 303 leaves, 4 steps):

$$\frac{6 a (17 A b-8 a B) x \sqrt{a+b x^3}}{935 b^2}+\frac{2 (17 A b-8 a B) x^4 \sqrt{a+b x^3}}{187 b}+\frac{2 B x^4 \left(a+b x^3\right)^{3/2}}{17 b}-\left(4\times 3^{3/4} \sqrt{2+\sqrt{3}}\, a^2 (17 A b-8 a B) \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/\left(935 b^{7/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 209 leaves):

$$\sqrt{a+b x^3} \left(-\frac{6 a (-17 A b+8 a B) x}{935 b^2}+\frac{2 (17 A b+3 a B) x^4}{187 b}+\frac{2 B x^7}{17}\right)-\left(4 i\, 3^{3/4} a^{7/3} (17 A b-8 a B) \sqrt{(-1)^{5/6} \left(-1+\frac{(-b)^{1/3} x}{a^{1/3}}\right)} \sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\Big/\left(935 (-b)^{1/3} b^2 \sqrt{a+b x^3}\right)$$

Problem 186: Result unnecessarily involves imaginary or complex numbers.

$$\int \sqrt{a+b x^3} \left(A+B x^3\right) dx$$

Optimal (type 4, 268 leaves, 3 steps):

$$\frac{2\,(11\,A\,b-2\,a\,B)\,x\,\sqrt{a+b\,x^3}}{55\,b}+\frac{2\,B\,x\,\left(a+b\,x^3\right)^{3/2}}{11\,b}+\left(2\times 3^{3/4}\,\sqrt{2+\sqrt{3}}\;a\,(11\,A\,b-2\,a\,B)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Bigg/\left(55\,b^{4/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 182 leaves):

$$-\left(\left(2\left((-b)^{1/3}\,x\,\left(a+b\,x^3\right)\,\left(11\,A\,b+3\,a\,B+5\,b\,B\,x^3\right)+i\,3^{3/4}\,a^{4/3}\,(11\,A\,b-2\,a\,B)\,\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\right)\Bigg/\left(55\,(-b)^{4/3}\,\sqrt{a+b\,x^3}\right)\right)$$

Problem 187: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a+b\,x^3}\,\left(A+B\,x^3\right)}{x^3}\,dx$$

Optimal (type 4, 269 leaves, 3 steps):

$$\frac{(5 A b+4 a B)\, x \sqrt{a+b x^3}}{10 a}-\frac{A \left(a+b x^3\right)^{3/2}}{2 a x^2}+\left(3^{3/4} \sqrt{2+\sqrt{3}}\, (5 A b+4 a B) \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Bigg/\left(10 b^{1/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 175 leaves):

$$\left(-\frac{A}{2 x^2}+\frac{2 B x}{5}\right) \sqrt{a+b x^3}+\left(i\, 3^{3/4} a^{1/3} (5 A b+4 a B) \sqrt{(-1)^{5/6} \left(-1+\frac{(-b)^{1/3} x}{a^{1/3}}\right)} \sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\Bigg/\left(10 (-b)^{1/3} \sqrt{a+b x^3}\right)$$

Problem 188: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a+b x^3} \left(A+B x^3\right)}{x^6}\, dx$$

Optimal (type 4, 272 leaves, 3 steps):

$$\frac{(A b-10 a B)\sqrt{a+b x^3}}{20 a x^2}-\frac{A\left(a+b x^3\right)^{3/2}}{5 a x^5}-\left(3^{3/4}\sqrt{2+\sqrt{3}}\, b^{2/3}(A b-10 a B)\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/\left(20 a\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\sqrt{a+b x^3}\right)$$

Result (type 4, 189 leaves):

$$\left(-\frac{A}{5 x^5}+\frac{-3 A b-10 a B}{20 a x^2}\right)\sqrt{a+b x^3}+\left(i\, 3^{3/4} b(-A b+10 a B)\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}x}{a^{1/3}}\right)}\sqrt{1+\frac{(-b)^{1/3}x}{a^{1/3}}+\frac{(-b)^{2/3}x^2}{a^{2/3}}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-b)^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\Big/\left(20 a^{2/3}(-b)^{1/3}\sqrt{a+b x^3}\right)$$

Problem 189: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\sqrt{a+b x^3}\left(A+B x^3\right)}{x^9}\,dx$$

Optimal (type 4, 305 leaves, 4 steps):

$$\frac{(7 A b-16 a B)\sqrt{a+b x^3}}{80 a x^5}+\frac{3 b (7 A b-16 a B)\sqrt{a+b x^3}}{320 a^2 x^2}-\frac{A (a+b x^3)^{3/2}}{8 a x^8}+\left(3^{3/4}\sqrt{2+\sqrt{3}}\, b^{5/3} (7 A b-16 a B)\left(a^{1/3}+b^{1/3} x\right)\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4\sqrt{3}\right]\right)\Bigg/\left(320 a^2\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\sqrt{a+b x^3}\right)$$

Result (type 4, 206 leaves):

$$-\frac{\sqrt{a+b x^3}\left(40 a^2 A+4 a (3 A b+16 a B) x^3-3 b (7 A b-16 a B) x^6\right)}{320 a^2 x^8}+\left(i\, 3^{3/4} (-b)^{5/3} (7 A b-16 a B)\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3} x}{a^{1/3}}\right)}\sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\Bigg/\left(320 a^{5/3}\sqrt{a+b x^3}\right)$$

Problem 190: Result unnecessarily involves imaginary or complex numbers.

$$\int x^4\sqrt{a+b x^3}\,\left(A+B x^3\right)\,dx$$

Optimal (type 4, 581 leaves, 6 steps):

$$\frac{6 a (19 A b - 10 a B) x^2 \sqrt{a + b x^3}}{1729 b^2} + \frac{2 (19 A b - 10 a B) x^5 \sqrt{a + b x^3}}{247 b} - \frac{24 a^2 (19 A b - 10 a B) \sqrt{a + b x^3}}{1729 b^{8/3} \left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)} + \frac{2 B x^5 \left(a + b x^3\right)^{3/2}}{19 b} +$$

$$\left(12 \times 3^{1/4} \sqrt{2 - \sqrt{3}}\, a^{7/3} (19 A b - 10 a B) \left(a^{1/3} + b^{1/3} x\right) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)^2}}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1 - \sqrt{3}\right) a^{1/3} + b^{1/3} x}{\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x}\right], -7 - 4 \sqrt{3}\right]\right) \Big/$$

$$\left(1729 b^{8/3} \sqrt{\frac{a^{1/3} \left(a^{1/3} + b^{1/3} x\right)}{\left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)^2}} \sqrt{a + b x^3}\right) -$$

$$\left(8 \sqrt{2}\, 3^{3/4} a^{7/3} (19 A b - 10 a B) \left(a^{1/3} + b^{1/3} x\right) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)^2}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1 - \sqrt{3}\right) a^{1/3} + b^{1/3} x}{\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x}\right], -7 - 4 \sqrt{3}\right]\right) \Big/$$

$$\left(1729 b^{8/3} \sqrt{\frac{a^{1/3} \left(a^{1/3} + b^{1/3} x\right)}{\left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)^2}} \sqrt{a + b x^3}\right)$$

Result (type 4, 263 leaves) :

$$\frac{1}{1729\,(-b)^{8/3}\sqrt{a+b\,x^3}}\left(2\,(-b)^{2/3}\,(a+b\,x^3)\,\left(3\,a\,(19\,A\,b-10\,a\,B)\,x^2+7\,b\,(19\,A\,b+3\,a\,B)\,x^5+91\,b^2\,B\,x^8\right)+\right.$$
$$4\,(-1)^{2/3}\,3^{3/4}\,a^{8/3}\,(19\,A\,b-10\,a\,B)\,\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}$$
$$\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\left(\sqrt{3}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+\right.$$
$$\left.\left.(-1)^{5/6}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\right)$$

Problem 191: Result unnecessarily involves imaginary or complex numbers.

$$\int x\,\sqrt{a+b\,x^3}\,\left(A+B\,x^3\right)\,dx$$

Optimal (type 4, 548 leaves, 5 steps):

$$\frac{2\,(13 A b-4 a B)\,x^2\sqrt{a+b x^3}}{91 b}+\frac{6 a\,(13 A b-4 a B)\sqrt{a+b x^3}}{91 b^{5/3}\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)}+$$

$$\frac{2 B x^2\left(a+b x^3\right)^{3/2}}{13 b}-\left(3\times 3^{1/4}\sqrt{2-\sqrt{3}}\;a^{4/3}\,(13 A b-4 a B)\left(a^{1/3}+b^{1/3}x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(91 b^{5/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\sqrt{a+b x^3}\right)+$$

$$\left(2\sqrt{2}\;3^{3/4}a^{4/3}\,(13 A b-4 a B)\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(91 b^{5/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\sqrt{a+b x^3}\right)$$

Result (type 4, 246 leaves):

$$\frac{2 x^2\sqrt{a+b x^3}\left(13 A b+3 a B+7 b B x^3\right)}{91 b}-$$

$$\left(2\,(-1)^{1/6}3^{3/4}a^{5/3}\,(13 A b-4 a B)\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}x}{a^{1/3}}\right)}\sqrt{1+\frac{(-b)^{1/3}x}{a^{1/3}}+\frac{(-b)^{2/3}x^2}{a^{2/3}}}\right.$$

$$\left(-i\sqrt{3}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-b)^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+\right.$$

$$\left.\left.(-1)^{1/3}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-b)^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right)\Big/\left(91\,(-b)^{5/3}\sqrt{a+b x^3}\right)$$

Problem 192: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a+b x^3}\,\left(A+B x^3\right)}{x^2}\,dx$$

Optimal (type 4, 545 leaves, 5 steps):

$$\frac{(7 A b+2 a B)\, x^2 \sqrt{a+b x^3}}{7 a}+\frac{3\,(7 A b+2 a B) \sqrt{a+b x^3}}{7 b^{2/3} \left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}-\frac{A \left(a+b x^3\right)^{3/2}}{a x}-\left(3\times 3^{1/4} \sqrt{2-\sqrt{3}}\; a^{1/3}\,(7 A b+2 a B) \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Bigg/\left(14 b^{2/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\; \sqrt{a+b x^3}\right)+\left(\sqrt{2}\; 3^{3/4} a^{1/3}\,(7 A b+2 a B) \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Bigg/\left(7 b^{2/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\; \sqrt{a+b x^3}\right)$$

Result (type 4, 236 leaves):

$$\left(-\frac{A}{x}+\frac{2\,B\,x^2}{7}\right)\sqrt{a+b\,x^3}+$$

$$\left((-1)^{1/6}\,3^{3/4}\,a^{2/3}\,(7\,A\,b+2\,a\,B)\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\right.$$

$$\left(-\mathrm{i}\,\sqrt{3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{\mathrm{i}\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+\right.$$

$$\left.\left.(-1)^{1/3}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{\mathrm{i}\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\right)\Big/\left(7\,(-b)^{2/3}\sqrt{a+b\,x^3}\right)$$

Problem 193: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\sqrt{a+b\,x^3}\,\left(A+B\,x^3\right)}{x^5}\,dx$$

Optimal (type 4, 546 leaves, 5 steps):

$$-\frac{\left(A b+8 a B\right)\sqrt{a+b x^3}}{8 a x}+\frac{3 b^{1/3}\left(A b+8 a B\right)\sqrt{a+b x^3}}{8 a\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)}-\frac{A\left(a+b x^3\right)^{3/2}}{4 a x^4}-\left(3\times 3^{1/4}\sqrt{2-\sqrt{3}}\,b^{1/3}\left(A b+8 a B\right)\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/\left(16 a^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\sqrt{a+b x^3}\right)+\left(3^{3/4}b^{1/3}\left(A b+8 a B\right)\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/\left(4\sqrt{2}\,a^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\sqrt{a+b x^3}\right)$$

Result (type 4, 249 leaves):

$$\left(-\frac{A}{4 x^4}+\frac{-3 A b-8 a B}{8 a x}\right)\sqrt{a+b x^3}+\left((-1)^{1/6}\,3^{3/4}\,b\left(A b+8 a B\right)\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}x}{a^{1/3}}\right)}\sqrt{1+\frac{(-b)^{1/3}x}{a^{1/3}}+\frac{(-b)^{2/3}x^2}{a^{2/3}}}\left(-i\sqrt{3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-b)^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+(-1)^{1/3}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-b)^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right)\Big/\left(8 a^{1/3}(-b)^{2/3}\sqrt{a+b x^3}\right)$$

Problem 194: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a+b x^3}\,\left(A+B x^3\right)}{x^8}\,dx$$

Optimal (type 4, 581 leaves, 6 steps):

$$\frac{(5 A b-14 a B)\sqrt{a+b x^3}}{56 a x^4}+\frac{3 b (5 A b-14 a B)\sqrt{a+b x^3}}{112 a^2 x}-\frac{3 b^{4/3}(5 A b-14 a B)\sqrt{a+b x^3}}{112 a^2\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3} x\right)}-\frac{A\left(a+b x^3\right)^{3/2}}{7 a x^7}+\left(3\times 3^{1/4}\sqrt{2-\sqrt{3}}\, b^{4/3}(5 A b-14 a B)\left(a^{1/3}+b^{1/3} x\right)\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3} x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3} x}\right],-7-4\sqrt{3}\right]\right)\Big/\left(224 a^{5/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3} x\right)^2}}\sqrt{a+b x^3}\right)-\left(3^{3/4} b^{4/3}(5 A b-14 a B)\left(a^{1/3}+b^{1/3} x\right)\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3} x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3} x}\right],-7-4\sqrt{3}\right]\right)\Big/\left(56\sqrt{2}\, a^{5/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3} x\right)^2}}\sqrt{a+b x^3}\right)$$

Result (type 4, 272 leaves):

$$\left(-\frac{A}{7 x^7}+\frac{-3 A b-14 a B}{56 a x^4}-\frac{3 b (-5 A b+14 a B)}{112 a^2 x}\right) \sqrt{a+b x^3}+$$

$$\left((-1)^{1/6} 3^{3/4} b^2 (-5 A b+14 a B) \sqrt{(-1)^{5/6} \left(-1+\frac{(-b)^{1/3} x}{a^{1/3}}\right)} \sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}}\right.$$

$$\left(-i \sqrt{3}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right]+(-1)^{1/3}\right.$$

$$\left.\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right]\right)\right) \Big/ \left(112 a^{4/3} (-b)^{2/3} \sqrt{a+b x^3}\right)$$

Problem 195: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a+b x^3}\, (A+B x^3)}{x^{11}} \, dx$$

Optimal (type 4, 614 leaves, 7 steps):

$$\frac{(11 A b-20 a B)\sqrt{a+b x^3}}{140 a x^7}+\frac{3 b (11 A b-20 a B)\sqrt{a+b x^3}}{1120 a^2 x^4}-\frac{3 b^2 (11 A b-20 a B)\sqrt{a+b x^3}}{448 a^3 x}+\frac{3 b^{7/3} (11 A b-20 a B)\sqrt{a+b x^3}}{448 a^3 \left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}-\frac{A \left(a+b x^3\right)^{3/2}}{10 a x^{10}}-\left(3\times 3^{1/4}\sqrt{2-\sqrt{3}}\, b^{7/3} (11 A b-20 a B) \left(a^{1/3}+b^{1/3} x\right)\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4\sqrt{3}\right]\right)\Big/\left(896 a^{8/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\sqrt{a+b x^3}\right)+\left(3^{3/4} b^{7/3} (11 A b-20 a B)\left(a^{1/3}+b^{1/3} x\right)\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4\sqrt{3}\right]\right)\Big/\left(224\sqrt{2}\, a^{8/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\sqrt{a+b x^3}\right)$$

Result (type 4, 284 leaves) :

$$-\frac{1}{2240 a^3 x^{10}}\sqrt{a+b x^3}\left(224 a^3 A+16 a^2 (3 A b+20 a B) x^3+6 a b (-11 A b+20 a B) x^6+15 b^2 (11 A b-20 a B) x^9\right)+\left((-1)^{2/3} 3^{3/4} (-b)^{7/3} (11 A b-20 a B)\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3} x}{a^{1/3}}\right)}\sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}}\left(\sqrt{3}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+(-1)^{5/6}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right)\Big/\left(448 a^{7/3}\sqrt{a+b x^3}\right)$$

Problem 202: Result unnecessarily involves imaginary or complex numbers.

$$\int x^3 \left(a+b x^3\right)^{3/2} \left(A+B x^3\right) dx$$

Optimal (type 4, 336 leaves, 5 steps):

$$\frac{54 a^2 (23 A b-8 a B) x \sqrt{a+b x^3}}{21505 b^2}+\frac{18 a (23 A b-8 a B) x^4 \sqrt{a+b x^3}}{4301 b}+\frac{2 (23 A b-8 a B) x^4 \left(a+b x^3\right)^{3/2}}{391 b}+\frac{2 B x^4 \left(a+b x^3\right)^{5/2}}{23 b}-\left(36\times 3^{3/4} \sqrt{2+\sqrt{3}}\, a^3 (23 A b-8 a B) \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/\left(21505 b^{7/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 229 leaves):

$$\sqrt{a+b x^3} \left(-\frac{54 a^2 (-23 A b+8 a B) x}{21505 b^2}+\frac{2 a (460 A b+27 a B) x^4}{4301 b}+\frac{2}{391} (23 A b+26 a B) x^7+\frac{2}{23} b B x^{10}\right)-\left(36 i\, 3^{3/4} a^{10/3} (23 A b-8 a B) \sqrt{(-1)^{5/6} \left(-1+\frac{(-b)^{1/3} x}{a^{1/3}}\right)} \sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\Big/\left(21505 (-b)^{1/3} b^2 \sqrt{a+b x^3}\right)$$

Problem 203: Result unnecessarily involves imaginary or complex numbers.

$$\int \left(a+b x^3\right)^{3/2} \left(A+B x^3\right) dx$$

Optimal (type 4, 299 leaves, 4 steps):

$$\frac{18\, a\, (17 A b - 2 a B)\, x\, \sqrt{a + b x^3}}{935\, b} + \frac{2\, (17 A b - 2 a B)\, x\, \left(a + b x^3\right)^{3/2}}{187\, b} + \frac{2 B x \left(a + b x^3\right)^{5/2}}{17\, b} + \left(18 \times 3^{3/4} \sqrt{2 + \sqrt{3}}\; a^2\, (17 A b - 2 a B)\, \left(a^{1/3} + b^{1/3} x\right) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)^2}}\; \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1 - \sqrt{3}\right) a^{1/3} + b^{1/3} x}{\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x}\right], -7 - 4\sqrt{3}\right]\right) \Big/ \left(935\, b^{4/3} \sqrt{\frac{a^{1/3} \left(a^{1/3} + b^{1/3} x\right)}{\left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)^2}}\; \sqrt{a + b x^3}\right)$$

Result (type 4, 202 leaves):

$$-\left(\left(2 \left((-b)^{1/3} \left(a + b x^3\right) \left(a\, (238 A b + 27 a B)\, x + 5 b\, (17 A b + 20 a B)\, x^4 + 55 b^2 B x^7\right) + 9\, i\, 3^{3/4} a^{7/3}\, (17 A b - 2 a B) \sqrt{(-1)^{5/6} \left(-1 + \frac{(-b)^{1/3} x}{a^{1/3}}\right)} \sqrt{1 + \frac{(-b)^{1/3} x}{a^{1/3}} + \frac{(-b)^{2/3} x^2}{a^{2/3}}}\; \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right]\right)\right) \Big/ \left(935\, (-b)^{4/3} \sqrt{a + b x^3}\right)\right)$$

Problem 204: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a + b x^3\right)^{3/2} \left(A + B x^3\right)}{x^3}\, dx$$

Optimal (type 4, 295 leaves, 4 steps):

$$\frac{9}{110}\left(11\,A\,b+4\,a\,B\right)\,x\,\sqrt{a+b\,x^3}+\frac{\left(11\,A\,b+4\,a\,B\right)\,x\,\left(a+b\,x^3\right)^{3/2}}{22\,a}-\frac{A\,\left(a+b\,x^3\right)^{5/2}}{2\,a\,x^2}+\left(9\times 3^{3/4}\,\sqrt{2+\sqrt{3}}\,a\,\left(11\,A\,b+4\,a\,B\right)\,\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Bigg/\left(110\,b^{1/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 193 leaves):

$$\sqrt{a+b\,x^3}\,\left(-\frac{a\,A}{2\,x^2}+\frac{2}{55}\left(11\,A\,b+14\,a\,B\right)\,x+\frac{2}{11}\,b\,B\,x^4\right)+\left(9\,\mathrm{i}\,3^{3/4}\,a^{4/3}\,\left(11\,A\,b+4\,a\,B\right)\,\sqrt{\left(-1\right)^{5/6}\,\left(-1+\frac{\left(-b\right)^{1/3}\,x}{a^{1/3}}\right)}\,\sqrt{1+\frac{\left(-b\right)^{1/3}\,x}{a^{1/3}}+\frac{\left(-b\right)^{2/3}\,x^2}{a^{2/3}}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-\left(-1\right)^{5/6}-\frac{\mathrm{i}\,\left(-b\right)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,\left(-1\right)^{1/3}\right]\right)\Bigg/\left(110\,\left(-b\right)^{1/3}\,\sqrt{a+b\,x^3}\right)$$

Problem 205: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(a+b\,x^3\right)^{3/2}\,\left(A+B\,x^3\right)}{x^6}\,dx$$

Optimal (type 4, 297 leaves, 4 steps):

$$\frac{9 b (A b + 2 a B) x \sqrt{a + b x^3}}{20 a} - \frac{(A b + 2 a B) (a + b x^3)^{3/2}}{4 a x^2} - \frac{A (a + b x^3)^{5/2}}{5 a x^5} + \left(9 \times 3^{3/4} \sqrt{2 + \sqrt{3}}\, b^{2/3} (A b + 2 a B) (a^{1/3} + b^{1/3} x) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)^2}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1 - \sqrt{3}\right) a^{1/3} + b^{1/3} x}{\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x}\right], -7 - 4 \sqrt{3}\right]\right) \Big/ \left(20 \sqrt{\frac{a^{1/3} (a^{1/3} + b^{1/3} x)}{\left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)^2}} \sqrt{a + b x^3}\right)$$

Result (type 4, 193 leaves):

$$\left(-\frac{a A}{5 x^5} + \frac{-13 A b - 10 a B}{20 x^2} + \frac{2 b B x}{5}\right) \sqrt{a + b x^3} + \left(9 i\, 3^{3/4} a^{1/3} b (A b + 2 a B) \sqrt{(-1)^{5/6} \left(-1 + \frac{(-b)^{1/3} x}{a^{1/3}}\right)} \sqrt{1 + \frac{(-b)^{1/3} x}{a^{1/3}} + \frac{(-b)^{2/3} x^2}{a^{2/3}}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right]\right) \Big/ \left(20 (-b)^{1/3} \sqrt{a + b x^3}\right)$$

Problem 206: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a + b x^3)^{3/2} (A + B x^3)}{x^9} \, dx$$

Optimal (type 4, 302 leaves, 4 steps):

$$\frac{9 b (A b-16 a B) \sqrt{a+b x^3}}{320 a x^2}+\frac{(A b-16 a B) \left(a+b x^3\right)^{3/2}}{80 a x^5}-$$
$$\frac{A \left(a+b x^3\right)^{5/2}}{8 a x^8}-\left(9\times 3^{3/4} \sqrt{2+\sqrt{3}}\, b^{5/3} (A b-16 a B) \left(a^{1/3}+b^{1/3} x\right)\right.$$
$$\left.\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Bigg/$$
$$\left(320 a \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 209 leaves):

$$\left(-\frac{a A}{8 x^8}+\frac{-19 A b-16 a B}{80 x^5}-\frac{b (27 A b+208 a B)}{320 a x^2}\right) \sqrt{a+b x^3}+$$
$$\left(9 i\, 3^{3/4} b^2 (-A b+16 a B) \sqrt{(-1)^{5/6} \left(-1+\frac{(-b)^{1/3} x}{a^{1/3}}\right)} \sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}}\right.$$
$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\Bigg/\left(320 a^{2/3} (-b)^{1/3} \sqrt{a+b x^3}\right)$$

Problem 207: Result unnecessarily involves imaginary or complex numbers.

$$\int x^4 \left(a+b x^3\right)^{3/2} \left(A+B x^3\right) dx$$

Optimal (type 4, 614 leaves, 7 steps):

$$\frac{54 a^2 \left(5 A b-2 a B\right) x^2 \sqrt{a+b x^3}}{8645 b^2}+\frac{18 a \left(5 A b-2 a B\right) x^5 \sqrt{a+b x^3}}{1235 b}-\frac{216 a^3 \left(5 A b-2 a B\right) \sqrt{a+b x^3}}{8645 b^{8/3} \left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}+\frac{2 \left(5 A b-2 a B\right) x^5 \left(a+b x^3\right)^{3/2}}{95 b}+\frac{2 B x^5 \left(a+b x^3\right)^{5/2}}{25 b}+\left(108\times 3^{1/4} \sqrt{2-\sqrt{3}}\ a^{10/3} \left(5 A b-2 a B\right) \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Bigg/\left(8645 b^{8/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)-\left(72 \sqrt{2}\ 3^{3/4} a^{10/3} \left(5 A b-2 a B\right) \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Bigg/\left(8645 b^{8/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 283 leaves) :

$$\frac{1}{43225\,(-b)^{8/3}\sqrt{a+b x^3}}\,2\left((-b)^{2/3}\left(a+b x^3\right)\left(135 a^2 (5 A b-2 a B) x^2+7 a b (550 A b+27 a B) x^5+91 b^2 (25 A b+28 a B) x^8+1729 b^3 B x^{11}\right)+180 (-1)^{2/3} 3^{3/4} a^{11/3} (5 A b-2 a B)\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3} x}{a^{1/3}}\right)}\sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}}\left(\sqrt{3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+(-1)^{5/6}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right)$$

Problem 208: Result unnecessarily involves imaginary or complex numbers.

$$\int x \left(a+b x^3\right)^{3/2} \left(A+B x^3\right) dx$$

Optimal (type 4, 581 leaves, 6 steps):

$$\frac{18 a (19 A b-4 a B) x^2 \sqrt{a+b x^3}}{1729 b}+\frac{54 a^2 (19 A b-4 a B) \sqrt{a+b x^3}}{1729 b^{5/3} \left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}+\frac{2 (19 A b-4 a B) x^2 \left(a+b x^3\right)^{3/2}}{247 b}+$$

$$\frac{2 B x^2 \left(a+b x^3\right)^{5/2}}{19 b}-\left(27\times 3^{1/4} \sqrt{2-\sqrt{3}}\ a^{7/3} (19 A b-4 a B) \left(a^{1/3}+b^{1/3} x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$

$$\left(1729 b^{5/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)+$$

$$\left(18 \sqrt{2}\ 3^{3/4} a^{7/3} (19 A b-4 a B) \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$

$$\left(1729 b^{5/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 262 leaves) :

$$-\frac{1}{1729\,(-b)^{5/3}\sqrt{a+b\,x^3}}\left(2\left((-b)^{2/3}\left(a+b\,x^3\right)\left(a\,(304\,A\,b+27\,a\,B)\,x^2+7\,b\,(19\,A\,b+22\,a\,B)\,x^5+91\,b^2\,B\,x^8\right)-\right.\right.$$

$$9\,(-1)^{2/3}\,3^{3/4}\,a^{8/3}\,(19\,A\,b-4\,a\,B)\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}$$

$$\left(\sqrt{3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+\right.$$

$$\left.\left.\left.(-1)^{5/6}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\right)\right)$$

Problem 209: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(a+b\,x^3\right)^{3/2}\left(A+B\,x^3\right)}{x^2}\,dx$$

Optimal (type 4, 573 leaves, 6 steps):

$$\frac{9}{91}\left(13\,A\,b+2\,a\,B\right)x^2\sqrt{a+b\,x^3}+\frac{27\,a\left(13\,A\,b+2\,a\,B\right)\sqrt{a+b\,x^3}}{91\,b^{2/3}\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)}+\frac{\left(13\,A\,b+2\,a\,B\right)x^2\left(a+b\,x^3\right)^{3/2}}{13\,a}-$$

$$\frac{A\left(a+b\,x^3\right)^{5/2}}{a\,x}-\left(27\times3^{1/4}\sqrt{2-\sqrt{3}}\;a^{4/3}\left(13\,A\,b+2\,a\,B\right)\left(a^{1/3}+b^{1/3}\,x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],-7-4\sqrt{3}\right]\right)\Bigg/$$

$$\left(182\,b^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\sqrt{a+b\,x^3}\right)+$$

$$\left(9\sqrt{2}\;3^{3/4}\,a^{4/3}\left(13\,A\,b+2\,a\,B\right)\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],-7-4\sqrt{3}\right]\right)\Bigg/$$

$$\left(91\,b^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\sqrt{a+b\,x^3}\right)$$

Result (type 4, 254 leaves):

$$\sqrt{a+b\,x^3}\left(-\frac{a\,A}{x}+\frac{2}{91}\left(13\,A\,b+16\,a\,B\right)x^2+\frac{2}{13}\,b\,B\,x^5\right)+$$

$$\left(9\left(-1\right)^{1/6}3^{3/4}\,a^{5/3}\left(13\,A\,b+2\,a\,B\right)\sqrt{\left(-1\right)^{5/6}\left(-1+\frac{\left(-b\right)^{1/3}x}{a^{1/3}}\right)}\sqrt{1+\frac{\left(-b\right)^{1/3}x}{a^{1/3}}+\frac{\left(-b\right)^{2/3}x^2}{a^{2/3}}}\right.$$

$$\left(-i\sqrt{3}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-\left(-1\right)^{5/6}-\frac{i\,\left(-b\right)^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],\left(-1\right)^{1/3}\right]+\right.$$

$$\left.\left.\left(-1\right)^{1/3}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-\left(-1\right)^{5/6}-\frac{i\,\left(-b\right)^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],\left(-1\right)^{1/3}\right]\right)\right)\Bigg/\left(91\left(-b\right)^{2/3}\sqrt{a+b\,x^3}\right)$$

Problem 210: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+b x^3\right)^{3/2}\left(A+B x^3\right)}{x^5}\,dx$$

Optimal (type 4, 578 leaves, 6 steps):

$$\frac{9 b\,(7 A b+8 a B)\,x^2\sqrt{a+b x^3}}{56 a}+\frac{27 b^{1/3}\,(7 A b+8 a B)\sqrt{a+b x^3}}{56\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)}-\frac{(7 A b+8 a B)\left(a+b x^3\right)^{3/2}}{8 a x}-\frac{A\left(a+b x^3\right)^{5/2}}{4 a x^4}-$$
$$\left(27\times 3^{1/4}\sqrt{2-\sqrt{3}}\;a^{1/3}b^{1/3}\,(7 A b+8 a B)\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\right.$$
$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$
$$\left(112\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\sqrt{a+b x^3}\right)+\left(9\times 3^{3/4}a^{1/3}b^{1/3}\,(7 A b+8 a B)\left(a^{1/3}+b^{1/3}x\right)\right.$$
$$\left.\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$
$$\left(28\sqrt{2}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\sqrt{a+b x^3}\right)$$

Result (type 4, 254 leaves):

$$-\frac{\sqrt{a+b x^3}\,\left(b x^3\left(77 A-16 B x^3\right)+14 a\left(A+4 B x^3\right)\right)}{56 x^4}-\frac{1}{56\sqrt{a+b x^3}}9\,(-1)^{1/6}\,3^{3/4}\,a^{2/3}\,(-b)^{1/3}\,(7 A b+8 a B)\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}x}{a^{1/3}}\right)}$$

$$\sqrt{1+\frac{(-b)^{1/3}x}{a^{1/3}}+\frac{(-b)^{2/3}x^2}{a^{2/3}}}\left(-i\sqrt{3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-b)^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+\right.$$

$$\left.(-1)^{1/3}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-b)^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

Problem 211: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(a+b x^3\right)^{3/2}\left(A+B x^3\right)}{x^8}\,dx$$

Optimal (type 4, 576 leaves, 6 steps):

$$-\frac{9 b\,(A b+14 a B)\sqrt{a+b x^3}}{112 a x}+\frac{27 b^{4/3}\,(A b+14 a B)\sqrt{a+b x^3}}{112 a\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)}-\frac{(A b+14 a B)\left(a+b x^3\right)^{3/2}}{56 a x^4}-$$

$$\frac{A\left(a+b x^3\right)^{5/2}}{7 a x^7}-\left(27\times 3^{1/4}\sqrt{2-\sqrt{3}}\;b^{4/3}\,(A b+14 a B)\left(a^{1/3}+b^{1/3}x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(224 a^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\;\sqrt{a+b x^3}\right)+\left(9\times 3^{3/4}\,b^{4/3}\,(A b+14 a B)\left(a^{1/3}+b^{1/3}x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(56\sqrt{2}\;a^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\;\sqrt{a+b x^3}\right)$$

Result (type 4, 269 leaves):

$$\left(-\frac{a A}{7 x^7}+\frac{-17 A b-14 a B}{56 x^4}-\frac{b (27 A b+154 a B)}{112 a x}\right) \sqrt{a+b x^3}+$$

$$\left(9 (-1)^{1/6} 3^{3/4} b^2 (A b+14 a B) \sqrt{(-1)^{5/6} \left(-1+\frac{(-b)^{1/3} x}{a^{1/3}}\right)} \sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}}\right.$$

$$\left(-i \sqrt{3}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right]+(-1)^{1/3}\right.$$

$$\left.\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right]\right)\right) \Big/ \left(112 a^{1/3} (-b)^{2/3} \sqrt{a+b x^3}\right)$$

Problem 212: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a+b x^3)^{3/2} (A+B x^3)}{x^{11}} \, dx$$

Optimal (type 4, 608 leaves, 7 steps):

$$\frac{9 b (A b-4 a B) \sqrt{a+b x^3}}{224 a x^4}+\frac{27 b^2 (A b-4 a B) \sqrt{a+b x^3}}{448 a^2 x}-$$

$$\frac{27 b^{7/3} (A b-4 a B) \sqrt{a+b x^3}}{448 a^2 \left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}+\frac{(A b-4 a B) \left(a+b x^3\right)^{3/2}}{28 a x^7}-\frac{A \left(a+b x^3\right)^{5/2}}{10 a x^{10}}+$$

$$\left(27\times 3^{1/4} \sqrt{2-\sqrt{3}}\; b^{7/3} (A b-4 a B) \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$

$$\left(896 a^{5/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)-\left(9\times 3^{3/4} b^{7/3} (A b-4 a B) \left(a^{1/3}+b^{1/3} x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\; \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$

$$\left(224 \sqrt{2}\; a^{5/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 282 leaves):

$$-\frac{1}{2240 a^2 x^{10}}$$

$$\sqrt{a+b x^3} \left(224 a^3 A+16 a^2 (23 A b+20 a B) x^3+2 a b (27 A b+340 a B) x^6-135 b^2 (A b-4 a B) x^9\right)-$$

$$\left(9 (-1)^{2/3} 3^{3/4} (-b)^{7/3} (A b-4 a B) \sqrt{(-1)^{5/6} \left(-1+\frac{(-b)^{1/3} x}{a^{1/3}}\right)} \sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}}\right.$$

$$\left(\sqrt{3}\; \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+\right.$$

$$\left.\left.(-1)^{5/6}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right)\Big/\left(448 a^{4/3} \sqrt{a+b x^3}\right)$$

Problem 219: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^3 \left(A + B x^3\right)}{\sqrt{a + b x^3}} \, dx$$

Optimal (type 4, 270 leaves, 3 steps):

$$\frac{2 \left(11 A b - 8 a B\right) x \sqrt{a + b x^3}}{55 b^2} + \frac{2 B x^4 \sqrt{a + b x^3}}{11 b} - \left(4 \sqrt{2 + \sqrt{3}}\, a \left(11 A b - 8 a B\right) \left(a^{1/3} + b^{1/3} x\right) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1 - \sqrt{3}\right) a^{1/3} + b^{1/3} x}{\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x}\right], -7 - 4 \sqrt{3}\right]\right) \Bigg/ \left(55 \times 3^{1/4} b^{7/3} \sqrt{\frac{a^{1/3} \left(a^{1/3} + b^{1/3} x\right)}{\left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)^2}} \sqrt{a + b x^3}\right)$$

Result (type 4, 189 leaves):

$$\left(6 \left(-b\right)^{1/3} x \left(a + b x^3\right) \left(11 A b - 8 a B + 5 b B x^3\right) - 4 i\, 3^{3/4} a^{4/3} \left(11 A b - 8 a B\right) \sqrt{\frac{\left(-1\right)^{5/6} \left(-a^{1/3} + \left(-b\right)^{1/3} x\right)}{a^{1/3}}} \sqrt{1 + \frac{\left(-b\right)^{1/3} x}{a^{1/3}} + \frac{\left(-b\right)^{2/3} x^2}{a^{2/3}}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-\left(-1\right)^{5/6} - \frac{i \left(-b\right)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], \left(-1\right)^{1/3}\right]\right) \Bigg/ \left(165 \left(-b\right)^{7/3} \sqrt{a + b x^3}\right)$$

Problem 220: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{A + B x^3}{\sqrt{a + b x^3}} \, dx$$

Optimal (type 4, 239 leaves, 2 steps):

$$\frac{2 B x \sqrt{a+b x^3}}{5 b}+\left(2 \sqrt{2+\sqrt{3}}\, (5 A b-2 a B) \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/\left(5\times 3^{1/4} b^{4/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 168 leaves) :

$$\frac{2 B x \sqrt{a+b x^3}}{5 b}-\left(2 i a^{1/3} (5 A b-2 a B) \sqrt{(-1)^{5/6} \left(-1+\frac{(-b)^{1/3} x}{a^{1/3}}\right)} \sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\Big/\left(5\times 3^{1/4} (-b)^{4/3} \sqrt{a+b x^3}\right)$$

Problem 221: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{A+B x^3}{x^3 \sqrt{a+b x^3}}\, dx$$

Optimal (type 4, 243 leaves, 2 steps) :

$$-\frac{A \sqrt{a+b x^3}}{2 a x^2}-\left(\sqrt{2+\sqrt{3}}\, (A b-4 a B) \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/\left(2\times 3^{1/4} a\, b^{1/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 170 leaves) :

$$-\frac{A\sqrt{a+b x^3}}{2 a x^2}+\left(i\left(-A b+4 a B\right)\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3} x}{a^{1/3}}\right)}\sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}}\right.$$
$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\Big/\left(2\times 3^{1/4} a^{2/3}(-b)^{1/3}\sqrt{a+b x^3}\right)$$

Problem 222: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{A+B x^3}{x^6\sqrt{a+b x^3}}\,dx$$

Optimal (type 4, 274 leaves, 3 steps):

$$-\frac{A\sqrt{a+b x^3}}{5 a x^5}+\frac{\left(7 A b-10 a B\right)\sqrt{a+b x^3}}{20 a^2 x^2}+$$
$$\left(\sqrt{2+\sqrt{3}}\; b^{2/3}\left(7 A b-10 a B\right)\left(a^{1/3}+b^{1/3} x\right)\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\right.$$
$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4\sqrt{3}\right]\right)\Big/$$
$$\left(20\times 3^{1/4} a^2\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\sqrt{a+b x^3}\right)$$

Result (type 4, 188 leaves):

$$-\frac{\sqrt{a+b x^3}\left(4 a A-7 A b x^3+10 a B x^3\right)}{20 a^2 x^5}+$$
$$\left(i(-b)^{2/3}\left(-7 A b+10 a B\right)\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3} x}{a^{1/3}}\right)}\sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}}\right.$$
$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\Big/\left(20\times 3^{1/4} a^{5/3}\sqrt{a+b x^3}\right)$$

Problem 223: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4 \left(A + B x^3\right)}{\sqrt{a + b x^3}} \, dx$$

Optimal (type 4, 548 leaves, 5 steps):

$$\frac{2 \left(13 A b - 10 a B\right) x^2 \sqrt{a + b x^3}}{91 b^2} + \frac{2 B x^5 \sqrt{a + b x^3}}{13 b} - \frac{8 a \left(13 A b - 10 a B\right) \sqrt{a + b x^3}}{91 b^{8/3} \left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)} + \left(4 \times 3^{1/4} \sqrt{2 - \sqrt{3}}\ a^{4/3} \left(13 A b - 10 a B\right) \left(a^{1/3} + b^{1/3} x\right) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)^2}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1 - \sqrt{3}\right) a^{1/3} + b^{1/3} x}{\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x}\right], -7 - 4 \sqrt{3}\right]\right) \Bigg/ \left(91 b^{8/3} \sqrt{\frac{a^{1/3} \left(a^{1/3} + b^{1/3} x\right)}{\left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)^2}} \sqrt{a + b x^3}\right) - \left(8 \sqrt{2}\ a^{4/3} \left(13 A b - 10 a B\right) \left(a^{1/3} + b^{1/3} x\right) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1 - \sqrt{3}\right) a^{1/3} + b^{1/3} x}{\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x}\right], -7 - 4 \sqrt{3}\right]\right) \Bigg/ \left(91 \times 3^{1/4} b^{8/3} \sqrt{\frac{a^{1/3} \left(a^{1/3} + b^{1/3} x\right)}{\left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)^2}} \sqrt{a + b x^3}\right)$$

Result (type 4, 243 leaves):

$$\left(2\left(3\,(-b)^{2/3}\,x^2\,\left(a+b\,x^3\right)\,\left(13\,A\,b-10\,a\,B+7\,b\,B\,x^3\right)+\right.\right.$$

$$4\,(-1)^{2/3}\,3^{3/4}\,a^{5/3}\,(13\,A\,b-10\,a\,B)\,\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}$$

$$\left(\sqrt{3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+(-1)^{5/6}\right.$$

$$\left.\left.\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\right)\right)\Big/\left(273\,(-b)^{8/3}\,\sqrt{a+b\,x^3}\right)$$

Problem 224: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x\,\left(A+B\,x^3\right)}{\sqrt{a+b\,x^3}}\,dx$$

Optimal (type 4, 517 leaves, 4 steps):

$$\frac{2 B x^2 \sqrt{a+b x^3}}{7 b}+\frac{2 (7 A b-4 a B) \sqrt{a+b x^3}}{7 b^{5/3} \left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}-$$

$$\left(3^{1/4} \sqrt{2-\sqrt{3}}\, a^{1/3} (7 A b-4 a B) \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$

$$\left(7 b^{5/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)+\left(2 \sqrt{2}\, a^{1/3} (7 A b-4 a B) \left(a^{1/3}+b^{1/3} x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$

$$\left(7\times 3^{1/4} b^{5/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 231 leaves):

$$\frac{2 B x^2 \sqrt{a+b x^3}}{7 b}-$$

$$\left(2 (-1)^{1/6} a^{2/3} (7 A b-4 a B) \sqrt{(-1)^{5/6} \left(-1+\frac{(-b)^{1/3} x}{a^{1/3}}\right)} \sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}}\right.$$

$$\left(-i \sqrt{3}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+(-1)^{1/3}\right.$$

$$\left.\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right)\Big/\left(7\times 3^{1/4} (-b)^{5/3} \sqrt{a+b x^3}\right)$$

Problem 225: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{A + B x^3}{x^2 \sqrt{a + b x^3}} \, dx$$

Optimal (type 4, 509 leaves, 4 steps):

$$-\frac{A \sqrt{a + b x^3}}{a x} + \frac{(A b + 2 a B) \sqrt{a + b x^3}}{a b^{2/3} \left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)} - \left(3^{1/4} \sqrt{2 - \sqrt{3}} \, (A b + 2 a B) \left(a^{1/3} + b^{1/3} x\right) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)^2}} \; \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1 - \sqrt{3}\right) a^{1/3} + b^{1/3} x}{\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x}\right], -7 - 4 \sqrt{3}\right]\right) \Big/ \left(2 a^{2/3} b^{2/3} \sqrt{\frac{a^{1/3} \left(a^{1/3} + b^{1/3} x\right)}{\left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)^2}} \sqrt{a + b x^3}\right) + \left(\sqrt{2} \, (A b + 2 a B) \left(a^{1/3} + b^{1/3} x\right) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)^2}} \; \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1 - \sqrt{3}\right) a^{1/3} + b^{1/3} x}{\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x}\right], -7 - 4 \sqrt{3}\right]\right) \Big/ \left(3^{1/4} a^{2/3} b^{2/3} \sqrt{\frac{a^{1/3} \left(a^{1/3} + b^{1/3} x\right)}{\left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)^2}} \sqrt{a + b x^3}\right)$$

Result (type 4, 225 leaves):

$$-\frac{A \sqrt{a + b x^3}}{a x} + \left((-1)^{1/6} (A b + 2 a B) \sqrt{(-1)^{5/6} \left(-1 + \frac{(-b)^{1/3} x}{a^{1/3}}\right)} \sqrt{1 + \frac{(-b)^{1/3} x}{a^{1/3}} + \frac{(-b)^{2/3} x^2}{a^{2/3}}} \left(-i \sqrt{3} \; \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right] + (-1)^{1/3} \; \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right]\right)\right) \Big/ \left(3^{1/4} a^{1/3} (-b)^{2/3} \sqrt{a + b x^3}\right)$$

Problem 226: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{A + B x^3}{x^5 \sqrt{a + b x^3}} \, dx$$

Optimal (type 4, 550 leaves, 5 steps):

$$-\frac{A \sqrt{a + b x^3}}{4 a x^4} + \frac{(5 A b - 8 a B) \sqrt{a + b x^3}}{8 a^2 x} - \frac{b^{1/3} (5 A b - 8 a B) \sqrt{a + b x^3}}{8 a^2 \left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)} + \left(3^{1/4} \sqrt{2 - \sqrt{3}}\, b^{1/3} (5 A b - 8 a B) \left(a^{1/3} + b^{1/3} x\right) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)^2}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1 - \sqrt{3}\right) a^{1/3} + b^{1/3} x}{\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x}\right], -7 - 4 \sqrt{3}\right]\right) \Bigg/ \left(16 a^{5/3} \sqrt{\frac{a^{1/3} \left(a^{1/3} + b^{1/3} x\right)}{\left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)^2}} \sqrt{a + b x^3}\right) - \left(b^{1/3} (5 A b - 8 a B) \left(a^{1/3} + b^{1/3} x\right) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1 - \sqrt{3}\right) a^{1/3} + b^{1/3} x}{\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x}\right], -7 - 4 \sqrt{3}\right]\right) \Bigg/ \left(4 \sqrt{2}\, 3^{1/4} a^{5/3} \sqrt{\frac{a^{1/3} \left(a^{1/3} + b^{1/3} x\right)}{\left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)^2}} \sqrt{a + b x^3}\right)$$

Result (type 4, 249 leaves):

$$\frac{\sqrt{a+b x^3}\,\left(5 A b x^3-2 a\left(A+4 B x^3\right)\right)}{8 a^2 x^4}-$$

$$\left((-1)^{1/6}(-b)^{1/3}(-5 A b+8 a B)\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3} x}{a^{1/3}}\right)}\sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}}\right.$$

$$\left(-i\sqrt{3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+\right.$$

$$\left.\left.(-1)^{1/3}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right)\Big/\left(8\times 3^{1/4} a^{4/3}\sqrt{a+b x^3}\right)$$

Problem 227: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{A+B x^3}{x^8\sqrt{a+b x^3}}\,dx$$

Optimal (type 4, 581 leaves, 6 steps):

$$-\frac{A\sqrt{a+b x^3}}{7 a x^7}+\frac{(11 A b-14 a B)\sqrt{a+b x^3}}{56 a^2 x^4}-\frac{5 b (11 A b-14 a B)\sqrt{a+b x^3}}{112 a^3 x}+\frac{5 b^{4/3} (11 A b-14 a B)\sqrt{a+b x^3}}{112 a^3 \left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}-$$

$$\left(5\times 3^{1/4}\sqrt{2-\sqrt{3}}\, b^{4/3} (11 A b-14 a B) \left(a^{1/3}+b^{1/3} x\right)\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(224 a^{8/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\sqrt{a+b x^3}\right)+\left(5 b^{4/3} (11 A b-14 a B) \left(a^{1/3}+b^{1/3} x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(56\sqrt{2}\, 3^{1/4} a^{8/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\sqrt{a+b x^3}\right)$$

Result (type 4, 269 leaves):

$$\frac{1}{336 a^3\sqrt{a+b x^3}}\left(-\frac{1}{x^7}3 \left(a+b x^3\right) \left(16 a^2 A+2 a (-11 A b+14 a B) x^3+5 b (11 A b-14 a B) x^6\right)+\right.$$

$$5 (-1)^{1/6} 3^{3/4} a^{2/3} (-b)^{4/3} (11 A b-14 a B)\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3} x\right)}{a^{1/3}}}$$

$$\sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}}\left(-i\sqrt{3}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+\right.$$

$$\left.\left.(-1)^{1/3}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right)$$

Problem 234: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^6\,(A+B\,x^3)}{(a+b\,x^3)^{3/2}}\,dx$$

Optimal (type 4, 300 leaves, 4 steps):

$$-\frac{2\,(11\,A\,b-14\,a\,B)\,x^4}{33\,b^2\,\sqrt{a+b\,x^3}}+\frac{2\,B\,x^7}{11\,b\,\sqrt{a+b\,x^3}}+\frac{16\,(11\,A\,b-14\,a\,B)\,x\,\sqrt{a+b\,x^3}}{165\,b^3}-\left(32\,\sqrt{2+\sqrt{3}}\;a\,(11\,A\,b-14\,a\,B)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Bigg/\left(165\times 3^{1/4}\,b^{10/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\;\sqrt{a+b\,x^3}\right)$$

Result (type 4, 205 leaves):

$$\left(-6\,(-b)^{1/3}\,x\,\left(-112\,a^2\,B+3\,b^2\,x^3\,\left(11\,A+5\,B\,x^3\right)+a\,\left(88\,A\,b-42\,b\,B\,x^3\right)\right)+32\,i\,3^{3/4}\,a^{4/3}\,(11\,A\,b-14\,a\,B)\,\sqrt{\frac{(-1)^{5/6}\,\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\Bigg/\left(495\,(-b)^{10/3}\,\sqrt{a+b\,x^3}\right)$$

Problem 235: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^3\,(A+B\,x^3)}{(a+b\,x^3)^{3/2}}\,dx$$

Optimal (type 4, 269 leaves, 3 steps):

$$-\frac{2\,(5\,A\,b-8\,a\,B)\,x}{15\,b^2\,\sqrt{a+b\,x^3}}+\frac{2\,B\,x^4}{5\,b\,\sqrt{a+b\,x^3}}+\left(4\,\sqrt{2+\sqrt{3}}\,(5\,A\,b-8\,a\,B)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Bigg/\left(15\times 3^{1/4}\,b^{7/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 182 leaves):

$$\left(6\,(-b)^{1/3}\,x\,\left(-5\,A\,b+8\,a\,B+3\,b\,B\,x^3\right)+4\,i\,3^{3/4}\,a^{1/3}\,(5\,A\,b-8\,a\,B)\,\sqrt{\frac{(-1)^{5/6}\,\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\Bigg/\left(45\,(-b)^{7/3}\,\sqrt{a+b\,x^3}\right)$$

Problem 236: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{A+B\,x^3}{\left(a+b\,x^3\right)^{3/2}}\,dx$$

Optimal (type 4, 251 leaves, 2 steps):

$$\frac{2\,(A\,b-a\,B)\,x}{3\,a\,b\,\sqrt{a+b\,x^3}}+\left(2\,\sqrt{2+\sqrt{3}}\,(A\,b+2\,a\,B)\,(a^{1/3}+b^{1/3}\,x)\right.$$
$$\left.\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left((1+\sqrt{3})\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{(1-\sqrt{3})\,a^{1/3}+b^{1/3}\,x}{(1+\sqrt{3})\,a^{1/3}+b^{1/3}\,x}\right],-7-4\,\sqrt{3}\right]\right)\Big/$$
$$\left(3\times 3^{1/4}\,a\,b^{4/3}\,\sqrt{\frac{a^{1/3}\,(a^{1/3}+b^{1/3}\,x)}{\left((1+\sqrt{3})\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 176 leaves):

$$-\left(\left(6\,(-b)^{1/3}\,(A\,b-a\,B)\,x+\right.\right.$$
$$2\,\mathrm{i}\,3^{3/4}\,a^{1/3}\,(A\,b+2\,a\,B)\,\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}$$
$$\left.\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{\mathrm{i}\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\Big/\left(9\,a\,(-b)^{4/3}\,\sqrt{a+b\,x^3}\right)\right)$$

Problem 237: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{A+B\,x^3}{x^3\,(a+b\,x^3)^{3/2}}\,dx$$

Optimal (type 4, 272 leaves, 3 steps):

$$-\frac{A}{2\,a\,x^2\,\sqrt{a+b\,x^3}}-\frac{(7\,A\,b-4\,a\,B)\,x}{6\,a^2\,\sqrt{a+b\,x^3}}-\left(\sqrt{2+\sqrt{3}}\,(7\,A\,b-4\,a\,B)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/\left(6\times 3^{1/4}\,a^2\,b^{1/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 193 leaves):

$$\left(-3\,(-b)^{1/3}\,\left(3\,a\,A+7\,A\,b\,x^3-4\,a\,B\,x^3\right)-i\,3^{3/4}\,a^{1/3}\,(7\,A\,b-4\,a\,B)\,x^2\,\sqrt{\frac{(-1)^{5/6}\,\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\Big/\left(18\,a^2\,(-b)^{1/3}\,x^2\,\sqrt{a+b\,x^3}\right)$$

Problem 238: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{A+B\,x^3}{x^6\,\left(a+b\,x^3\right)^{3/2}}\,dx$$

Optimal (type 4, 304 leaves, 4 steps):

$$-\frac{A}{5\,a\,x^5\,\sqrt{a+b\,x^3}}-\frac{13\,A\,b-10\,a\,B}{15\,a^2\,x^2\,\sqrt{a+b\,x^3}}+\frac{7\,\left(13\,A\,b-10\,a\,B\right)\,\sqrt{a+b\,x^3}}{60\,a^3\,x^2}+$$

$$\left(7\,\sqrt{2+\sqrt{3}}\,\,b^{2/3}\,\left(13\,A\,b-10\,a\,B\right)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\,\right]\right)\Big/$$

$$\left(60\times 3^{1/4}\,a^3\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 218 leaves):

$$\sqrt{a+b\,x^3}\,\left(-\frac{A}{5\,a^2\,x^5}+\frac{17\,A\,b-10\,a\,B}{20\,a^3\,x^2}-\frac{2\,b\,\left(-A\,b+a\,B\right)\,x}{3\,a^3\,\left(a+b\,x^3\right)}\right)-$$

$$\left(7\,\mathrm{i}\,b\,\left(-13\,A\,b+10\,a\,B\right)\,\sqrt{\left(-1\right)^{5/6}\,\left(-1+\frac{\left(-b\right)^{1/3}\,x}{a^{1/3}}\right)}\,\,\sqrt{1+\frac{\left(-b\right)^{1/3}\,x}{a^{1/3}}+\frac{\left(-b\right)^{2/3}\,x^2}{a^{2/3}}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-\left(-1\right)^{5/6}-\frac{\mathrm{i}\,\left(-b\right)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,\left(-1\right)^{1/3}\right]\right)\Big/\left(60\times 3^{1/4}\,a^{8/3}\,\left(-b\right)^{1/3}\,\sqrt{a+b\,x^3}\right)$$

Problem 239: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^4\,\left(A+B\,x^3\right)}{\left(a+b\,x^3\right)^{3/2}}\,dx$$

Optimal (type 4, 547 leaves, 5 steps):

$$-\frac{2\,(7\,A\,b-10\,a\,B)\,x^2}{21\,b^2\,\sqrt{a+b\,x^3}}+\frac{2\,B\,x^5}{7\,b\,\sqrt{a+b\,x^3}}+\frac{8\,(7\,A\,b-10\,a\,B)\,\sqrt{a+b\,x^3}}{21\,b^{8/3}\,\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)}-$$

$$\left(4\,\sqrt{2-\sqrt{3}}\;a^{1/3}\,(7\,A\,b-10\,a\,B)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/$$

$$\left(7\times 3^{3/4}\,b^{8/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\;\sqrt{a+b\,x^3}\right)+$$

$$\left(8\,\sqrt{2}\;a^{1/3}\,(7\,A\,b-10\,a\,B)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/$$

$$\left(21\times 3^{1/4}\,b^{8/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\;\sqrt{a+b\,x^3}\right)$$

Result (type 4, 236 leaves) :

$$-\left(\left(2\,\left(-3\,(-b)^{2/3}\,x^2\,\left(-7\,A\,b+10\,a\,B+3\,b\,B\,x^3\right)+\right.\right.\right.$$

$$4\,(-1)^{2/3}\,3^{3/4}\,a^{2/3}\,(7\,A\,b-10\,a\,B)\,\sqrt{(-1)^{5/6}\,\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\;\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}$$

$$\left(\sqrt{3}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+(-1)^{5/6}\right.$$

$$\left.\left.\left.\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\right)\right)\Big/\left(63\,(-b)^{8/3}\,\sqrt{a+b\,x^3}\right)\right)$$

Problem 240: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x\,(A+B\,x^3)}{(a+b\,x^3)^{3/2}}\,dx$$

Optimal (type 4, 524 leaves, 4 steps):

$$\frac{2\,(A\,b-a\,B)\,x^2}{3\,a\,b\,\sqrt{a+b\,x^3}}-\frac{2\,(A\,b-4\,a\,B)\,\sqrt{a+b\,x^3}}{3\,a\,b^{5/3}\,\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)}+$$

$$\left(\sqrt{2-\sqrt{3}}\,(A\,b-4\,a\,B)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Bigg/$$

$$\left(3^{3/4}\,a^{2/3}\,b^{5/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)-$$

$$\left(2\,\sqrt{2}\,(A\,b-4\,a\,B)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Bigg/$$

$$\left(3\times 3^{1/4}\,a^{2/3}\,b^{5/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 235 leaves):

$$\frac{1}{9\,a\,b\,\sqrt{a+b\,x^3}}2\left(3\,(A\,b-a\,B)\,x^2+\frac{1}{(-b)^{5/3}}(-1)^{1/6}\,3^{3/4}\,a^{2/3}\,b\,(A\,b-4\,a\,B)\,\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}\right.$$

$$\left.\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\left(-i\,\sqrt{3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+\right.\right.$$

$$\left.\left.(-1)^{1/3}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\right)$$

Problem 241: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{A+B\,x^3}{x^2\,\left(a+b\,x^3\right)^{3/2}}\,dx$$

Optimal (type 4, 548 leaves, 5 steps):

$$-\frac{A}{a x \sqrt{a+b x^3}}-\frac{(5 A b-2 a B) x^2}{3 a^2 \sqrt{a+b x^3}}+\frac{(5 A b-2 a B) \sqrt{a+b x^3}}{3 a^2 b^{2/3} \left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}-$$

$$\left(\sqrt{2-\sqrt{3}}\, (5 A b-2 a B) \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right) \Bigg/$$

$$\left(2\times 3^{3/4} a^{5/3} b^{2/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)+$$

$$\left(\sqrt{2}\, (5 A b-2 a B) \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right) \Bigg/$$

$$\left(3\times 3^{1/4} a^{5/3} b^{2/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 243 leaves) :

$$\left(-3 (-b)^{2/3} \left(3 a A+5 A b x^3-2 a B x^3\right)-(-1)^{2/3} 3^{3/4} a^{2/3} (5 A b-2 a B) x \sqrt{(-1)^{5/6} \left(-1+\frac{(-b)^{1/3} x}{a^{1/3}}\right)}\right.$$

$$\sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}} \left(\sqrt{3}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+\right.$$

$$\left.\left.(-1)^{5/6} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right) \Bigg/ \left(9 a^2 (-b)^{2/3} x \sqrt{a+b x^3}\right)$$

Problem 242: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{A + B x^3}{x^5 \left(a + b x^3\right)^{3/2}} \, dx$$

Optimal (type 4, 580 leaves, 6 steps):

$$-\frac{A}{4 a x^4 \sqrt{a + b x^3}} - \frac{11 A b - 8 a B}{12 a^2 x \sqrt{a + b x^3}} + \frac{5 \left(11 A b - 8 a B\right) \sqrt{a + b x^3}}{24 a^3 x} - \frac{5 b^{1/3} \left(11 A b - 8 a B\right) \sqrt{a + b x^3}}{24 a^3 \left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)} + \left(5 \sqrt{2 - \sqrt{3}} \, b^{1/3} \left(11 A b - 8 a B\right) \left(a^{1/3} + b^{1/3} x\right) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)^2}} \, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1 - \sqrt{3}\right) a^{1/3} + b^{1/3} x}{\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x}\right], -7 - 4 \sqrt{3}\right]\right) \Big/ \left(16 \times 3^{3/4} a^{8/3} \sqrt{\frac{a^{1/3} \left(a^{1/3} + b^{1/3} x\right)}{\left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)^2}} \sqrt{a + b x^3}\right) - \left(5 b^{1/3} \left(11 A b - 8 a B\right) \left(a^{1/3} + b^{1/3} x\right) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)^2}} \, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1 - \sqrt{3}\right) a^{1/3} + b^{1/3} x}{\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x}\right], -7 - 4 \sqrt{3}\right]\right) \Big/ \left(12 \sqrt{2} \, 3^{1/4} a^{8/3} \sqrt{\frac{a^{1/3} \left(a^{1/3} + b^{1/3} x\right)}{\left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)^2}} \sqrt{a + b x^3}\right)$$

Result (type 4, 266 leaves):

$$\left(3\,(-b)^{2/3}\left(55\,A\,b^2\,x^6+a\,b\,x^3\left(33\,A-40\,B\,x^3\right)-6\,a^2\left(A+4\,B\,x^3\right)\right)+\right.$$

$$5\,(-1)^{2/3}\,3^{3/4}\,a^{2/3}\,b\,(11\,A\,b-8\,a\,B)\,x^4\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}$$

$$\left(\sqrt{3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+(-1)^{5/6}\right.$$

$$\left.\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right)\Big/\left(72\,a^3\,(-b)^{2/3}\,x^4\sqrt{a+b\,x^3}\right)$$

Problem 243: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{A+B\,x^3}{x^8\,\left(a+b\,x^3\right)^{3/2}}\,dx$$

Optimal (type 4, 611 leaves, 7 steps):

$$-\frac{A}{7\,a\,x^7\,\sqrt{a+b\,x^3}}-\frac{17\,A\,b-14\,a\,B}{21\,a^2\,x^4\,\sqrt{a+b\,x^3}}+\frac{11\,\left(17\,A\,b-14\,a\,B\right)\,\sqrt{a+b\,x^3}}{168\,a^3\,x^4}-\frac{55\,b\,\left(17\,A\,b-14\,a\,B\right)\,\sqrt{a+b\,x^3}}{336\,a^4\,x}+\frac{55\,b^{4/3}\,\left(17\,A\,b-14\,a\,B\right)\,\sqrt{a+b\,x^3}}{336\,a^4\,\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)}-\left(55\,\sqrt{2-\sqrt{3}}\,b^{4/3}\,\left(17\,A\,b-14\,a\,B\right)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],-7-4\,\sqrt{3}\right]\right)\Bigg/\left(224\times 3^{3/4}\,a^{11/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)+\left(55\,b^{4/3}\,\left(17\,A\,b-14\,a\,B\right)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],-7-4\,\sqrt{3}\right]\right)\Bigg/\left(168\,\sqrt{2}\,3^{1/4}\,a^{11/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 292 leaves) :

$$\frac{1}{1008\,a^4\,(-b)^{2/3}\,x^7\,\sqrt{a+b\,x^3}}\left(-3\,(-b)^{2/3}\left(935\,A\,b^3\,x^9+11\,a\,b^2\,x^6\,\left(51\,A-70\,B\,x^3\right)+12\,a^3\,\left(4\,A+7\,B\,x^3\right)-6\,a^2\,b\,x^3\,\left(17\,A+77\,B\,x^3\right)\right)-\right.$$

$$55\,(-1)^{2/3}\,3^{3/4}\,a^{2/3}\,b^2\,(17\,A\,b-14\,a\,B)\,x^7\,\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}$$

$$\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\left(\sqrt{3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+\right.$$

$$\left.\left.(-1)^{5/6}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\right)$$

Problem 249: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^6\,\left(A+B\,x^3\right)}{\left(a+b\,x^3\right)^{5/2}}\,dx$$

Optimal (type 4, 299 leaves, 4 steps):

$$-\frac{2\,(5\,A\,b-14\,a\,B)\,x^4}{45\,b^2\,\left(a+b\,x^3\right)^{3/2}}+\frac{2\,B\,x^7}{5\,b\,\left(a+b\,x^3\right)^{3/2}}-\frac{16\,(5\,A\,b-14\,a\,B)\,x}{135\,b^3\,\sqrt{a+b\,x^3}}+$$

$$\left(32\,\sqrt{2+\sqrt{3}}\,(5\,A\,b-14\,a\,B)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Bigg/$$

$$\left(135\times 3^{1/4}\,b^{10/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 205 leaves):

$$-\left(\left(2\left(3\left(-b\right)^{1/3}x\left(112\,a^2 B+b^2 x^3\left(-55\,A+27\,B\,x^3\right)+a\left(-40\,A\,b+154\,b\,B\,x^3\right)\right)+16\,i\,3^{3/4}a^{1/3}\right.\right.\right.$$

$$\left(5\,A\,b-14\,a\,B\right)\sqrt{\left(-1\right)^{5/6}\left(-1+\frac{\left(-b\right)^{1/3}x}{a^{1/3}}\right)}\sqrt{1+\frac{\left(-b\right)^{1/3}x}{a^{1/3}}+\frac{\left(-b\right)^{2/3}x^2}{a^{2/3}}}\left(a+b\,x^3\right)$$

$$\left.\left.\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-\left(-1\right)^{5/6}-\frac{i\,(-b)^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],\left(-1\right)^{1/3}\right]\right)\right)\Big/\left(405\left(-b\right)^{10/3}\left(a+b\,x^3\right)^{3/2}\right)\right)$$

Problem 250: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^3\left(A+B\,x^3\right)}{\left(a+b\,x^3\right)^{5/2}}\,dx$$

Optimal (type 4, 283 leaves, 3 steps):

$$\frac{2\left(A\,b-a\,B\right)x^4}{9\,a\,b\left(a+b\,x^3\right)^{3/2}}-\frac{2\left(A\,b+8\,a\,B\right)x}{27\,a\,b^2\sqrt{a+b\,x^3}}+$$

$$\left(4\sqrt{2+\sqrt{3}}\left(A\,b+8\,a\,B\right)\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(27\times3^{1/4}\,a\,b^{7/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\sqrt{a+b\,x^3}\right)$$

Result (type 4, 199 leaves):

$$\left(2\,i\left(-3\,i\,(-b)^{1/3}\,x\left(-8\,a^2\,B+2\,A\,b^2\,x^3-a\,b\left(A+11\,B\,x^3\right)\right)+\right.\right.$$
$$2\times 3^{3/4}\,a^{1/3}\,(A\,b+8\,a\,B)\,\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\,\left(a+b\,x^3\right)$$
$$\left.\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\right)\Big/\left(81\,a\,(-b)^{7/3}\left(a+b\,x^3\right)^{3/2}\right)$$

Problem 251: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{A+B\,x^3}{\left(a+b\,x^3\right)^{5/2}}\,dx$$

Optimal (type 4, 283 leaves, 3 steps):

$$\frac{2\,(A\,b-a\,B)\,x}{9\,a\,b\,\left(a+b\,x^3\right)^{3/2}}+\frac{2\,(7\,A\,b+2\,a\,B)\,x}{27\,a^2\,b\,\sqrt{a+b\,x^3}}+$$
$$\left(2\,\sqrt{2+\sqrt{3}}\,(7\,A\,b+2\,a\,B)\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\right.$$
$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/$$
$$\left(27\times 3^{1/4}\,a^2\,b^{4/3}\,\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 199 leaves):

$$-\left(\left(2\left(3\,(-b)^{1/3}\,x\left(-a^2 B+7 A b^2 x^3+2 a b\left(5 A+B x^3\right)\right)+\right.\right.\right.$$
$$i\,3^{3/4}\,a^{1/3}\,(7 A b+2 a B)\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\,\left(a+b x^3\right)$$
$$\left.\left.\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\right)\Big/\left(81\,a^2\,(-b)^{4/3}\,\left(a+b x^3\right)^{3/2}\right)\right)$$

Problem 252: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{A+B x^3}{x^3\,\left(a+b x^3\right)^{5/2}}\,dx$$

Optimal (type 4, 300 leaves, 4 steps):

$$-\frac{A}{2 a x^2\,\left(a+b x^3\right)^{3/2}}-\frac{(13 A b-4 a B)\,x}{18 a^2\,\left(a+b x^3\right)^{3/2}}-\frac{7\,(13 A b-4 a B)\,x}{54 a^3\,\sqrt{a+b x^3}}-$$
$$\left(7\sqrt{2+\sqrt{3}}\,(13 A b-4 a B)\left(a^{1/3}+b^{1/3} x\right)\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\right.$$
$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],\,-7-4\sqrt{3}\right]\right)\Big/$$
$$\left(54\times 3^{1/4}\,a^3\,b^{1/3}\,\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\,\sqrt{a+b x^3}\right)$$

Result (type 4, 210 leaves):

$$\frac{-91 A b^2 x^6+a^2 \left(-27 A+40 B x^3\right)+a \left(-130 A b x^3+28 b B x^6\right)}{54 a^3 x^2 \left(a+b x^3\right)^{3/2}}+$$

$$\left(7 i \left(-13 A b+4 a B\right) \sqrt{(-1)^{5/6} \left(-1+\frac{(-b)^{1/3} x}{a^{1/3}}\right)} \sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right) \Big/ \left(54\times 3^{1/4} a^{8/3} (-b)^{1/3} \sqrt{a+b x^3}\right)$$

Problem 253: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{A+B x^3}{x^6 \left(a+b x^3\right)^{5/2}} \, dx$$

Optimal (type 4, 334 leaves, 5 steps):

$$-\frac{A}{5 a x^5 \left(a+b x^3\right)^{3/2}}-\frac{19 A b-10 a B}{45 a^2 x^2 \left(a+b x^3\right)^{3/2}}-\frac{13 (19 A b-10 a B)}{135 a^3 x^2 \sqrt{a+b x^3}}+$$

$$\frac{91 (19 A b-10 a B) \sqrt{a+b x^3}}{540 a^4 x^2}+\left(91 \sqrt{2+\sqrt{3}}\, b^{2/3} (19 A b-10 a B) \left(a^{1/3}+b^{1/3} x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right) \Big/$$

$$\left(540\times 3^{1/4} a^4 \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 228 leaves):

$$\left(5187\,A\,b^3\,x^9+3\,a^2\,b\,x^3\,\left(513\,A-1300\,B\,x^3\right)+390\,a\,b^2\,x^6\,\left(19\,A-7\,B\,x^3\right)-162\,a^3\,\left(2\,A+5\,B\,x^3\right)-91\,\mathrm{i}\,3^{3/4}\,a^{1/3}\,(-b)^{2/3}\,\left(19\,A\,b-10\,a\,B\right)\,x^5\,\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\,\left(a+b\,x^3\right)\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{\mathrm{i}\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\Big/\left(1620\,a^4\,x^5\,\left(a+b\,x^3\right)^{3/2}\right)$$

Problem 254: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^7\,\left(A+B\,x^3\right)}{\left(a+b\,x^3\right)^{5/2}}\,dx$$

Optimal (type 4, 577 leaves, 6 steps):

$$-\frac{2\,\left(7\,A\,b-16\,a\,B\right)\,x^5}{63\,b^2\,\left(a+b\,x^3\right)^{3/2}}+\frac{2\,B\,x^8}{7\,b\,\left(a+b\,x^3\right)^{3/2}}-\frac{20\,\left(7\,A\,b-16\,a\,B\right)\,x^2}{189\,b^3\,\sqrt{a+b\,x^3}}+\frac{80\,\left(7\,A\,b-16\,a\,B\right)\,\sqrt{a+b\,x^3}}{189\,b^{11/3}\,\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)}-\left(40\,\sqrt{2-\sqrt{3}}\,a^{1/3}\,\left(7\,A\,b-16\,a\,B\right)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/\left(63\times3^{3/4}\,b^{11/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)+\left(80\,\sqrt{2}\,a^{1/3}\,\left(7\,A\,b-16\,a\,B\right)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/\left(189\times3^{1/4}\,b^{11/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 265 leaves) :

$$-\frac{1}{567\,(-b)^{11/3}\,(a+b\,x^3)^{3/2}}\left(2\left(3\,(-b)^{2/3}\,x^2\,\left(160\,a^2\,B+b^2\,x^3\,\left(-91\,A+27\,B\,x^3\right)+a\,\left(-70\,A\,b+208\,b\,B\,x^3\right)\right)-40\,(-1)^{2/3}\,3^{3/4}\,a^{2/3}\,(7\,A\,b-16\,a\,B)\,\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\,(a+b\,x^3)\left(\sqrt{3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+(-1)^{5/6}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\right)\right)$$

Problem 255: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4\,(A+B\,x^3)}{(a+b\,x^3)^{5/2}}\,dx$$

Optimal (type 4, 559 leaves, 5 steps) :

$$\frac{2\,(A\,b-a\,B)\,x^5}{9\,a\,b\,(a+b\,x^3)^{3/2}}+\frac{2\,(A\,b-10\,a\,B)\,x^2}{27\,a\,b^2\,\sqrt{a+b\,x^3}}-\frac{8\,(A\,b-10\,a\,B)\,\sqrt{a+b\,x^3}}{27\,a\,b^{8/3}\,\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)}+$$

$$\left(4\,\sqrt{2-\sqrt{3}}\,(A\,b-10\,a\,B)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/$$

$$\left(9\times 3^{3/4}\,a^{2/3}\,b^{8/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)-$$

$$\left(8\,\sqrt{2}\,(A\,b-10\,a\,B)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/$$

$$\left(27\times 3^{1/4}\,a^{2/3}\,b^{8/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 256 leaves) :

$$\left(2\left(3\,(-b)^{2/3}\,x^2\,\left(-10\,a^2\,B+4\,A\,b^2\,x^3+a\,b\,\left(A-13\,B\,x^3\right)\right)+\right.\right.$$

$$4\,(-1)^{2/3}\,3^{3/4}\,a^{2/3}\,(A\,b-10\,a\,B)\,\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}$$

$$\left(a+b\,x^3\right)\left(\sqrt{3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+(-1)^{5/6}\right.$$

$$\left.\left.\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\right)\right)\Big/\left(81\,a\,(-b)^{8/3}\,\left(a+b\,x^3\right)^{3/2}\right)$$

Problem 256: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x\,(A+B\,x^3)}{(a+b\,x^3)^{5/2}}\,dx$$

Optimal (type 4, 563 leaves, 5 steps):

$$\frac{2\,(A\,b-a\,B)\,x^2}{9\,a\,b\,(a+b\,x^3)^{3/2}}+\frac{2\,(5\,A\,b+4\,a\,B)\,x^2}{27\,a^2\,b\,\sqrt{a+b\,x^3}}-\frac{2\,(5\,A\,b+4\,a\,B)\,\sqrt{a+b\,x^3}}{27\,a^2\,b^{5/3}\,\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)}+$$
$$\left(\sqrt{2-\sqrt{3}}\,(5\,A\,b+4\,a\,B)\,(a^{1/3}+b^{1/3}\,x)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\right.$$
$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\,\right]\right)\Big/$$
$$\left(9\times 3^{3/4}\,a^{5/3}\,b^{5/3}\,\sqrt{\frac{a^{1/3}\,(a^{1/3}+b^{1/3}\,x)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)-$$
$$\left(2\,\sqrt{2}\,(5\,A\,b+4\,a\,B)\,(a^{1/3}+b^{1/3}\,x)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\right.$$
$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\,\right]\right)\Big/$$
$$\left(27\times 3^{1/4}\,a^{5/3}\,b^{5/3}\,\sqrt{\frac{a^{1/3}\,(a^{1/3}+b^{1/3}\,x)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 257 leaves):

$$-\Bigg(\Bigg(2\Bigg(3\,(-b)^{2/3}x^2\left(a^2 B+5 A b^2 x^3+4 a b\left(2 A+B x^3\right)\right)+$$

$$(-1)^{2/3}\,3^{3/4}a^{2/3}(5 A b+4 a B)\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}x}{a^{1/3}}\right)}\sqrt{1+\frac{(-b)^{1/3}x}{a^{1/3}}+\frac{(-b)^{2/3}x^2}{a^{2/3}}}$$

$$\left(a+b x^3\right)\Bigg(\sqrt{3}\,\text{EllipticE}\Big[\text{ArcSin}\Big[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}x}{a^{1/3}}}}{3^{1/4}}\Big],(-1)^{1/3}\Big]+(-1)^{5/6}\,\text{EllipticF}\Big[$$

$$\text{ArcSin}\Big[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}x}{a^{1/3}}}}{3^{1/4}}\Big],(-1)^{1/3}\Big]\Bigg)\Bigg)\Bigg)\Bigg/\left(81\,a^2\,(-b)^{5/3}\left(a+b x^3\right)^{3/2}\right)\Bigg)$$

Problem 257: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{A+B x^3}{x^2\left(a+b x^3\right)^{5/2}}\,dx$$

Optimal (type 4, 578 leaves, 6 steps):

$$-\frac{A}{a x \left(a+b x^3\right)^{3/2}}-\frac{\left(11 A b-2 a B\right) x^2}{9 a^2 \left(a+b x^3\right)^{3/2}}-\frac{5 \left(11 A b-2 a B\right) x^2}{27 a^3 \sqrt{a+b x^3}}+\frac{5 \left(11 A b-2 a B\right) \sqrt{a+b x^3}}{27 a^3 b^{2/3} \left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}-\left(5 \sqrt{2-\sqrt{3}} \left(11 A b-2 a B\right) \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Bigg/\left(18\times 3^{3/4} a^{8/3} b^{2/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)+\left(5 \sqrt{2} \left(11 A b-2 a B\right) \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Bigg/\left(27\times 3^{1/4} a^{8/3} b^{2/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 273 leaves):

$$\frac{1}{81 a^3 \left(a+b x^3\right)^{3/2}}\left(-\frac{3 \left(55 A b^2 x^6+a^2 \left(27 A-16 B x^3\right)+2 a b x^3 \left(44 A-5 B x^3\right)\right)}{x}+\frac{1}{(-b)^{2/3}} 5 (-1)^{1/6} 3^{3/4} a^{2/3} \left(11 A b-2 a B\right) \sqrt{\frac{(-1)^{5/6} \left(-a^{1/3}+(-b)^{1/3} x\right)}{a^{1/3}}} \sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}} \left(a+b x^3\right)\left(-i \sqrt{3} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+(-1)^{1/3} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right)$$

Problem 258: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{A+B x^3}{x^5 \left(a+b x^3\right)^{5/2}}\, dx$$

Optimal (type 4, 610 leaves, 7 steps):

$$-\frac{A}{4 a x^4 \left(a+b x^3\right)^{3/2}}-\frac{17 A b-8 a B}{36 a^2 x \left(a+b x^3\right)^{3/2}}-\frac{11 (17 A b-8 a B)}{108 a^3 x \sqrt{a+b x^3}}+\frac{55 (17 A b-8 a B) \sqrt{a+b x^3}}{216 a^4 x}-\frac{55 b^{1/3} (17 A b-8 a B) \sqrt{a+b x^3}}{216 a^4 \left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}+\left(55 \sqrt{2-\sqrt{3}}\, b^{1/3} (17 A b-8 a B) \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right) \Bigg/ \left(144 \times 3^{3/4} a^{11/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)-\left(55 b^{1/3} (17 A b-8 a B) \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right) \Bigg/ \left(108 \sqrt{2}\, 3^{1/4} a^{11/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 293 leaves):

$$\frac{1}{648\, a^4 \left(a+b x^3\right)^{3/2}} \left(-\frac{1}{x^4} 3 \left(-935 A b^3 x^9+54 a^3 \left(A+4 B x^3\right)+88 a b^2 x^6 \left(-17 A+5 B x^3\right)+a^2 \left(-459 A b x^3+704 b B x^6\right)\right)+\right.$$
$$55 (-1)^{1/6}\, 3^{3/4} a^{2/3} (-b)^{1/3} (17 A b-8 a B) \sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3} x\right)}{a^{1/3}}} \sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}} \left(a+b x^3\right)$$
$$\left(-i \sqrt{3}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right]+\right.$$
$$\left.\left.(-1)^{1/3}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right]\right)\right)$$

Problem 262: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{c+d x^3}}{x \left(4 c+d x^3\right)}\, dx$$

Optimal (type 3, 65 leaves, 6 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{c+d x^3}}{\sqrt{3}\, \sqrt{c}}\right]}{2 \sqrt{3}\, \sqrt{c}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{6 \sqrt{c}}$$

Result (type 6, 158 leaves):

$$-\left(\left(2 d x^3 \sqrt{c+d x^3}\, \text{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, 1, \frac{3}{2}, -\frac{c}{d x^3}, -\frac{4 c}{d x^3}\right]\right) \middle/ \right.$$
$$\left(\left(4 c+d x^3\right) \left(3 d x^3\, \text{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, 1, \frac{3}{2}, -\frac{c}{d x^3}, -\frac{4 c}{d x^3}\right]+\right.\right.$$
$$\left.\left.\left.c \left(-8\, \text{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}, 2, \frac{5}{2}, -\frac{c}{d x^3}, -\frac{4 c}{d x^3}\right]+\text{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, 1, \frac{5}{2}, -\frac{c}{d x^3}, -\frac{4 c}{d x^3}\right]\right)\right)\right)\right)$$

Problem 263: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{c+d x^3}}{x^4 \left(4 c+d x^3\right)} \, dx$$

Optimal (type 3, 88 leaves, 7 steps):

$$-\frac{\sqrt{c+d x^3}}{12 c x^3}-\frac{d \operatorname{ArcTan}\left[\frac{\sqrt{c+d x^3}}{\sqrt{3} \sqrt{c}}\right]}{8 \sqrt{3} c^{3/2}}-\frac{d \operatorname{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{24 c^{3/2}}$$

Result (type 6, 319 leaves):

$$\frac{1}{36 x^3 \sqrt{c+d x^3}}\left(-3-\frac{3 d x^3}{c}+\left(12 d^2 x^6 \operatorname{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]\right) \Big/ \left(\left(4 c+d x^3\right)\left(-8 c \operatorname{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]+d x^3\left(\operatorname{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]+2 \operatorname{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]\right)\right)\right)+\left(10 d^2 x^6 \operatorname{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},-\frac{4 c}{d x^3}\right]\right) \Big/ \left(\left(4 c+d x^3\right)\left(-5 d x^3 \operatorname{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},-\frac{4 c}{d x^3}\right]+c\left(8 \operatorname{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^3},-\frac{4 c}{d x^3}\right]+\operatorname{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^3},-\frac{4 c}{d x^3}\right]\right)\right)\right)\right)$$

Problem 264: Result unnecessarily involves higher level functions.

$$\int \frac{x^4 \sqrt{c+d x^3}}{4 c+d x^3} \, dx$$

Optimal (type 4, 689 leaves, 7 steps):

$$\frac{2x^2\sqrt{c+dx^3}}{7d}-\frac{50c\sqrt{c+dx^3}}{7d^{5/3}\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)}-\frac{2\times 2^{1/3}c^{7/6}\,\text{ArcTan}\left[\frac{\sqrt{3}\,c^{1/6}\left(c^{1/3}+2^{1/3}d^{1/3}x\right)}{\sqrt{c+dx^3}}\right]}{\sqrt{3}\,d^{5/3}}+$$

$$\frac{2\times 2^{1/3}c^{7/6}\,\text{ArcTan}\left[\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right]}{\sqrt{3}\,d^{5/3}}-\frac{2\times 2^{1/3}c^{7/6}\,\text{ArcTanh}\left[\frac{c^{1/6}\left(c^{1/3}-2^{1/3}d^{1/3}x\right)}{\sqrt{c+dx^3}}\right]}{d^{5/3}}+$$

$$\frac{2\times 2^{1/3}c^{7/6}\,\text{ArcTanh}\left[\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right]}{3d^{5/3}}+\left(25\times 3^{1/4}\sqrt{2-\sqrt{3}}\,c^{4/3}\left(c^{1/3}+d^{1/3}x\right)\right.$$

$$\left.\sqrt{\frac{c^{2/3}-c^{1/3}d^{1/3}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(7d^{5/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\sqrt{c+dx^3}\right)-\left(50\sqrt{2}\,c^{4/3}\left(c^{1/3}+d^{1/3}x\right)\right.$$

$$\left.\sqrt{\frac{c^{2/3}-c^{1/3}d^{1/3}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(7\times 3^{1/4}d^{5/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\sqrt{c+dx^3}\right)$$

Result (type 6, 343 leaves):

$$\frac{1}{7\sqrt{c+dx^3}}2x^2\left(\frac{c}{d}+x^3+\left(80c^3\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},-\frac{dx^3}{4c}\right]\right)\Big/\right.$$

$$\left(d\left(4c+dx^3\right)\left(-20c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},-\frac{dx^3}{4c}\right]+3dx^3\right.\right.$$

$$\left.\left.\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{dx^3}{c},-\frac{dx^3}{4c}\right]+2\,\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{dx^3}{c},-\frac{dx^3}{4c}\right]\right)\right)\right)-$$

$$\left(80c^2x^3\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{dx^3}{c},-\frac{dx^3}{4c}\right]\right)\Big/$$

$$\left(\left(4c+dx^3\right)\left(32c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{dx^3}{c},-\frac{dx^3}{4c}\right]-3dx^3\right.\right.$$

$$\left.\left.\left.\left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{dx^3}{c},-\frac{dx^3}{4c}\right]+2\,\text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{dx^3}{c},-\frac{dx^3}{4c}\right]\right)\right)\right)\right)$$

Problem 265: Result unnecessarily involves higher level functions.

$$\int \frac{x\sqrt{c+d x^3}}{4 c+d x^3}\,dx$$

Optimal (type 4, 659 leaves, 5 steps):

$$\frac{2\sqrt{c+d x^3}}{d^{2/3}\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)}+\frac{c^{1/6}\,\text{ArcTan}\left[\frac{\sqrt{3}\,c^{1/6}\left(c^{1/3}+2^{1/3}d^{1/3}x\right)}{\sqrt{c+d x^3}}\right]}{2^{2/3}\sqrt{3}\,d^{2/3}}-\frac{c^{1/6}\,\text{ArcTan}\left[\frac{\sqrt{c+d x^3}}{\sqrt{3}\sqrt{c}}\right]}{2^{2/3}\sqrt{3}\,d^{2/3}}+\frac{c^{1/6}\,\text{ArcTanh}\left[\frac{c^{1/6}\left(c^{1/3}-2^{1/3}d^{1/3}x\right)}{\sqrt{c+d x^3}}\right]}{2^{2/3}d^{2/3}}-\frac{c^{1/6}\,\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{3\times 2^{2/3}d^{2/3}}-$$

$$\left(3^{1/4}\sqrt{2-\sqrt{3}}\,c^{1/3}\left(c^{1/3}+d^{1/3}x\right)\sqrt{\frac{c^{2/3}-c^{1/3}d^{1/3}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(d^{2/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\sqrt{c+d x^3}\right)+\left(2\sqrt{2}\,c^{1/3}\left(c^{1/3}+d^{1/3}x\right)\sqrt{\frac{c^{2/3}-c^{1/3}d^{1/3}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(3^{1/4}d^{2/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\sqrt{c+d x^3}\right)$$

Result (type 6, 167 leaves):

$$\left(10\,c\,x^2\sqrt{c+d x^3}\;\text{AppellF1}\left[\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]\right)\Big/$$

$$\left(\left(4 c+d x^3\right)\left(20\,c\,\text{AppellF1}\left[\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]-3\,d\,x^3\left(\text{AppellF1}\left[\frac{5}{3},-\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]-2\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]\right)\right)\right)$$

Problem 266: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{c+d x^3}}{x^2 \left(4 c+d x^3\right)} \, dx$$

Optimal (type 4, 697 leaves, 7 steps):

$$-\frac{\sqrt{c+d x^3}}{4 c x}+\frac{d^{1/3} \sqrt{c+d x^3}}{4 c \left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)}-\frac{d^{1/3} \text{ArcTan}\left[\frac{\sqrt{3} c^{1/6} \left(c^{1/3}+2^{1/3} d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]}{4\times 2^{2/3} \sqrt{3} c^{5/6}}+\frac{d^{1/3} \text{ArcTan}\left[\frac{\sqrt{c+d x^3}}{\sqrt{3} \sqrt{c}}\right]}{4\times 2^{2/3} \sqrt{3} c^{5/6}}-\frac{d^{1/3} \text{ArcTanh}\left[\frac{c^{1/6} \left(c^{1/3}-2^{1/3} d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]}{4\times 2^{2/3} c^{5/6}}+\frac{d^{1/3} \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{12\times 2^{2/3} c^{5/6}}-\left(3^{1/4} \sqrt{2-\sqrt{3}} d^{1/3} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/\left(8 c^{2/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}\right)+\left(d^{1/3} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/\left(2 \sqrt{2} 3^{1/4} c^{2/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}\right)$$

Result (type 6, 344 leaves):

$$\frac{1}{20\,x\,\sqrt{c+d\,x^3}}\left(-5-\frac{5\,d\,x^3}{c}+\left(250\,c\,d\,x^3\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d\,x^3}{c},-\frac{d\,x^3}{4\,c}\right]\right)\Big/\left(\left(4\,c+d\,x^3\right)\left(20\,c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d\,x^3}{c},-\frac{d\,x^3}{4\,c}\right]-3\,d\,x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d\,x^3}{c},-\frac{d\,x^3}{4\,c}\right]+2\,\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},-\frac{d\,x^3}{4\,c}\right]\right)\right)\right)+\left(16\,d^2\,x^6\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},-\frac{d\,x^3}{4\,c}\right]\right)\Big/\left(\left(4\,c+d\,x^3\right)\left(32\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},-\frac{d\,x^3}{4\,c}\right]-3\,d\,x^3\left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d\,x^3}{c},-\frac{d\,x^3}{4\,c}\right]+2\,\text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d\,x^3}{c},-\frac{d\,x^3}{4\,c}\right]\right)\right)\right)\right)$$

Problem 267: Result more than twice size of optimal antiderivative.

$$\int\frac{x^3\,\sqrt{c+d\,x^3}}{4\,c+d\,x^3}\,dx$$

Optimal (type 6, 66 leaves, 2 steps):

$$\frac{x^4\,\sqrt{c+d\,x^3}\,\text{AppellF1}\left[\frac{4}{3},1,-\frac{1}{2},\frac{7}{3},-\frac{d\,x^3}{4\,c},-\frac{d\,x^3}{c}\right]}{16\,c\,\sqrt{1+\frac{d\,x^3}{c}}}$$

Result (type 6, 344 leaves):

$$\frac{1}{5\,\sqrt{c+d\,x^3}}x\left(2\left(\frac{c}{d}+x^3\right)+\left(128\,c^3\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d\,x^3}{c},-\frac{d\,x^3}{4\,c}\right]\right)\Big/\left(d\left(4\,c+d\,x^3\right)\left(-16\,c\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d\,x^3}{c},-\frac{d\,x^3}{4\,c}\right]+3\,d\,x^3\left(\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{d\,x^3}{4\,c}\right]+2\,\text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{d\,x^3}{4\,c}\right]\right)\right)\right)-\left(119\,c^2\,x^3\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{d\,x^3}{4\,c}\right]\right)\Big/\left(\left(4\,c+d\,x^3\right)\left(28\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{d\,x^3}{4\,c}\right]-3\,d\,x^3\left(\text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d\,x^3}{c},-\frac{d\,x^3}{4\,c}\right]+2\,\text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d\,x^3}{c},-\frac{d\,x^3}{4\,c}\right]\right)\right)\right)\right)$$

Problem 268: Result more than twice size of optimal antiderivative.

$$\int\frac{\sqrt{c+d\,x^3}}{4\,c+d\,x^3}\,dx$$

Optimal (type 6, 64 leaves, 2 steps):

$$\frac{x\sqrt{c+d x^3}\ \text{AppellF1}\left[\frac{1}{3},1,-\frac{1}{2},\frac{4}{3},-\frac{d x^3}{4 c},-\frac{d x^3}{c}\right]}{4 c\sqrt{1+\frac{d x^3}{c}}}$$

Result (type 6, 165 leaves):

$$\left(16 c x\sqrt{c+d x^3}\ \text{AppellF1}\left[\frac{1}{3},-\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]\right)\Big/\left(\left(4 c+d x^3\right)\left(16 c\,\text{AppellF1}\left[\frac{1}{3},-\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]-3 d x^3\left(\text{AppellF1}\left[\frac{4}{3},-\frac{1}{2},2,\frac{7}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]-2\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]\right)\right)\right)$$

Problem 269: Result more than twice size of optimal antiderivative.

$$\int\frac{\sqrt{c+d x^3}}{x^3\left(4 c+d x^3\right)}\,dx$$

Optimal (type 6, 66 leaves, 2 steps):

$$-\frac{\sqrt{c+d x^3}\ \text{AppellF1}\left[-\frac{2}{3},1,-\frac{1}{2},\frac{1}{3},-\frac{d x^3}{4 c},-\frac{d x^3}{c}\right]}{8 c x^2\sqrt{1+\frac{d x^3}{c}}}$$

Result (type 6, 344 leaves):

$$\frac{1}{16 x^2\sqrt{c+d x^3}}\left(-2-\frac{2 d x^3}{c}+\left(128 c d x^3\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]\right)\Big/\left(\left(4 c+d x^3\right)\left(16 c\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]-3 d x^3\left(\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]+2\,\text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]\right)\right)\right)+\left(7 d^2 x^6\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]\right)\Big/\left(\left(4 c+d x^3\right)\left(-28 c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]+2\,\text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]\right)\right)\right)\right)$$

Problem 273: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\frac{1}{x\sqrt{c+d x^3}\left(4 c+d x^3\right)}\,dx$$

Optimal (type 3, 65 leaves, 6 steps):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{c+d x^3}}{\sqrt{3}\sqrt{c}}\right]}{6\sqrt{3}\,c^{3/2}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{6\,c^{3/2}}$$

Result (type 6, 160 leaves):

$$\left(10\,d\,x^3\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^3},-\frac{4\,c}{d\,x^3}\right]\right)\Big/\left(9\sqrt{c+d\,x^3}\,\left(4\,c+d\,x^3\right)\left(-5\,d\,x^3\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^3},-\frac{4\,c}{d\,x^3}\right]+c\left(8\,\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d\,x^3},-\frac{4\,c}{d\,x^3}\right]+\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d\,x^3},-\frac{4\,c}{d\,x^3}\right]\right)\right)\right)$$

Problem 274: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\frac{1}{x^4\sqrt{c+d\,x^3}\,\left(4\,c+d\,x^3\right)}\,dx$$

Optimal (type 3, 88 leaves, 7 steps):

$$-\frac{\sqrt{c+d\,x^3}}{12\,c^2\,x^3}+\frac{d\,\text{ArcTan}\left[\frac{\sqrt{c+d x^3}}{\sqrt{3}\sqrt{c}}\right]}{24\sqrt{3}\,c^{5/2}}+\frac{d\,\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{8\,c^{5/2}}$$

Result (type 6, 324 leaves):

$$\frac{1}{12\,c^2\,x^3\sqrt{c+d\,x^3}}\left(-c-d\,x^3-\left(4\,c\,d^2\,x^6\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d\,x^3}{c},-\frac{d\,x^3}{4\,c}\right]\right)\Big/\left(\left(4\,c+d\,x^3\right)\left(8\,c\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d\,x^3}{c},-\frac{d\,x^3}{4\,c}\right]-d\,x^3\left(\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d\,x^3}{c},-\frac{d\,x^3}{4\,c}\right]+2\,\text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d\,x^3}{c},-\frac{d\,x^3}{4\,c}\right]\right)\right)\right)-\left(10\,c\,d^2\,x^6\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^3},-\frac{4\,c}{d\,x^3}\right]\right)\Big/\left(\left(4\,c+d\,x^3\right)\left(-5\,d\,x^3\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^3},-\frac{4\,c}{d\,x^3}\right]+8\,c\,\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d\,x^3},-\frac{4\,c}{d\,x^3}\right]+c\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d\,x^3},-\frac{4\,c}{d\,x^3}\right]\right)\right)\right)$$

Problem 275: Result unnecessarily involves higher level functions.

$$\int\frac{x^4}{\sqrt{c+d\,x^3}\,\left(4\,c+d\,x^3\right)}\,dx$$

Optimal (type 4, 667 leaves, 5 steps):

$$\frac{2\sqrt{c+d x^3}}{d^{5/3}\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)}+\frac{2\times 2^{1/3}c^{1/6}\,\text{ArcTan}\left[\frac{\sqrt{3}\,c^{1/6}\left(c^{1/3}+2^{1/3}d^{1/3}x\right)}{\sqrt{c+d x^3}}\right]}{3\sqrt{3}\,d^{5/3}}-\frac{2\times 2^{1/3}c^{1/6}\,\text{ArcTan}\left[\frac{\sqrt{c+d x^3}}{\sqrt{3}\sqrt{c}}\right]}{3\sqrt{3}\,d^{5/3}}+\frac{2\times 2^{1/3}c^{1/6}\,\text{ArcTanh}\left[\frac{c^{1/6}\left(c^{1/3}-2^{1/3}d^{1/3}x\right)}{\sqrt{c+d x^3}}\right]}{3\,d^{5/3}}-\frac{2\times 2^{1/3}c^{1/6}\,\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{9\,d^{5/3}}-\left(3^{1/4}\sqrt{2-\sqrt{3}}\;c^{1/3}\left(c^{1/3}+d^{1/3}x\right)\sqrt{\frac{c^{2/3}-c^{1/3}d^{1/3}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/\left(d^{5/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\sqrt{c+d x^3}\right)+\left(2\sqrt{2}\;c^{1/3}\left(c^{1/3}+d^{1/3}x\right)\sqrt{\frac{c^{2/3}-c^{1/3}d^{1/3}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/\left(3^{1/4}d^{5/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\sqrt{c+d x^3}\right)$$

Result (type 6, 169 leaves):

$$\left(32\,c\,x^5\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]\right)\Big/\left(5\sqrt{c+d x^3}\left(4 c+d x^3\right)\left(32\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]-3\,d\,x^3\left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]+2\,\text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]\right)\right)\right)$$

Problem 276: Result unnecessarily involves higher level functions.

$$\int \frac{x}{\sqrt{c+d x^3}\left(4 c+d x^3\right)}\,dx$$

Optimal (type 3, 206 leaves, 1 step):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{3}\, c^{1/6}\left(c^{1/3}+2^{1/3} d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]}{3\times 2^{2/3}\sqrt{3}\, c^{5/6} d^{2/3}}+\frac{\text{ArcTan}\left[\frac{\sqrt{c+d x^3}}{\sqrt{3}\sqrt{c}}\right]}{3\times 2^{2/3}\sqrt{3}\, c^{5/6} d^{2/3}}-\frac{\text{ArcTanh}\left[\frac{c^{1/6}\left(c^{1/3}-2^{1/3} d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]}{3\times 2^{2/3} c^{5/6} d^{2/3}}+\frac{\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{9\times 2^{2/3} c^{5/6} d^{2/3}}$$

Result (type 6, 167 leaves):

$$\left(10\, c\, x^2\, \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]\right)\Big/ \left(\sqrt{c+d x^3}\left(4 c+d x^3\right)\left(20\, c\, \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]-3\, d\, x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]+2\, \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]\right)\right)\right)$$

Problem 277: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^2\sqrt{c+d x^3}\left(4 c+d x^3\right)}\, dx$$

Optimal (type 4, 697 leaves, 7 steps):

$$-\frac{\sqrt{c+d x^3}}{4 c^2 x}+\frac{d^{1/3} \sqrt{c+d x^3}}{4 c^2 \left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)}+\frac{d^{1/3} \text{ArcTan}\left[\frac{\sqrt{3}\, c^{1/6} \left(c^{1/3}+2^{1/3} d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]}{12 \times 2^{2/3} \sqrt{3}\, c^{11/6}}-$$

$$\frac{d^{1/3} \text{ArcTan}\left[\frac{\sqrt{c+d x^3}}{\sqrt{3} \sqrt{c}}\right]}{12 \times 2^{2/3} \sqrt{3}\, c^{11/6}}+\frac{d^{1/3} \text{ArcTanh}\left[\frac{c^{1/6} \left(c^{1/3}-2^{1/3} d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]}{12 \times 2^{2/3} c^{11/6}}-\frac{d^{1/3} \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{36 \times 2^{2/3} c^{11/6}}-$$

$$\left(3^{1/4} \sqrt{2-\sqrt{3}}\; d^{1/3} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right], -7-4 \sqrt{3}\right]\right) \Big/$$

$$\left(8 c^{5/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}\right)+\left(d^{1/3} \left(c^{1/3}+d^{1/3} x\right)\right.$$

$$\left.\sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\; \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right], -7-4 \sqrt{3}\right]\right) \Big/$$

$$\left(2 \sqrt{2}\; 3^{1/4} c^{5/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}\right)$$

Result (type 6, 348 leaves):

$$\frac{1}{20 x \sqrt{c+d x^3}}\left(\left(50 d x^3 \text{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{d x^3}{c}, -\frac{d x^3}{4 c}\right]\right) \Big/\right.$$

$$\left(\left(4 c+d x^3\right) \left(20 c \text{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{d x^3}{c}, -\frac{d x^3}{4 c}\right]-3 d x^3\right.\right.$$

$$\left.\left.\left.\left(\text{AppellF1}\left[\frac{5}{3}, \frac{1}{2}, 2, \frac{8}{3}, -\frac{d x^3}{c}, -\frac{d x^3}{4 c}\right]+2 \text{AppellF1}\left[\frac{5}{3}, \frac{3}{2}, 1, \frac{8}{3}, -\frac{d x^3}{c}, -\frac{d x^3}{4 c}\right]\right)\right)\right)\right)+$$

$$\frac{1}{c^2}\left(-5 \left(c+d x^3\right)+\left(16 c d^2 x^6 \text{AppellF1}\left[\frac{5}{3}, \frac{1}{2}, 1, \frac{8}{3}, -\frac{d x^3}{c}, -\frac{d x^3}{4 c}\right]\right) \Big/\right.$$

$$\left(\left(4 c+d x^3\right) \left(32 c \text{AppellF1}\left[\frac{5}{3}, \frac{1}{2}, 1, \frac{8}{3}, -\frac{d x^3}{c}, -\frac{d x^3}{4 c}\right]-3 d x^3 \left(\text{AppellF1}\left[\frac{8}{3}, \frac{1}{2},\right.\right.\right.\right.$$

$$\left.\left.\left.\left.\left.2, \frac{11}{3}, -\frac{d x^3}{c}, -\frac{d x^3}{4 c}\right]+2 \text{AppellF1}\left[\frac{8}{3}, \frac{3}{2}, 1, \frac{11}{3}, -\frac{d x^3}{c}, -\frac{d x^3}{4 c}\right]\right)\right)\right)\right)\right)$$

Problem 278: Result more than twice size of optimal antiderivative.

$$\int \frac{x^3}{\sqrt{c+d x^3}\,\left(4 c+d x^3\right)}\,dx$$

Optimal (type 6, 66 leaves, 2 steps):

$$\frac{x^4 \sqrt{1+\frac{d x^3}{c}}\ \text{AppellF1}\left[\frac{4}{3},1,\frac{1}{2},\frac{7}{3},-\frac{d x^3}{4 c},-\frac{d x^3}{c}\right]}{16 c \sqrt{c+d x^3}}$$

Result (type 6, 167 leaves):

$$\left(7 c x^4 \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]\right)\Big/ \left(\sqrt{c+d x^3}\,\left(4 c+d x^3\right)\left(28 c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]-3 d x^3\left(\text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]+2\,\text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]\right)\right)\right)$$

Problem 279: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{c+d x^3}\,\left(4 c+d x^3\right)}\,dx$$

Optimal (type 6, 64 leaves, 2 steps):

$$\frac{x \sqrt{1+\frac{d x^3}{c}}\ \text{AppellF1}\left[\frac{1}{3},1,\frac{1}{2},\frac{4}{3},-\frac{d x^3}{4 c},-\frac{d x^3}{c}\right]}{4 c \sqrt{c+d x^3}}$$

Result (type 6, 165 leaves):

$$\left(16 c x\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]\right)\Big/ \left(\sqrt{c+d x^3}\,\left(4 c+d x^3\right)\left(16 c\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]-3 d x^3\left(\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]+2\,\text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]\right)\right)\right)$$

Problem 280: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^3 \sqrt{c+d x^3}\,\left(4 c+d x^3\right)}\,dx$$

Optimal (type 6, 66 leaves, 2 steps):

$$-\frac{\sqrt{1+\frac{d x^3}{c}}\ \text{AppellF1}\left[-\frac{2}{3},1,\frac{1}{2},\frac{1}{3},-\frac{d x^3}{4 c},-\frac{d x^3}{c}\right]}{8 c x^2\sqrt{c+d x^3}}$$

Result (type 6, 348 leaves):

$$\frac{1}{16 x^2\sqrt{c+d x^3}}\left(\left(128 d x^3\ \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]\right)\Big/\left(\left(4 c+d x^3\right)\left(-16 c\ \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]+2\ \text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]\right)\right)\right)+\frac{1}{c^2}\left(-2\left(c+d x^3\right)-\left(7 c d^2 x^6\ \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]\right)\Big/\left(\left(4 c+d x^3\right)\left(28 c\ \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]-3 d x^3\left(\text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]+2\ \text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d x^3}{c},-\frac{d x^3}{4 c}\right]\right)\right)\right)\right)\right)$$

Problem 281: Result unnecessarily involves higher level functions.

$$\int\frac{x}{\sqrt{1-x^3}\left(4-x^3\right)}\,dx$$

Optimal (type 3, 127 leaves, 1 step):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{3}\left(1-2^{1/3}x\right)}{\sqrt{1-x^3}}\right]}{3\times 2^{2/3}\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{\sqrt{1-x^3}}{\sqrt{3}}\right]}{3\times 2^{2/3}\sqrt{3}}-\frac{\text{ArcTanh}\left[\frac{1+2^{1/3}x}{\sqrt{1-x^3}}\right]}{3\times 2^{2/3}}+\frac{\text{ArcTanh}\left[\sqrt{1-x^3}\right]}{9\times 2^{2/3}}$$

Result (type 6, 120 leaves):

$$-\left(\left(10 x^2\ \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},x^3,\frac{x^3}{4}\right]\right)\Big/\left(\sqrt{1-x^3}\left(-4+x^3\right)\left(20\ \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},x^3,\frac{x^3}{4}\right]+3 x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},x^3,\frac{x^3}{4}\right]+2\ \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},x^3,\frac{x^3}{4}\right]\right)\right)\right)\right)$$

Problem 286: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\frac{\sqrt{c+d x^3}}{x\left(8 c-d x^3\right)}\,dx$$

Optimal (type 3, 58 leaves, 6 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{4 \sqrt{c}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{12 \sqrt{c}}$$

Result (type 6, 158 leaves):

$$\left(2 d x^3 \sqrt{c+d x^3}\ \text{AppellF1}\left[\frac{1}{2},-\frac{1}{2},1,\frac{3}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]\right) \Big/ \left(\left(-8 c+d x^3\right)\left(3 d x^3\ \text{AppellF1}\left[\frac{1}{2},-\frac{1}{2},1,\frac{3}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]+c\left(16\ \text{AppellF1}\left[\frac{3}{2},-\frac{1}{2},2,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]+\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]\right)\right)\right)$$

Problem 287: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{c+d x^3}}{x^4\left(8 c-d x^3\right)}\, dx$$

Optimal (type 3, 81 leaves, 7 steps):

$$-\frac{\sqrt{c+d x^3}}{24 c x^3}+\frac{d\ \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{32 c^{3/2}}-\frac{5 d\ \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{96 c^{3/2}}$$

Result (type 6, 321 leaves):

$$\frac{1}{72 x^3 \sqrt{c+d x^3}}\left(-3-\frac{3 d x^3}{c}+\left(24 d^2 x^6\ \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right) \Big/ \left(\left(8 c-d x^3\right)\left(16 c\ \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+d x^3\left(\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\ \text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)+\left(50 d^2 x^6\ \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]\right) \Big/ \left(\left(-8 c+d x^3\right)\left(5 d x^3\ \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]+16 c\ \text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]-c\ \text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]\right)\right)\right)$$

Problem 288: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{c+d x^3}}{x^7\left(8 c-d x^3\right)}\, dx$$

Optimal (type 3, 107 leaves, 8 steps):

$$-\frac{\sqrt{c+d x^3}}{48 c x^6}-\frac{d \sqrt{c+d x^3}}{64 c^2 x^3}+\frac{d^2 \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{256 c^{5/2}}+\frac{d^2 \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{256 c^{5/2}}$$

Result (type 6, 341 leaves):

$$\frac{1}{96 \sqrt{c+d x^3}}\left(-\frac{3 d^2}{2 c^2}-\frac{2}{x^6}-\frac{7 d}{2 c x^3}+\left(12 d^3 x^3 \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(c\left(8 c-d x^3\right)\left(16 c \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+d x^3\left(\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)+\left(5 d^3 x^3 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]\right)\Big/\left(c\left(8 c-d x^3\right)\left(5 d x^3 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]+16 c \text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]-c \text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]\right)\right)\right)$$

Problem 289: Result unnecessarily involves higher level functions.

$$\int \frac{x^7 \sqrt{c+d x^3}}{8 c-d x^3}\, dx$$

Optimal (type 4, 648 leaves, 15 steps):

$$-\frac{214\,c\,x^2\sqrt{c+d\,x^3}}{91\,d^2}-\frac{2\,x^5\sqrt{c+d\,x^3}}{13\,d}-\frac{12\,248\,c^2\sqrt{c+d\,x^3}}{91\,d^{8/3}\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x\right)}-$$

$$\frac{32\sqrt{3}\,c^{13/6}\,\text{ArcTan}\left[\frac{\sqrt{3}\,c^{1/6}\left(c^{1/3}+d^{1/3}\,x\right)}{\sqrt{c+d\,x^3}}\right]}{d^{8/3}}+\frac{32\,c^{13/6}\,\text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3}\,x\right)^2}{3\,c^{1/6}\sqrt{c+d\,x^3}}\right]}{d^{8/3}}-$$

$$\frac{32\,c^{13/6}\,\text{ArcTanh}\left[\frac{\sqrt{c+d\,x^3}}{3\sqrt{c}}\right]}{d^{8/3}}+\left(6124\times 3^{1/4}\sqrt{2-\sqrt{3}}\;c^{7/3}\left(c^{1/3}+d^{1/3}\,x\right)\right.$$

$$\left.\sqrt{\frac{c^{2/3}-c^{1/3}\,d^{1/3}\,x+d^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}\,x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x}\right],-7-4\sqrt{3}\right]\right)\Bigg/$$

$$\left(91\,d^{8/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x\right)^2}}\;\sqrt{c+d\,x^3}\right)-\left(12\,248\sqrt{2}\;c^{7/3}\left(c^{1/3}+d^{1/3}\,x\right)\right.$$

$$\left.\sqrt{\frac{c^{2/3}-c^{1/3}\,d^{1/3}\,x+d^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}\,x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x}\right],-7-4\sqrt{3}\right]\right)\Bigg/$$

$$\left(91\times 3^{1/4}\,d^{8/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x\right)^2}}\;\sqrt{c+d\,x^3}\right)$$

Result (type 6, 361 leaves):

$$\frac{1}{455\,d^2\sqrt{c+d\,x^3}}$$

$$2\,x^2\left(-5\left(107\,c^2+114\,c\,d\,x^3+7\,d^2\,x^6\right)+\left(171\,200\,c^4\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\Big/\right.$$

$$\left(\left(8\,c-d\,x^3\right)\left(40\,c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]+\right.\right.$$

$$\left.\left.3\,d\,x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]-4\,\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\right)\right)+$$

$$\left(195\,968\,c^3\,d\,x^3\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\Big/$$

$$\left(\left(8\,c-d\,x^3\right)\left(64\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]+3\,d\,x^3\right.\right.$$

$$\left.\left.\left.\left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]-4\,\text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\right)\right)\right)$$

Problem 290: Result unnecessarily involves higher level functions.

$$\int \frac{x^4 \sqrt{c+d x^3}}{8 c-d x^3} \, dx$$

Optimal (type 4, 624 leaves, 14 steps):

$$-\frac{2 x^2 \sqrt{c+d x^3}}{7 d}-\frac{118 c \sqrt{c+d x^3}}{7 d^{5/3} \left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)}-\frac{4 \sqrt{3}\, c^{7/6} \operatorname{ArcTan}\left[\frac{\sqrt{3}\, c^{1/6} \left(c^{1/3}+d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]}{d^{5/3}}+\frac{4 c^{7/6} \operatorname{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3} x\right)^2}{3 c^{1/6} \sqrt{c+d x^3}}\right]}{d^{5/3}}-\frac{4 c^{7/6} \operatorname{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{d^{5/3}}+\left(59 \times 3^{1/4} \sqrt{2-\sqrt{3}}\, c^{4/3} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\ \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Bigg/\left(7 d^{5/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\ \sqrt{c+d x^3}\right)-\left(118 \sqrt{2}\, c^{4/3} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\ \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Bigg/\left(7 \times 3^{1/4} d^{5/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\ \sqrt{c+d x^3}\right)$$

Result (type 6, 349 leaves):

$$\frac{1}{35\sqrt{c+d x^3}}2 x^2 \left(-\frac{5\left(c+d x^3\right)}{d}+\left(1600 c^3 \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(d\left(8 c-d x^3\right)\left(40 c\, \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\, \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)+\left(1888 c^2 x^3 \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\left(64 c\, \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\, \text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)$$

Problem 291: Result unnecessarily involves higher level functions.

$$\int \frac{x\sqrt{c+d x^3}}{8 c-d x^3}\,dx$$

Optimal (type 4, 601 leaves, 12 steps):

$$-\frac{2\sqrt{c+d x^3}}{d^{2/3}\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)}-\frac{\sqrt{3}\,c^{1/6}\,\text{ArcTan}\left[\frac{\sqrt{3}\,c^{1/6}\left(c^{1/3}+d^{1/3}x\right)}{\sqrt{c+d x^3}}\right]}{2 d^{2/3}}+\frac{c^{1/6}\,\text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3}x\right)^2}{3 c^{1/6}\sqrt{c+d x^3}}\right]}{2 d^{2/3}}-\frac{c^{1/6}\,\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3\sqrt{c}}\right]}{2 d^{2/3}}+\left(3^{1/4}\sqrt{2-\sqrt{3}}\,c^{1/3}\left(c^{1/3}+d^{1/3}x\right)\sqrt{\frac{c^{2/3}-c^{1/3}d^{1/3}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/\left(d^{2/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\sqrt{c+d x^3}\right)-\left(2\sqrt{2}\,c^{1/3}\left(c^{1/3}+d^{1/3}x\right)\sqrt{\frac{c^{2/3}-c^{1/3}d^{1/3}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/\left(3^{1/4}d^{2/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\sqrt{c+d x^3}\right)$$

Result (type 6, 168 leaves):

$$\left(20\,c\,x^2\sqrt{c+d\,x^3}\;\text{AppellF1}\left[\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\Big/\left(\left(8\,c-d\,x^3\right)\left(40\,c\,\text{AppellF1}\left[\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]+3\,d\,x^3\left(\text{AppellF1}\left[\frac{5}{3},-\frac{1}{2},2,\frac{8}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]+4\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\right)\right)$$

Problem 292: Result unnecessarily involves higher level functions.

$$\int\frac{\sqrt{c+d\,x^3}}{x^2\left(8\,c-d\,x^3\right)}\,dx$$

Optimal (type 4, 632 leaves, 14 steps):

$$-\frac{\sqrt{c+d\,x^3}}{8\,c\,x}+\frac{d^{1/3}\sqrt{c+d\,x^3}}{8\,c\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x\right)}-\frac{\sqrt{3}\,d^{1/3}\,\text{ArcTan}\left[\frac{\sqrt{3}\,c^{1/6}\left(c^{1/3}+d^{1/3}\,x\right)}{\sqrt{c+d\,x^3}}\right]}{16\,c^{5/6}}+\frac{d^{1/3}\,\text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3}\,x\right)^2}{3\,c^{1/6}\sqrt{c+d\,x^3}}\right]}{16\,c^{5/6}}-\frac{d^{1/3}\,\text{ArcTanh}\left[\frac{\sqrt{c+d\,x^3}}{3\sqrt{c}}\right]}{16\,c^{5/6}}-\left(3^{1/4}\sqrt{2-\sqrt{3}}\,d^{1/3}\left(c^{1/3}+d^{1/3}\,x\right)\sqrt{\frac{c^{2/3}-c^{1/3}\,d^{1/3}\,x+d^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}\,x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x}\right],-7-4\sqrt{3}\right]\right)\Big/\left(16\,c^{2/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x\right)^2}}\sqrt{c+d\,x^3}\right)+\left(d^{1/3}\left(c^{1/3}+d^{1/3}\,x\right)\sqrt{\frac{c^{2/3}-c^{1/3}\,d^{1/3}\,x+d^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}\,x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x}\right],-7-4\sqrt{3}\right]\right)\Big/\left(4\sqrt{2}\,3^{1/4}\,c^{2/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x\right)^2}}\sqrt{c+d\,x^3}\right)$$

Result (type 6, 345 leaves):

$$\frac{1}{40 x \sqrt{c+d x^3}}\left(-5-\frac{5 d x^3}{c}+\left(1300 c d x^3 \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\left(40 c \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)+\left(32 d^2 x^6 \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(-8 c+d x^3\right)\left(64 c \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)$$

Problem 293: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{c+d x^3}}{x^5\left(8 c-d x^3\right)}\,dx$$

Optimal (type 4, 654 leaves, 15 steps):

$$-\frac{\sqrt{c+d x^3}}{32 c x^4}-\frac{d \sqrt{c+d x^3}}{16 c^2 x}+\frac{d^{4/3} \sqrt{c+d x^3}}{16 c^2\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)}-\frac{\sqrt{3}\, d^{4/3} \text{ArcTan}\left[\frac{\sqrt{3}\, c^{1/6}\left(c^{1/3}+d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]}{128 c^{11/6}}+\frac{d^{4/3} \text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3} x\right)^2}{3 c^{1/6} \sqrt{c+d x^3}}\right]}{128 c^{11/6}}-\frac{d^{4/3} \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{128 c^{11/6}}-\left(3^{1/4} \sqrt{2-\sqrt{3}}\, d^{4/3}\left(c^{1/3}+d^{1/3} x\right)\sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\; \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/\left(32 c^{5/3} \sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\sqrt{c+d x^3}\right)+\left(d^{4/3}\left(c^{1/3}+d^{1/3} x\right)\sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\; \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/\left(8 \sqrt{2}\, 3^{1/4} c^{5/3} \sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\sqrt{c+d x^3}\right)$$

Result (type 6, 367 leaves):

$$\frac{1}{80\sqrt{c+d x^3}}\left(\left(625 d^2 x^2 \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\left(40 c \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)-\frac{1}{2 c^2 x^4}\left(5\left(c^2+3 c d x^3+2 d^2 x^6\right)+\left(64 c d^3 x^9 \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\left(64 c \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)\right)$$

Problem 294: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{c+d x^3}}{x^8\left(8 c-d x^3\right)}\, dx$$

Optimal (type 4, 678 leaves, 16 steps):

$$-\frac{\sqrt{c+d x^3}}{56 c x^7}-\frac{19 d \sqrt{c+d x^3}}{1792 c^2 x^4}+\frac{d^2 \sqrt{c+d x^3}}{112 c^3 x}-\frac{d^{7/3} \sqrt{c+d x^3}}{112 c^3 \left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)}-\frac{\sqrt{3}\, d^{7/3} \text{ArcTan}\left[\frac{\sqrt{3}\, c^{1/6} \left(c^{1/3}+d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]}{1024 c^{17/6}}+\frac{d^{7/3} \text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3} x\right)^2}{3 c^{1/6} \sqrt{c+d x^3}}\right]}{1024 c^{17/6}}-\frac{d^{7/3} \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{1024 c^{17/6}}+\left(3^{1/4} \sqrt{2-\sqrt{3}}\, d^{7/3} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right], -7-4 \sqrt{3}\right]\right) \Big/ \left(224 c^{8/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}\right)-\left(d^{7/3} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right], -7-4 \sqrt{3}\right]\right) \Big/ \left(56 \sqrt{2}\, 3^{1/4} c^{8/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}\right)$$

Result (type 6, 378 leaves):

$$\left(-5 \left(32 c^3+51 c^2 d x^3+3 c d^2 x^6-16 d^3 x^9\right)-\left(3250 c^2 d^3 x^9 \text{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right]\right) \Big/ \left(\left(8 c-d x^3\right) \left(40 c \text{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right]+3 d x^3 \left(\text{AppellF1}\left[\frac{5}{3}, \frac{1}{2}, 2, \frac{8}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{5}{3}, \frac{3}{2}, 1, \frac{8}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right]\right)\right)\right)+\left(512 c d^4 x^{12} \text{AppellF1}\left[\frac{5}{3}, \frac{1}{2}, 1, \frac{8}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right]\right) \Big/ \left(\left(8 c-d x^3\right) \left(64 c \text{AppellF1}\left[\frac{5}{3}, \frac{1}{2}, 1, \frac{8}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right]+3 d x^3 \left(\text{AppellF1}\left[\frac{8}{3}, \frac{1}{2}, 2, \frac{11}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{8}{3}, \frac{3}{2}, 1, \frac{11}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right]\right)\right)\right)\right) \Big/ \left(8960 c^3 x^7 \sqrt{c+d x^3}\right)$$

Problem 299: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(c+d\,x^3\right)^{3/2}}{x\,\left(8\,c-d\,x^3\right)}\,dx$$

Optimal (type 3, 73 leaves, 7 steps):

$$-\frac{2}{3}\sqrt{c+d\,x^3}+\frac{9}{4}\sqrt{c}\;\text{ArcTanh}\left[\frac{\sqrt{c+d\,x^3}}{3\,\sqrt{c}}\right]-\frac{1}{12}\sqrt{c}\;\text{ArcTanh}\left[\frac{\sqrt{c+d\,x^3}}{\sqrt{c}}\right]$$

Result (type 6, 319 leaves):

$$\frac{1}{9\,\sqrt{c+d\,x^3}}2\left(-3\,\left(c+d\,x^3\right)+\left(240\,c^2\,d\,x^3\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\Big/\left(\left(8\,c-d\,x^3\right)\left(16\,c\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]+d\,x^3\left(\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]-4\,\text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\right)\right)+\left(5\,c^2\,d\,x^3\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^3},\frac{8\,c}{d\,x^3}\right]\right)\Big/\left(\left(-8\,c+d\,x^3\right)\left(5\,d\,x^3\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^3},\frac{8\,c}{d\,x^3}\right]+16\,c\,\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d\,x^3},\frac{8\,c}{d\,x^3}\right]-c\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d\,x^3},\frac{8\,c}{d\,x^3}\right]\right)\right)\right)$$

Problem 300: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(c+d\,x^3\right)^{3/2}}{x^4\,\left(8\,c-d\,x^3\right)}\,dx$$

Optimal (type 3, 78 leaves, 7 steps):

$$-\frac{\sqrt{c+d\,x^3}}{24\,x^3}+\frac{9\,d\,\text{ArcTanh}\left[\frac{\sqrt{c+d\,x^3}}{3\,\sqrt{c}}\right]}{32\,\sqrt{c}}-\frac{13\,d\,\text{ArcTanh}\left[\frac{\sqrt{c+d\,x^3}}{\sqrt{c}}\right]}{96\,\sqrt{c}}$$

Result (type 6, 322 leaves):

$$\frac{1}{72 x^3 \sqrt{c+d x^3}}\left(-3\left(c+d x^3\right)+\left(408 c d^2 x^6 \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\left(16 c \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+d x^3\left(\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)+\left(130 c d^2 x^6 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]\right)\Big/\left(\left(-8 c+d x^3\right)\left(5 d x^3 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]+16 c \text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]-c \text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]\right)\right)\right)$$

Problem 301: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(c+d x^3\right)^{3/2}}{x^7\left(8 c-d x^3\right)} \, dx$$

Optimal (type 3, 104 leaves, 8 steps):

$$-\frac{\sqrt{c+d x^3}}{48 x^6}-\frac{11 d \sqrt{c+d x^3}}{192 c x^3}+\frac{9 d^2 \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{256 c^{3/2}}-\frac{37 d^2 \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{768 c^{3/2}}$$

Result (type 6, 332 leaves):

$$\frac{1}{288 \sqrt{c+d x^3}}\left(-\frac{33 d^2}{2 c}-\frac{6 c}{x^6}-\frac{45 d}{2 x^3}+\left(132 d^3 x^3 \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\left(16 c \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+d x^3\left(\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)+\left(185 d^3 x^3 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]\right)\Big/\left(\left(-8 c+d x^3\right)\left(5 d x^3 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]+16 c \text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]-c \text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]\right)\right)\right)$$

Problem 302: Result unnecessarily involves higher level functions.

$$\int \frac{x^7\left(c+d x^3\right)^{3/2}}{8 c-d x^3} \, dx$$

Optimal (type 4, 669 leaves, 16 steps):

$$-\frac{36534\,c^2 x^2\sqrt{c+d x^3}}{1729\,d^2}-\frac{348\,c\,x^5\sqrt{c+d x^3}}{247\,d}-\frac{2}{19}x^8\sqrt{c+d x^3}-\frac{2094648\,c^3\sqrt{c+d x^3}}{1729\,d^{8/3}\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)}-\frac{288\sqrt{3}\,c^{19/6}\,\text{ArcTan}\left[\frac{\sqrt{3}\,c^{1/6}\left(c^{1/3}+d^{1/3}x\right)}{\sqrt{c+d x^3}}\right]}{d^{8/3}}+\frac{288\,c^{19/6}\,\text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3}x\right)^2}{3\,c^{1/6}\sqrt{c+d x^3}}\right]}{d^{8/3}}-\frac{288\,c^{19/6}\,\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3\sqrt{c}}\right]}{d^{8/3}}+\left(1047324\times 3^{1/4}\sqrt{2-\sqrt{3}}\,c^{10/3}\left(c^{1/3}+d^{1/3}x\right)\sqrt{\frac{c^{2/3}-c^{1/3}d^{1/3}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/\left(1729\,d^{8/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\sqrt{c+d x^3}\right)-\left(698216\sqrt{2}\,3^{3/4}\,c^{10/3}\left(c^{1/3}+d^{1/3}x\right)\sqrt{\frac{c^{2/3}-c^{1/3}d^{1/3}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/\left(1729\,d^{8/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\sqrt{c+d x^3}\right)$$

Result (type 6, 371 leaves):

$$\frac{1}{8645\sqrt{c+d x^3}}2x^2\left(-\frac{91335\,c^3}{d^2}-\frac{97425\,c^2x^3}{d}-6545\,c\,x^6-455\,d\,x^9+\left(29227200\,c^5\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8c}\right]\right)\Big/\left(d^2\left(8c-d x^3\right)\left(40\,c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8c}\right]+3\,d\,x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8c}\right]-4\,\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8c}\right]\right)\right)\right)+\left(33514368\,c^4x^3\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8c}\right]\right)\Big/\left(d\left(8c-d x^3\right)\left(64\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8c}\right]+3\,d\,x^3\left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8c}\right]-4\,\text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8c}\right]\right)\right)\right)\right)$$

Problem 303: Result unnecessarily involves higher level functions.

$$\int \frac{x^4 \left(c+d x^3\right)^{3/2}}{8 c-d x^3} \, dx$$

Optimal (type 4, 645 leaves, 15 steps):

$$-\frac{240 c x^2 \sqrt{c+d x^3}}{91 d}-\frac{2}{13} x^5 \sqrt{c+d x^3}-\frac{13782 c^2 \sqrt{c+d x^3}}{91 d^{5/3} \left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)}-\frac{36 \sqrt{3} c^{13/6} \text{ArcTan}\left[\frac{\sqrt{3} c^{1/6} \left(c^{1/3}+d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]}{d^{5/3}}+\frac{36 c^{13/6} \text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3} x\right)^2}{3 c^{1/6} \sqrt{c+d x^3}}\right]}{d^{5/3}}-\frac{36 c^{13/6} \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{d^{5/3}}+\left(6891 \times 3^{1/4} \sqrt{2-\sqrt{3}} c^{7/3} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]\right) \Big/ \left(91 d^{5/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}\right)-\left(4594 \sqrt{2} \, 3^{3/4} c^{7/3} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]\right) \Big/ \left(91 d^{5/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}\right)$$

Result (type 6, 357 leaves):

$$\frac{1}{455\sqrt{c+d x^3}}2 x^2\left(-\frac{600 c^2}{d}-635 c x^3-35 d x^6+\left(192000 c^4 \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(d\left(8 c-d x^3\right)\left(40 c \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)+\left(220512 c^3 x^3 \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\left(64 c \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)$$

Problem 304: Result unnecessarily involves higher level functions.

$$\int \frac{x\left(c+d x^3\right)^{3/2}}{8 c-d x^3}\,dx$$

Optimal (type 4, 627 leaves, 14 steps):

$$-\frac{2}{7} x^2 \sqrt{c+d x^3}-\frac{132 c \sqrt{c+d x^3}}{7 d^{2/3}\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)}-\frac{9 \sqrt{3}\, c^{7/6} \text{ArcTan}\left[\frac{\sqrt{3}\, c^{1/6}\left(c^{1/3}+d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]}{2 d^{2/3}}+\frac{9 c^{7/6} \text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3} x\right)^2}{3 c^{1/6}\sqrt{c+d x^3}}\right]}{2 d^{2/3}}-\frac{9 c^{7/6} \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3\sqrt{c}}\right]}{2 d^{2/3}}+\left(66\times 3^{1/4}\sqrt{2-\sqrt{3}}\, c^{4/3}\left(c^{1/3}+d^{1/3} x\right)\sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4\sqrt{3}\right]\right)\Big/\left(7 d^{2/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\sqrt{c+d x^3}\right)-\left(44\sqrt{2}\, 3^{3/4} c^{4/3}\left(c^{1/3}+d^{1/3} x\right)\sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4\sqrt{3}\right]\right)\Big/\left(7 d^{2/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\sqrt{c+d x^3}\right)$$

Result (type 6, 344 leaves):

$$\frac{1}{35\sqrt{c+d x^3}}2 x^2 \left(-5 \left(c+d x^3\right)+\left(1950 c^3 \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right) \left(40 c \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3 \left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)+\left(2112 c^2 d x^3 \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right) \left(64 c \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3 \left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)$$

Problem 305: Result unnecessarily involves higher level functions.

$$\int \frac{\left(c+d x^3\right)^{3/2}}{x^2 \left(8 c-d x^3\right)} \, dx$$

Optimal (type 4, 626 leaves, 14 steps):

$$-\frac{\sqrt{c+d x^3}}{8 x}-\frac{15 d^{1/3} \sqrt{c+d x^3}}{8 \left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)}-\frac{9}{16} \sqrt{3}\, c^{1/6} d^{1/3} \text{ArcTan}\left[\frac{\sqrt{3}\, c^{1/6} \left(c^{1/3}+d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]+\frac{9}{16} c^{1/6} d^{1/3} \text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3} x\right)^2}{3 c^{1/6} \sqrt{c+d x^3}}\right]-\frac{9}{16} c^{1/6} d^{1/3} \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]+\left(15\times 3^{1/4} \sqrt{2-\sqrt{3}}\, c^{1/3} d^{1/3} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/\left(16 \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}\right)-\left(5\times 3^{3/4} c^{1/3} d^{1/3} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/\left(4 \sqrt{2} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}\right)$$

Result (type 6, 348 leaves):

$$\frac{1}{8 x \sqrt{c+d x^3}}\left(-c-d x^3+\left(420 c^2 d x^3 \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\left(40 c \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)+\left(96 c d^2 x^6 \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\left(64 c \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)$$

Problem 306: Result unnecessarily involves higher level functions.

$$\int \frac{\left(c+d x^3\right)^{3/2}}{x^5 \left(8 c-d x^3\right)} \, dx$$

Optimal (type 4, 651 leaves, 15 steps):

$$-\frac{\sqrt{c+d x^3}}{32 x^4}-\frac{3 d \sqrt{c+d x^3}}{16 c x}+\frac{3 d^{4/3} \sqrt{c+d x^3}}{16 c \left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)}-\frac{9 \sqrt{3}\, d^{4/3}\, \text{ArcTan}\left[\frac{\sqrt{3}\, c^{1/6} \left(c^{1/3}+d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]}{128 c^{5/6}}+\frac{9 d^{4/3}\, \text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3} x\right)^2}{3 c^{1/6} \sqrt{c+d x^3}}\right]}{128 c^{5/6}}-\frac{9 d^{4/3}\, \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{128 c^{5/6}}-\left(3\times 3^{1/4} \sqrt{2-\sqrt{3}}\, d^{4/3} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/\left(32 c^{2/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}\right)+\left(3^{3/4} d^{4/3} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/\left(8 \sqrt{2}\, c^{2/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}\right)$$

Result (type 6, 363 leaves):

$$\frac{1}{80 \sqrt{c+d x^3}}\left(-\frac{5 \left(c^2+7 c d x^3+6 d^2 x^6\right)}{2 c x^4}+\left(3225 c d^2 x^2\, \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right) \left(40 c\, \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3 \left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\, \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)+\left(96 d^3 x^5\, \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(-8 c+d x^3\right) \left(64 c\, \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3 \left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\, \text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)$$

Problem 307: Result unnecessarily involves higher level functions.

$$\int \frac{\left(c+d x^3\right)^{3/2}}{x^8 \left(8 c-d x^3\right)} \, dx$$

Optimal (type 4, 675 leaves, 16 steps):

$$-\frac{\sqrt{c+d x^3}}{56 x^7}-\frac{75 d \sqrt{c+d x^3}}{1792 c x^4}-\frac{3 d^2 \sqrt{c+d x^3}}{56 c^2 x}+\frac{3 d^{7/3} \sqrt{c+d x^3}}{56 c^2 \left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)}-\frac{9 \sqrt{3}\, d^{7/3} \text{ArcTan}\left[\frac{\sqrt{3}\, c^{1/6} \left(c^{1/3}+d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]}{1024 c^{11/6}}+\frac{9 d^{7/3} \text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3} x\right)^2}{3 c^{1/6} \sqrt{c+d x^3}}\right]}{1024 c^{11/6}}-\frac{9 d^{7/3} \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{1024 c^{11/6}}-\left(3\times 3^{1/4} \sqrt{2-\sqrt{3}}\, d^{7/3} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right], -7-4 \sqrt{3}\right]\right) \Big/ \left(112 c^{5/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}\right)+\left(3^{3/4} d^{7/3} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right], -7-4 \sqrt{3}\right]\right) \Big/ \left(28 \sqrt{2}\, c^{5/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}\right)$$

Result (type 6, 379 leaves):

$$\frac{1}{4480 \sqrt{c+d x^3}}\left(-\frac{5 \left(32 c^3+107 c^2 d x^3+171 c d^2 x^6+96 d^3 x^9\right)}{2 c^2 x^7}+\left(33375 d^3 x^2 \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right) \Big/ \left(\left(8 c-d x^3\right) \left(40 c \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3 \left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)-\left(1536 d^4 x^5 \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right) \Big/ \left(c \left(8 c-d x^3\right) \left(64 c \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3 \left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)$$

Problem 312: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{x\left(8 c-d x^3\right) \sqrt{c+d x^3}} \, dx$$

Optimal (type 3, 58 leaves, 6 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{36 c^{3/2}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{12 c^{3/2}}$$

Result (type 6, 161 leaves):

$$\left(10 d x^3 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]\right) \Big/ \left(9 \left(-8 c+d x^3\right) \sqrt{c+d x^3} \left(5 d x^3 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]+16 c \text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]-c \text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]\right)\right)$$

Problem 313: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{x^4\left(8 c-d x^3\right) \sqrt{c+d x^3}} \, dx$$

Optimal (type 3, 81 leaves, 7 steps):

$$-\frac{\sqrt{c+d x^3}}{24 c^2 x^3}+\frac{d \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{288 c^{5/2}}+\frac{d \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{32 c^{5/2}}$$

Result (type 6, 326 leaves):

$$\frac{1}{24 c^2 x^3 \sqrt{c+d x^3}}\left(-c-d x^3+\left(8 c d^2 x^6 \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right) \Big/ \left(\left(8 c-d x^3\right) \left(16 c \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+d x^3 \left(\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)+\left(10 c d^2 x^6 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]\right) \Big/ \left(\left(8 c-d x^3\right) \left(5 d x^3 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]+16 c \text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]-c \text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]\right)\right)\right)$$

Problem 314: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{x^7 \left(8 c - d x^3\right) \sqrt{c + d x^3}} \, dx$$

Optimal (type 3, 107 leaves, 8 steps):

$$-\frac{\sqrt{c+d x^3}}{48 c^2 x^6}+\frac{5 d \sqrt{c+d x^3}}{192 c^3 x^3}+\frac{d^2 \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{2304 c^{7/2}}-\frac{7 d^2 \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{256 c^{7/2}}$$

Result (type 6, 332 leaves):

$$\frac{1}{192 c^3 \sqrt{c+d x^3}}\left(5 d^2-\frac{4 c^2}{x^6}+\frac{c d}{x^3}-\left(40 c d^3 x^3 \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\right.$$
$$\left(\left(8 c-d x^3\right)\left(16 c \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+\right.\right.$$
$$\left.\left.d x^3\left(\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)+$$
$$\left(70 c d^3 x^3 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]\right)\Big/$$
$$\left(\left(-8 c+d x^3\right)\left(5 d x^3 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]+\right.\right.$$
$$\left.\left.\left.16 c \text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]-c \text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]\right)\right)\right)$$

Problem 315: Result unnecessarily involves higher level functions.

$$\int \frac{x^7}{\left(8 c - d x^3\right) \sqrt{c + d x^3}} \, dx$$

Optimal (type 4, 630 leaves, 14 steps):

$$-\frac{2 x^2 \sqrt{c+d x^3}}{7 d^2}-\frac{104 c \sqrt{c+d x^3}}{7 d^{8/3} \left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)}-\frac{32 c^{7/6} \text{ArcTan}\left[\frac{\sqrt{3} c^{1/6} \left(c^{1/3}+d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]}{3 \sqrt{3} d^{8/3}}+$$

$$\frac{32 c^{7/6} \text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3} x\right)^2}{3 c^{1/6} \sqrt{c+d x^3}}\right]}{9 d^{8/3}}-\frac{32 c^{7/6} \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{9 d^{8/3}}+$$

$$\left(52\times 3^{1/4} \sqrt{2-\sqrt{3}}\; c^{4/3} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$

$$\left(7 d^{8/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}\right)-\left(104 \sqrt{2}\; c^{4/3} \left(c^{1/3}+d^{1/3} x\right)\right.$$

$$\left.\sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\; \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$

$$\left(7\times 3^{1/4} d^{8/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}\right)$$

Result (type 6, 347 leaves):

$$\frac{1}{35 d^2 \sqrt{c+d x^3}} 2 x^2 \left(-5 \left(c+d x^3\right)+\left(1600 c^3 \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\right.$$

$$\left(\left(8 c-d x^3\right) \left(40 c\, \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+\right.\right.$$

$$\left.\left.3 d x^3 \left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\, \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)+$$

$$\left(1664 c^2 d x^3 \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/$$

$$\left(\left(8 c-d x^3\right) \left(64 c\, \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\right.\right.$$

$$\left.\left.\left.\left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\, \text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)$$

Problem 316: Result unnecessarily involves higher level functions.

$$\int \frac{x^4}{\left(8 c-d x^3\right) \sqrt{c+d x^3}}\, dx$$

Optimal (type 4, 601 leaves, 12 steps):

$$-\frac{2 \sqrt{c+d x^3}}{d^{5/3} \left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)}-\frac{4 c^{1/6} \text{ArcTan}\left[\frac{\sqrt{3}\, c^{1/6} \left(c^{1/3}+d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]}{3 \sqrt{3}\, d^{5/3}}+\frac{4 c^{1/6} \text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3} x\right)^2}{3 c^{1/6} \sqrt{c+d x^3}}\right]}{9 d^{5/3}}-\frac{4 c^{1/6} \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{9 d^{5/3}}+\left(3^{1/4} \sqrt{2-\sqrt{3}}\, c^{1/3} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/\left(d^{5/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\, \sqrt{c+d x^3}\right)-\left(2 \sqrt{2}\, c^{1/3} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/\left(3^{1/4} d^{5/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\, \sqrt{c+d x^3}\right)$$

Result (type 6, 170 leaves):

$$\left(64 c x^5 \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(5 \left(8 c-d x^3\right) \sqrt{c+d x^3} \left(64 c \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3 \left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)$$

Problem 317: Result unnecessarily involves higher level functions.

$$\int \frac{x}{\left(8 c-d x^3\right) \sqrt{c+d x^3}}\, dx$$

Optimal (type 3, 141 leaves, 8 steps):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{3}\,c^{1/6}\left(c^{1/3}+d^{1/3}x\right)}{\sqrt{c+d x^3}}\right]}{6\sqrt{3}\,c^{5/6}\,d^{2/3}}+\frac{\text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3}x\right)^2}{3\,c^{1/6}\sqrt{c+d x^3}}\right]}{18\,c^{5/6}\,d^{2/3}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3\sqrt{c}}\right]}{18\,c^{5/6}\,d^{2/3}}$$

Result (type 6, 168 leaves):

$$\left(20\,c\,x^2\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/$$
$$\left(\left(8 c-d x^3\right)\sqrt{c+d x^3}\,\left(40\,c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+\right.\right.$$
$$\left.\left.3\,d\,x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\,\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)$$

Problem 318: Result unnecessarily involves higher level functions.

$$\int\frac{1}{x^2\left(8 c-d x^3\right)\sqrt{c+d x^3}}\,dx$$

Optimal (type 4, 632 leaves, 14 steps):

$$-\frac{\sqrt{c+d x^3}}{8\,c^2\,x}+\frac{d^{1/3}\sqrt{c+d x^3}}{8\,c^2\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)}-$$
$$\frac{d^{1/3}\,\text{ArcTan}\left[\frac{\sqrt{3}\,c^{1/6}\left(c^{1/3}+d^{1/3}x\right)}{\sqrt{c+d x^3}}\right]}{48\sqrt{3}\,c^{11/6}}+\frac{d^{1/3}\,\text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3}x\right)^2}{3\,c^{1/6}\sqrt{c+d x^3}}\right]}{144\,c^{11/6}}-\frac{d^{1/3}\,\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3\sqrt{c}}\right]}{144\,c^{11/6}}-$$
$$\left(3^{1/4}\sqrt{2-\sqrt{3}}\,d^{1/3}\left(c^{1/3}+d^{1/3}x\right)\sqrt{\frac{c^{2/3}-c^{1/3}d^{1/3}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\right.$$
$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$
$$\left(16\,c^{5/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\sqrt{c+d x^3}\right)+\left(d^{1/3}\left(c^{1/3}+d^{1/3}x\right)\right.$$
$$\left.\sqrt{\frac{c^{2/3}-c^{1/3}d^{1/3}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$
$$\left(4\sqrt{2}\,3^{1/4}\,c^{5/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\sqrt{c+d x^3}\right)$$

Result (type 6, 350 leaves):

$$\frac{1}{40\,x\,\sqrt{c+d\,x^3}}\left(\left(500\,d\,x^3\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\Big/\left(\left(8\,c-d\,x^3\right)\left(40\,c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]+3\,d\,x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]-4\,\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\right)\right)+\frac{1}{c^2}\left(-5\,\left(c+d\,x^3\right)-\left(32\,c\,d^2\,x^6\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\Big/\left(\left(8\,c-d\,x^3\right)\left(64\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]+3\,d\,x^3\left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]-4\,\text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\right)\right)\right)\right)$$

Problem 319: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^5\,\left(8\,c-d\,x^3\right)\,\sqrt{c+d\,x^3}}\,dx$$

Optimal (type 4, 654 leaves, 15 steps):

$$-\frac{\sqrt{c+d\,x^3}}{32\,c^2\,x^4}+\frac{d\,\sqrt{c+d\,x^3}}{16\,c^3\,x}-\frac{d^{4/3}\,\sqrt{c+d\,x^3}}{16\,c^3\,\left(\left(1+\sqrt{3}\right)\,c^{1/3}+d^{1/3}\,x\right)}-\frac{d^{4/3}\,\text{ArcTan}\left[\frac{\sqrt{3}\,c^{1/6}\,\left(c^{1/3}+d^{1/3}\,x\right)}{\sqrt{c+d\,x^3}}\right]}{384\,\sqrt{3}\,c^{17/6}}+\frac{d^{4/3}\,\text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3}\,x\right)^2}{3\,c^{1/6}\,\sqrt{c+d\,x^3}}\right]}{1152\,c^{17/6}}-\frac{d^{4/3}\,\text{ArcTanh}\left[\frac{\sqrt{c+d\,x^3}}{3\,\sqrt{c}}\right]}{1152\,c^{17/6}}+$$

$$\left(3^{1/4}\,\sqrt{2-\sqrt{3}}\,d^{4/3}\,\left(c^{1/3}+d^{1/3}\,x\right)\,\sqrt{\frac{c^{2/3}-c^{1/3}\,d^{1/3}\,x+d^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,c^{1/3}+d^{1/3}\,x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,c^{1/3}+d^{1/3}\,x}{\left(1+\sqrt{3}\right)\,c^{1/3}+d^{1/3}\,x}\right],-7-4\,\sqrt{3}\right]\right)\Big/\left(32\,c^{8/3}\,\sqrt{\frac{c^{1/3}\,\left(c^{1/3}+d^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,c^{1/3}+d^{1/3}\,x\right)^2}}\,\sqrt{c+d\,x^3}\right)-$$

$$\left(d^{4/3}\,\left(c^{1/3}+d^{1/3}\,x\right)\,\sqrt{\frac{c^{2/3}-c^{1/3}\,d^{1/3}\,x+d^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,c^{1/3}+d^{1/3}\,x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,c^{1/3}+d^{1/3}\,x}{\left(1+\sqrt{3}\right)\,c^{1/3}+d^{1/3}\,x}\right],-7-4\,\sqrt{3}\right]\right)\Big/\left(8\,\sqrt{2}\,3^{1/4}\,c^{8/3}\,\sqrt{\frac{c^{1/3}\,\left(c^{1/3}+d^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,c^{1/3}+d^{1/3}\,x\right)^2}}\,\sqrt{c+d\,x^3}\right)$$

Result (type 6, 364 leaves) :

$$\left(-5 c^2+5 c d x^3+10 d^2 x^6-\left(750 c^2 d^2 x^6 \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\left(40 c \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)+\left(64 c d^3 x^9 \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\left(64 c \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)\Big/\left(160 c^3 x^4 \sqrt{c+d x^3}\right)$$

Problem 320: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^8 \left(8 c-d x^3\right) \sqrt{c+d x^3}}\, dx$$

Optimal (type 4, 678 leaves, 16 steps) :

$$-\frac{\sqrt{c+d x^3}}{56 c^2 x^7}+\frac{37 d \sqrt{c+d x^3}}{1792 c^3 x^4}-\frac{3 d^2 \sqrt{c+d x^3}}{56 c^4 x}+\frac{3 d^{7/3} \sqrt{c+d x^3}}{56 c^4 \left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)}-$$

$$\frac{d^{7/3} \operatorname{ArcTan}\left[\frac{\sqrt{3}\, c^{1/6}\left(c^{1/3}+d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]}{3072 \sqrt{3}\, c^{23/6}}+\frac{d^{7/3} \operatorname{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3} x\right)^2}{3 c^{1/6} \sqrt{c+d x^3}}\right]}{9216 c^{23/6}}-$$

$$\frac{d^{7/3} \operatorname{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{9216 c^{23/6}}-\left(3\times 3^{1/4} \sqrt{2-\sqrt{3}}\, d^{7/3} \left(c^{1/3}+d^{1/3} x\right)\right.$$

$$\left.\sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\ \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$

$$\left(112 c^{11/3} \sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\ \sqrt{c+d x^3}\right)+\left(3^{3/4} d^{7/3} \left(c^{1/3}+d^{1/3} x\right)\right.$$

$$\left.\sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\ \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$

$$\left(28 \sqrt{2}\, c^{11/3} \sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\ \sqrt{c+d x^3}\right)$$

Result (type 6, 378 leaves):

$$\left(-5 \left(32 c^3-5 c^2 d x^3+59 c d^2 x^6+96 d^3 x^9\right)+\left(38750 c^2 d^3 x^9 \operatorname{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\right.$$

$$\left(\left(8 c-d x^3\right) \left(40 c \operatorname{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+\right.\right.$$

$$\left.\left.3 d x^3 \left(\operatorname{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \operatorname{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)-$$

$$\left(3072 c d^4 x^{12} \operatorname{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\right.$$

$$\left(64 c \operatorname{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3 \left(\operatorname{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-\right.\right.$$

$$\left.\left.\left.\left.4 \operatorname{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)\Big/\left(8960 c^4 x^7 \sqrt{c+d x^3}\right)$$

Problem 321: Result more than twice size of optimal antiderivative.

$$\int \frac{x^3}{\left(8 c-d x^3\right) \sqrt{c+d x^3}}\, dx$$

Optimal (type 6, 66 leaves, 2 steps):

$$\frac{x^4 \sqrt{1+\frac{d x^3}{c}}\ \text{AppellF1}\left[\frac{4}{3},1,\frac{1}{2},\frac{7}{3},\frac{d x^3}{8 c},-\frac{d x^3}{c}\right]}{32 c \sqrt{c+d x^3}}$$

Result (type 6, 168 leaves):

$$\left(14 c x^4 \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/ \left(\left(8 c-d x^3\right) \sqrt{c+d x^3} \left(56 c \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3 \left(\text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)$$

Problem 322: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(8 c-d x^3\right) \sqrt{c+d x^3}}\, dx$$

Optimal (type 6, 64 leaves, 2 steps):

$$\frac{x \sqrt{1+\frac{d x^3}{c}}\ \text{AppellF1}\left[\frac{1}{3},1,\frac{1}{2},\frac{4}{3},\frac{d x^3}{8 c},-\frac{d x^3}{c}\right]}{8 c \sqrt{c+d x^3}}$$

Result (type 6, 166 leaves):

$$\left(32 c x \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/ \left(\left(8 c-d x^3\right) \sqrt{c+d x^3} \left(32 c \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3 \left(\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)$$

Problem 323: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^3 \left(8 c-d x^3\right) \sqrt{c+d x^3}}\, dx$$

Optimal (type 6, 66 leaves, 2 steps):

$$-\frac{\sqrt{1+\frac{d x^3}{c}}\ \text{AppellF1}\left[-\frac{2}{3},1,\frac{1}{2},\frac{1}{3},\frac{d x^3}{8 c},-\frac{d x^3}{c}\right]}{16 c x^2 \sqrt{c+d x^3}}$$

Result (type 6, 347 leaves):

$$\frac{1}{16 x^2 \sqrt{c+d x^3}}\left(\left(64 d x^3 \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(-8 c+d x^3\right)\left(32 c\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\,\text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)-\frac{1}{c^2}\left(c+d x^3-\left(7 c d^2 x^6\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\left(56 c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\,\text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)\right)$$

Problem 324: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^6 \left(8 c-d x^3\right) \sqrt{c+d x^3}}\,dx$$

Optimal (type 6, 66 leaves, 2 steps):

$$-\frac{\sqrt{1+\frac{d x^3}{c}}\,\text{AppellF1}\left[-\frac{5}{3},1,\frac{1}{2},-\frac{2}{3},\frac{d x^3}{8 c},-\frac{d x^3}{c}\right]}{40 c x^5 \sqrt{c+d x^3}}$$

Result (type 6, 364 leaves):

$$\left(-16 c^2+7 c d x^3+23 d^2 x^6+\left(3264 c^2 d^2 x^6\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\left(32 c\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\,\text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)-\left(161 c d^3 x^9\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\left(56 c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\,\text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)\Big/\left(640 c^3 x^5 \sqrt{c+d x^3}\right)$$

Problem 329: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{x \left(8 c-d x^3\right) \left(c+d x^3\right)^{3/2}}\,dx$$

Optimal (type 3, 76 leaves, 7 steps):

$$\frac{2}{27 c^2 \sqrt{c+d x^3}}+\frac{\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{324 c^{5/2}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{12 c^{5/2}}$$

Result (type 6, 310 leaves):

$$\frac{1}{27 c^2 \sqrt{c+d x^3}} 2 \left(1-\left(8 c d x^3 \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\right.$$
$$\left(\left(8 c-d x^3\right)\left(16 c \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+\right.\right.$$
$$\left.\left.d x^3\left(\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)+$$
$$\left(15 c d x^3 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]\right)\Big/$$
$$\left(\left(-8 c+d x^3\right)\left(5 d x^3 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]+\right.\right.$$
$$\left.\left.\left.16 c \text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]-c \text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]\right)\right)\right)$$

Problem 330: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{x^4 \left(8 c-d x^3\right) \left(c+d x^3\right)^{3/2}} \, dx$$

Optimal (type 3, 100 leaves, 8 steps):

$$-\frac{25 d}{216 c^3 \sqrt{c+d x^3}}-\frac{1}{24 c^2 x^3 \sqrt{c+d x^3}}+\frac{d \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{2592 c^{7/2}}+\frac{11 d \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{96 c^{7/2}}$$

Result (type 6, 326 leaves):

$$\left(-9 c-25 d x^3+\left(200 c d^2 x^6 \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\right.$$
$$\left(\left(8 c-d x^3\right)\left(16 c \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+\right.\right.$$
$$\left.\left.d x^3\left(\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)+$$
$$\left(330 c d^2 x^6 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]\right)\Big/\left(\left(8 c-d x^3\right)\right.$$
$$\left(5 d x^3 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]+16 c \text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]-\right.$$
$$\left.\left.\left.c \text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]\right)\right)\right)\Big/\left(216 c^3 x^3 \sqrt{c+d x^3}\right)$$

Problem 331: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{x^7 \left(8 c - d x^3\right) \left(c + d x^3\right)^{3/2}} \, dx$$

Optimal (type 3, 128 leaves, 9 steps):

$$\frac{245 d^2}{1728 c^4 \sqrt{c + d x^3}} - \frac{1}{48 c^2 x^6 \sqrt{c + d x^3}} + \frac{3 d}{64 c^3 x^3 \sqrt{c + d x^3}} + \frac{d^2 \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{20736 c^{9/2}} - \frac{109 d^2 \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{768 c^{9/2}}$$

Result (type 6, 336 leaves):

$$\left(-36 c^2 + 81 c d x^3 + 245 d^2 x^6 - \left(1960 c d^3 x^9 \text{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right]\right) \Big/ \left(\left(8 c - d x^3\right) \left(16 c \text{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right] + d x^3 \left(\text{AppellF1}\left[2, \frac{1}{2}, 2, 3, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right] - 4 \text{AppellF1}\left[2, \frac{3}{2}, 1, 3, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right]\right)\right)\right) + \left(3270 c d^3 x^9 \text{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, 1, \frac{5}{2}, -\frac{c}{d x^3}, \frac{8 c}{d x^3}\right]\right) \Big/ \left(\left(-8 c + d x^3\right) \left(5 d x^3 \text{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, 1, \frac{5}{2}, -\frac{c}{d x^3}, \frac{8 c}{d x^3}\right] + 16 c \text{AppellF1}\left[\frac{5}{2}, \frac{1}{2}, 2, \frac{7}{2}, -\frac{c}{d x^3}, \frac{8 c}{d x^3}\right] - c \text{AppellF1}\left[\frac{5}{2}, \frac{3}{2}, 1, \frac{7}{2}, -\frac{c}{d x^3}, \frac{8 c}{d x^3}\right]\right)\right)\right) \Big/ \left(1728 c^4 x^6 \sqrt{c + d x^3}\right)$$

Problem 332: Result unnecessarily involves higher level functions.

$$\int \frac{x^7}{\left(8 c - d x^3\right) \left(c + d x^3\right)^{3/2}} \, dx$$

Optimal (type 4, 629 leaves, 14 steps):

$$\frac{2x^2}{27 d^2 \sqrt{c+d x^3}} - \frac{56\sqrt{c+d x^3}}{27 d^{8/3}\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)} - \frac{32 c^{1/6}\,\text{ArcTan}\left[\frac{\sqrt{3}\,c^{1/6}\left(c^{1/3}+d^{1/3}x\right)}{\sqrt{c+d x^3}}\right]}{27\sqrt{3}\,d^{8/3}} + \frac{32 c^{1/6}\,\text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3}x\right)^2}{3 c^{1/6}\sqrt{c+d x^3}}\right]}{81 d^{8/3}} - \frac{32 c^{1/6}\,\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3\sqrt{c}}\right]}{81 d^{8/3}} + \left(28\sqrt{2-\sqrt{3}}\,c^{1/3}\left(c^{1/3}+d^{1/3}x\right)\sqrt{\frac{c^{2/3}-c^{1/3}d^{1/3}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x}\right],-7-4\sqrt{3}\right]\right) \Big/ \left(9\times 3^{3/4} d^{8/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\sqrt{c+d x^3}\right) - \left(56\sqrt{2}\,c^{1/3}\left(c^{1/3}+d^{1/3}x\right)\sqrt{\frac{c^{2/3}-c^{1/3}d^{1/3}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x}\right],-7-4\sqrt{3}\right]\right) \Big/ \left(27\times 3^{1/4} d^{8/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\sqrt{c+d x^3}\right)$$

Result (type 6, 337 leaves):

$$\frac{1}{135 d^2\sqrt{c+d x^3}} 2x^2\left(5-\left(1600 c^2\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8c}\right]\right)\Big/\left(\left(8c-d x^3\right)\left(40 c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8c}\right]-4\,\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8c}\right]\right)\right)\right)+\left(896 c d x^3\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8c}\right]\right)\Big/\left(\left(8c-d x^3\right)\left(64 c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8c}\right]-4\,\text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8c}\right]\right)\right)\right)\right)$$

Problem 333: Result unnecessarily involves higher level functions.

$$\int \frac{x^4}{\left(8c-d x^3\right)\left(c+d x^3\right)^{3/2}}\,dx$$

Optimal (type 4, 635 leaves, 14 steps):

$$-\frac{2 x^2}{27 c d \sqrt{c+d x^3}}+\frac{2 \sqrt{c+d x^3}}{27 c d^{5/3} \left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)}-\frac{4 \text{ArcTan}\left[\frac{\sqrt{3} c^{1/6} \left(c^{1/3}+d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]}{27 \sqrt{3} c^{5/6} d^{5/3}}+\frac{4 \text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3} x\right)^2}{3 c^{1/6} \sqrt{c+d x^3}}\right]}{81 c^{5/6} d^{5/3}}-\frac{4 \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{81 c^{5/6} d^{5/3}}-\left(\sqrt{2-\sqrt{3}} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/\left(9\times 3^{3/4} c^{2/3} d^{5/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}\right)+\left(2 \sqrt{2} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/\left(27\times 3^{1/4} c^{2/3} d^{5/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}\right)$$

Result (type 6, 340 leaves):

$$\frac{1}{135 \sqrt{c+d x^3}} 2 x^2 \left(-\frac{5}{c d}+\left(1600 c \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(d \left(8 c-d x^3\right) \left(40 c \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3 \left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)-\left(32 x^3 \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right) \left(64 c \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3 \left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)$$

Problem 334: Result unnecessarily involves higher level functions.

$$\int \frac{x}{\left(8 c-d x^3\right) \left(c+d x^3\right)^{3/2}} \, dx$$

Optimal (type 4, 632 leaves, 14 steps):

$$\frac{2 x^2}{27 c^2 \sqrt{c+d x^3}}-\frac{2 \sqrt{c+d x^3}}{27 c^2 d^{2/3} \left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)}-\frac{\text{ArcTan}\left[\frac{\sqrt{3} c^{1/6} \left(c^{1/3}+d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]}{54 \sqrt{3} c^{11/6} d^{2/3}}+\frac{\text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3} x\right)^2}{3 c^{1/6} \sqrt{c+d x^3}}\right]}{162 c^{11/6} d^{2/3}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{162 c^{11/6} d^{2/3}}+\left(\sqrt{2-\sqrt{3}} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]\right) \Bigg/ \left(9\times 3^{3/4} c^{5/3} d^{2/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}\right)-\left(2 \sqrt{2} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]\right) \Bigg/ \left(27\times 3^{1/4} c^{5/3} d^{2/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}\right)$$

Result (type 6, 336 leaves):

$$\frac{1}{135 \sqrt{c+d x^3}} 2 x^2 \left(-\left(\left(250 \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right) \Big/ \left(\left(8 c-d x^3\right) \left(40 c \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3 \left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)+\frac{1}{c^2}\left(5+\left(32 c d x^3 \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right) \Big/ \left(\left(8 c-d x^3\right) \left(64 c \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3 \left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)\right)$$

Problem 335: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^2 \left(8 c - d x^3\right) \left(c + d x^3\right)^{3/2}} \, dx$$

Optimal (type 4, 653 leaves, 15 steps):

$$\frac{2}{27 c^2 x \sqrt{c + d x^3}} - \frac{43 \sqrt{c + d x^3}}{216 c^3 x} + \frac{43 d^{1/3} \sqrt{c + d x^3}}{216 c^3 \left(\left(1 + \sqrt{3}\right) c^{1/3} + d^{1/3} x\right)} - \frac{d^{1/3} \text{ArcTan}\left[\frac{\sqrt{3} c^{1/6} \left(c^{1/3} + d^{1/3} x\right)}{\sqrt{c + d x^3}}\right]}{432 \sqrt{3} c^{17/6}} + \frac{d^{1/3} \text{ArcTanh}\left[\frac{\left(c^{1/3} + d^{1/3} x\right)^2}{3 c^{1/6} \sqrt{c + d x^3}}\right]}{1296 c^{17/6}} - \frac{d^{1/3} \text{ArcTanh}\left[\frac{\sqrt{c + d x^3}}{3 \sqrt{c}}\right]}{1296 c^{17/6}} - \left(43 \sqrt{2 - \sqrt{3}} d^{1/3} \left(c^{1/3} + d^{1/3} x\right) \sqrt{\frac{c^{2/3} - c^{1/3} d^{1/3} x + d^{2/3} x^2}{\left(\left(1 + \sqrt{3}\right) c^{1/3} + d^{1/3} x\right)^2}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1 - \sqrt{3}\right) c^{1/3} + d^{1/3} x}{\left(1 + \sqrt{3}\right) c^{1/3} + d^{1/3} x}\right], -7 - 4 \sqrt{3}\right]\right) \Bigg/ \left(144 \times 3^{3/4} c^{8/3} \sqrt{\frac{c^{1/3} \left(c^{1/3} + d^{1/3} x\right)}{\left(\left(1 + \sqrt{3}\right) c^{1/3} + d^{1/3} x\right)^2}} \sqrt{c + d x^3}\right) + \left(43 d^{1/3} \left(c^{1/3} + d^{1/3} x\right) \sqrt{\frac{c^{2/3} - c^{1/3} d^{1/3} x + d^{2/3} x^2}{\left(\left(1 + \sqrt{3}\right) c^{1/3} + d^{1/3} x\right)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1 - \sqrt{3}\right) c^{1/3} + d^{1/3} x}{\left(1 + \sqrt{3}\right) c^{1/3} + d^{1/3} x}\right], -7 - 4 \sqrt{3}\right]\right) \Bigg/ \left(108 \sqrt{2} \, 3^{1/4} c^{8/3} \sqrt{\frac{c^{1/3} \left(c^{1/3} + d^{1/3} x\right)}{\left(\left(1 + \sqrt{3}\right) c^{1/3} + d^{1/3} x\right)^2}} \sqrt{c + d x^3}\right)$$

Result (type 6, 356 leaves):

$$\frac{1}{270\sqrt{c+d x^3}}\left(\left(4375\, d\, x^2\, \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(c\left(8 c-d x^3\right)\left(40\, c\, \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3\, d\, x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\, \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)-\frac{1}{4 c^3 x}\left(135 c+215 d x^3+\left(1376\, c\, d^2 x^6\, \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\left(64\, c\, \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3\, d\, x^3\left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\, \text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)$$

Problem 336: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^5\left(8 c-d x^3\right)\left(c+d x^3\right)^{3/2}}\, dx$$

Optimal (type 4, 675 leaves, 16 steps):

$$\frac{2}{27 c^2 x^4 \sqrt{c+d x^3}}-\frac{91 \sqrt{c+d x^3}}{864 c^3 x^4}+\frac{113 d \sqrt{c+d x^3}}{432 c^4 x}-\frac{113 d^{4/3} \sqrt{c+d x^3}}{432 c^4 \left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)}-\frac{d^{4/3} \text{ArcTan}\left[\frac{\sqrt{3} c^{1/6} \left(c^{1/3}+d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]}{3456 \sqrt{3} c^{23/6}}+\frac{d^{4/3} \text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3} x\right)^2}{3 c^{1/6} \sqrt{c+d x^3}}\right]}{10368 c^{23/6}}-\frac{d^{4/3} \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{10368 c^{23/6}}+\left(113 \sqrt{2-\sqrt{3}} d^{4/3} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]\right) \Bigg/ \left(288\times 3^{3/4} c^{11/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}\right)-\left(113 d^{4/3} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]\right) \Bigg/ \left(216 \sqrt{2}\, 3^{1/4} c^{11/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}\right)$$

Result (type 6, 364 leaves):

$$\left(-135 c^2+675 c d x^3+1130 d^2 x^6-\left(90250 c^2 d^2 x^6 \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right) \Bigg/ \left(\left(8 c-d x^3\right) \left(40 c \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3 \left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)+\left(7232 c d^3 x^9 \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right) \Bigg/ \left(\left(8 c-d x^3\right) \left(64 c \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3 \left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right) \Bigg/ \left(4320 c^4 x^4 \sqrt{c+d x^3}\right)$$

Problem 337: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^8 \left(8 c-d x^3\right) \left(c+d x^3\right)^{3/2}} \, dx$$

Optimal (type 4, 699 leaves, 17 steps):

$$\frac{2}{27 c^2 x^7 \sqrt{c+d x^3}}-\frac{139 \sqrt{c+d x^3}}{1512 c^3 x^7}+\frac{6095 d \sqrt{c+d x^3}}{48384 c^4 x^4}-\frac{953 d^2 \sqrt{c+d x^3}}{3024 c^5 x}+$$
$$\frac{953 d^{7/3} \sqrt{c+d x^3}}{3024 c^5 \left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)}-\frac{d^{7/3} \text{ArcTan}\left[\frac{\sqrt{3} c^{1/6} \left(c^{1/3}+d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]}{27648 \sqrt{3} c^{29/6}}+$$
$$\frac{d^{7/3} \text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3} x\right)^2}{3 c^{1/6} \sqrt{c+d x^3}}\right]}{82944 c^{29/6}}-\frac{d^{7/3} \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{82944 c^{29/6}}-\left(953 \sqrt{2-\sqrt{3}} d^{7/3} \left(c^{1/3}+d^{1/3} x\right)\right.$$
$$\left.\sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$
$$\left(2016\times 3^{3/4} c^{14/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}\right)+\left(953 d^{7/3} \left(c^{1/3}+d^{1/3} x\right)\right.$$
$$\left.\sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$
$$\left(1512 \sqrt{2} 3^{1/4} c^{14/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}\right)$$

Result (type 6, 378 leaves):

$$\left(-5 \left(864 c^3-1647 c^2 d x^3+9153 c d^2 x^6+15248 d^3 x^9\right)+\right.$$
$$\left(6100250 c^2 d^3 x^9 \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/$$
$$\left(\left(8 c-d x^3\right) \left(40 c \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+\right.\right.$$
$$\left.\left.3 d x^3 \left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)-$$
$$\left(487936 c d^4 x^{12} \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/$$
$$\left(\left(8 c-d x^3\right) \left(64 c \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3 \left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\right.\right.\right.\right.$$
$$\left.\left.\left.\left.\left.\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)\Big/\left(241920 c^5 x^7 \sqrt{c+d x^3}\right)$$

Problem 338: Result more than twice size of optimal antiderivative.

$$\int \frac{x^3}{\left(8\,c-d\,x^3\right)\,\left(c+d\,x^3\right)^{3/2}}\,dx$$

Optimal (type 6, 66 leaves, 2 steps):

$$\frac{x^4\,\sqrt{1+\frac{d\,x^3}{c}}\;\text{AppellF1}\left[\frac{4}{3},1,\frac{3}{2},\frac{7}{3},\frac{d\,x^3}{8\,c},-\frac{d\,x^3}{c}\right]}{32\,c^2\,\sqrt{c+d\,x^3}}$$

Result (type 6, 338 leaves):

$$\frac{1}{27\,\sqrt{c+d\,x^3}}2\,x\left(-\frac{1}{c\,d}+\left(256\,c\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\Big/\right.$$
$$\left(d\,\left(8\,c-d\,x^3\right)\,\left(32\,c\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]+\right.\right.$$
$$\left.\left.3\,d\,x^3\,\left(\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]-4\,\text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\right)\right)+$$
$$\left(7\,x^3\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\Big/$$
$$\left(\left(8\,c-d\,x^3\right)\,\left(56\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]+3\,d\,x^3\right.\right.$$
$$\left.\left.\left.\left(\text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]-4\,\text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\right)\right)\right)$$

Problem 339: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(8\,c-d\,x^3\right)\,\left(c+d\,x^3\right)^{3/2}}\,dx$$

Optimal (type 6, 64 leaves, 2 steps):

$$\frac{x\,\sqrt{1+\frac{d\,x^3}{c}}\;\text{AppellF1}\left[\frac{1}{3},1,\frac{3}{2},\frac{4}{3},\frac{d\,x^3}{8\,c},-\frac{d\,x^3}{c}\right]}{8\,c^2\,\sqrt{c+d\,x^3}}$$

Result (type 6, 334 leaves):

$$\frac{1}{27\sqrt{c+d x^3}}2 x \left(\left(176 \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\left(32 c \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)+\frac{1}{c^2}\left(1-\left(7 c d x^3 \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\left(56 c \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)\right)$$

Problem 340: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^3\left(8 c-d x^3\right)\left(c+d x^3\right)^{3/2}}\, dx$$

Optimal (type 6, 66 leaves, 2 steps):

$$-\frac{\sqrt{1+\frac{d x^3}{c}}\ \text{AppellF1}\left[-\frac{2}{3},1,\frac{3}{2},\frac{1}{3},\frac{d x^3}{8 c},-\frac{d x^3}{c}\right]}{16 c^2 x^2\sqrt{c+d x^3}}$$

Result (type 6, 351 leaves):

$$\left(-27 c-59 d x^3-\left(7360 c^2 d x^3 \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\left(32 c \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)+\left(413 c d^2 x^6 \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\left(56 c \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)\Big/\left(432 c^3 x^2\sqrt{c+d x^3}\right)$$

Problem 341: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^6\left(8 c-d x^3\right)\left(c+d x^3\right)^{3/2}}\, dx$$

Optimal (type 6, 66 leaves, 2 steps):

$$-\frac{\sqrt{1+\frac{d x^3}{c}}\ \text{AppellF1}\left[-\frac{5}{3},1,\frac{3}{2},-\frac{2}{3},\frac{d x^3}{8 c},-\frac{d x^3}{c}\right]}{40 c^2 x^5 \sqrt{c+d x^3}}$$

Result (type 6, 364 leaves):

$$\left(-432 c^2+1269 c d x^3+2981 d^2 x^6+\left(382528 c^2 d^2 x^6 \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\left(32 c \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)-\left(20867 c d^3 x^9 \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\left(56 c \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)\Big/\left(17280 c^4 x^5 \sqrt{c+d x^3}\right)$$

Problem 342: Result unnecessarily involves higher level functions.

$$\int \frac{x \sqrt{a+b x^3}}{2\left(5+3 \sqrt{3}\right) a+b x^3}\,dx$$

Optimal (type 4, 737 leaves, 5 steps):

$$\frac{2\sqrt{a+b\,x^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)}+\frac{3^{3/4}\,a^{1/6}\,\text{ArcTan}\left[\frac{3^{1/4}\left(1+\sqrt{3}\right)a^{1/6}\left(a^{1/3}+b^{1/3}\,x\right)}{\sqrt{2}\,\sqrt{a+b\,x^3}}\right]}{2\sqrt{2}\,b^{2/3}}+$$

$$\frac{a^{1/6}\,\text{ArcTan}\left[\frac{\left(1-\sqrt{3}\right)\sqrt{a+b\,x^3}}{\sqrt{2}\,3^{3/4}\,\sqrt{a}}\right]}{\sqrt{2}\,3^{1/4}\,b^{2/3}}+\frac{3^{1/4}\,a^{1/6}\,\text{ArcTanh}\left[\frac{3^{1/4}\,a^{1/6}\left(\left(1+\sqrt{3}\right)a^{1/3}-2\,b^{1/3}\,x\right)}{\sqrt{2}\,\sqrt{a+b\,x^3}}\right]}{\sqrt{2}\,b^{2/3}}+$$

$$\frac{3^{1/4}\,a^{1/6}\,\text{ArcTanh}\left[\frac{3^{1/4}\left(1-\sqrt{3}\right)a^{1/6}\left(a^{1/3}+b^{1/3}\,x\right)}{\sqrt{2}\,\sqrt{a+b\,x^3}}\right]}{2\sqrt{2}\,b^{2/3}}-\left(3^{1/4}\sqrt{2-\sqrt{3}}\;a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],\,-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(b^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\sqrt{a+b\,x^3}\right)+\left(2\sqrt{2}\;a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],\,-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(3^{1/4}\,b^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\sqrt{a+b\,x^3}\right)$$

Result (type 6, 250 leaves) :

$$\left(10\left(26+15\sqrt{3}\right)a\,x^2\sqrt{a+b\,x^3}\;\text{AppellF1}\left[\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},-\frac{b\,x^3}{a},-\frac{b\,x^3}{10\,a+6\sqrt{3}\,a}\right]\right)\Big/$$

$$\left(\left(5+3\sqrt{3}\right)\left(2\left(5+3\sqrt{3}\right)a+b\,x^3\right)\right.$$

$$\left(10\left(5+3\sqrt{3}\right)a\,\text{AppellF1}\left[\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},-\frac{b\,x^3}{a},-\frac{b\,x^3}{10\,a+6\sqrt{3}\,a}\right]-\right.$$

$$3\,b\,x^3\left(\text{AppellF1}\left[\frac{5}{3},-\frac{1}{2},2,\frac{8}{3},-\frac{b\,x^3}{a},-\frac{b\,x^3}{10\,a+6\sqrt{3}\,a}\right]-\right.$$

$$\left.\left.\left.\left(5+3\sqrt{3}\right)\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{b\,x^3}{a},-\frac{b\,x^3}{10\,a+6\sqrt{3}\,a}\right]\right)\right)\right)$$

Problem 343: Result unnecessarily involves higher level functions.

$$\int\frac{x\sqrt{a-b\,x^3}}{2\left(5+3\sqrt{3}\right)a-b\,x^3}\,dx$$

Optimal (type 4, 757 leaves, 5 steps):

$$\frac{2\sqrt{a-b x^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}x\right)}+\frac{3^{3/4}a^{1/6}\,\text{ArcTan}\left[\frac{3^{1/4}\left(1+\sqrt{3}\right)a^{1/6}\left(a^{1/3}-b^{1/3}x\right)}{\sqrt{2}\sqrt{a-b x^3}}\right]}{2\sqrt{2}\,b^{2/3}}+$$

$$\frac{a^{1/6}\,\text{ArcTan}\left[\frac{\left(1-\sqrt{3}\right)\sqrt{a-b x^3}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right]}{\sqrt{2}\,3^{1/4}\,b^{2/3}}+\frac{3^{1/4}a^{1/6}\,\text{ArcTanh}\left[\frac{3^{1/4}\left(1-\sqrt{3}\right)a^{1/6}\left(a^{1/3}-b^{1/3}x\right)}{\sqrt{2}\sqrt{a-b x^3}}\right]}{2\sqrt{2}\,b^{2/3}}+$$

$$\frac{3^{1/4}a^{1/6}\,\text{ArcTanh}\left[\frac{3^{1/4}a^{1/6}\left(\left(1+\sqrt{3}\right)a^{1/3}+2b^{1/3}x\right)}{\sqrt{2}\sqrt{a-b x^3}}\right]}{\sqrt{2}\,b^{2/3}}-$$

$$\left(3^{1/4}\sqrt{2-\sqrt{3}}\,a^{1/3}\left(a^{1/3}-b^{1/3}x\right)\sqrt{\frac{a^{2/3}+a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}x\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(b^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}-b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}x\right)^2}}\sqrt{a-b x^3}\right)+\left(2\sqrt{2}\,a^{1/3}\left(a^{1/3}-b^{1/3}x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}+a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(3^{1/4}b^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}-b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}x\right)^2}}\sqrt{a-b x^3}\right)$$

Result (type 6, 244 leaves):

$$\left(10\left(26+15\sqrt{3}\right)a\,x^2\sqrt{a-b x^3}\,\text{AppellF1}\left[\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},\frac{b x^3}{a},\frac{b x^3}{10a+6\sqrt{3}\,a}\right]\right)\Big/$$

$$\left(\left(5+3\sqrt{3}\right)\left(2\left(5+3\sqrt{3}\right)a-b x^3\right)\right.$$

$$\left(10\left(5+3\sqrt{3}\right)a\,\text{AppellF1}\left[\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},\frac{b x^3}{a},\frac{b x^3}{10a+6\sqrt{3}\,a}\right]+3b x^3\left(\text{AppellF1}\left[\frac{5}{3},-\frac{1}{2},2,\frac{8}{3},\right.\right.\right.$$

$$\left.\left.\left.\frac{b x^3}{a},\frac{b x^3}{10a+6\sqrt{3}\,a}\right]-\left(5+3\sqrt{3}\right)\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},\frac{b x^3}{a},\frac{b x^3}{10a+6\sqrt{3}\,a}\right]\right)\right)\right)$$

Problem 344: Result unnecessarily involves higher level functions.

$$\int \frac{x\sqrt{-a+b x^3}}{-2\left(5+3\sqrt{3}\right)a+b x^3}\,dx$$

Optimal (type 4, 774 leaves, 5 steps):

$$-\frac{2\sqrt{-a+b x^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}x\right)}+\frac{3^{1/4}a^{1/6}\,\text{ArcTan}\left[\frac{3^{1/4}\left(1-\sqrt{3}\right)a^{1/6}\left(a^{1/3}-b^{1/3}x\right)}{\sqrt{2}\sqrt{-a+b x^3}}\right]}{2\sqrt{2}\,b^{2/3}}+\frac{3^{1/4}a^{1/6}\,\text{ArcTan}\left[\frac{3^{1/4}a^{1/6}\left(\left(1+\sqrt{3}\right)a^{1/3}+2b^{1/3}x\right)}{\sqrt{2}\sqrt{-a+b x^3}}\right]}{\sqrt{2}\,b^{2/3}}+\frac{3^{3/4}a^{1/6}\,\text{ArcTanh}\left[\frac{3^{1/4}\left(1+\sqrt{3}\right)a^{1/6}\left(a^{1/3}-b^{1/3}x\right)}{\sqrt{2}\sqrt{-a+b x^3}}\right]}{2\sqrt{2}\,b^{2/3}}-\frac{a^{1/6}\,\text{ArcTanh}\left[\frac{\left(1-\sqrt{3}\right)\sqrt{-a+b x^3}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right]}{\sqrt{2}\,3^{1/4}b^{2/3}}+\left(3^{1/4}\sqrt{2+\sqrt{3}}\,a^{1/3}\left(a^{1/3}-b^{1/3}x\right)\sqrt{\frac{a^{2/3}+a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}x}{\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}x}\right],-7+4\sqrt{3}\right]\right)\Bigg/\left(b^{2/3}\sqrt{-\frac{a^{1/3}\left(a^{1/3}-b^{1/3}x\right)}{\left(\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}x\right)^2}}\sqrt{-a+b x^3}\right)-\left(2\sqrt{2}\,a^{1/3}\left(a^{1/3}-b^{1/3}x\right)\sqrt{\frac{a^{2/3}+a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}x}{\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}x}\right],-7+4\sqrt{3}\right]\right)\Bigg/\left(3^{1/4}b^{2/3}\sqrt{-\frac{a^{1/3}\left(a^{1/3}-b^{1/3}x\right)}{\left(\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}x\right)^2}}\sqrt{-a+b x^3}\right)$$

Result (type 6, 245 leaves):

$$-\left(\left(10\left(26+15\sqrt{3}\right)a x^2\sqrt{-a+b x^3}\,\text{AppellF1}\left[\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},\frac{b x^3}{a},\frac{b x^3}{10a+6\sqrt{3}a}\right]\right)\Bigg/\left(\left(5+3\sqrt{3}\right)\left(2\left(5+3\sqrt{3}\right)a-b x^3\right)\left(10\left(5+3\sqrt{3}\right)a\,\text{AppellF1}\left[\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},\frac{b x^3}{a},\frac{b x^3}{10a+6\sqrt{3}a}\right]+3b x^3\left(\text{AppellF1}\left[\frac{5}{3},-\frac{1}{2},2,\frac{8}{3},\frac{b x^3}{a},\frac{b x^3}{10a+6\sqrt{3}a}\right]-\left(5+3\sqrt{3}\right)\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},\frac{b x^3}{a},\frac{b x^3}{10a+6\sqrt{3}a}\right]\right)\right)\right)\right)$$

Problem 345: Result unnecessarily involves higher level functions.

$$\int \frac{x \sqrt{-a-b x^3}}{-2 \left(5+3 \sqrt{3}\right) a-b x^3} \, dx$$

Optimal (type 4, 768 leaves, 5 steps):

$$-\frac{2 \sqrt{-a-b x^3}}{b^{2/3} \left(\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}+\frac{3^{1/4} a^{1/6} \text{ArcTan}\left[\frac{3^{1/4} a^{1/6} \left(\left(1+\sqrt{3}\right) a^{1/3}-2 b^{1/3} x\right)}{\sqrt{2} \sqrt{-a-b x^3}}\right]}{\sqrt{2} b^{2/3}}+$$

$$\frac{3^{1/4} a^{1/6} \text{ArcTan}\left[\frac{3^{1/4} \left(1-\sqrt{3}\right) a^{1/6} \left(a^{1/3}+b^{1/3} x\right)}{\sqrt{2} \sqrt{-a-b x^3}}\right]}{2 \sqrt{2} b^{2/3}}+\frac{3^{3/4} a^{1/6} \text{ArcTanh}\left[\frac{3^{1/4} \left(1+\sqrt{3}\right) a^{1/6} \left(a^{1/3}+b^{1/3} x\right)}{\sqrt{2} \sqrt{-a-b x^3}}\right]}{2 \sqrt{2} b^{2/3}}-$$

$$\frac{a^{1/6} \text{ArcTanh}\left[\frac{\left(1-\sqrt{3}\right) \sqrt{-a-b x^3}}{\sqrt{2}\, 3^{3/4} \sqrt{a}}\right]}{\sqrt{2}\, 3^{1/4} b^{2/3}}+\left(3^{1/4} \sqrt{2+\sqrt{3}}\, a^{1/3} \left(a^{1/3}+b^{1/3} x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7+4 \sqrt{3}\right]\right)\Big/$$

$$\left(b^{2/3} \sqrt{-\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{-a-b x^3}\right)-\left(2 \sqrt{2}\, a^{1/3} \left(a^{1/3}+b^{1/3} x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7+4 \sqrt{3}\right]\right)\Big/$$

$$\left(3^{1/4} b^{2/3} \sqrt{-\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{-a-b x^3}\right)$$

Result (type 6, 253 leaves):

$$-\left(\left(10\left(26+15\sqrt{3}\right)a x^2\sqrt{-a-b x^3}\ \text{AppellF1}\left[\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},-\frac{b x^3}{a},-\frac{b x^3}{10 a+6\sqrt{3}\ a}\right]\right)\Big/\right.$$
$$\left(\left(5+3\sqrt{3}\right)\left(2\left(5+3\sqrt{3}\right)a+b x^3\right)\right.$$
$$\left(10\left(5+3\sqrt{3}\right)a\ \text{AppellF1}\left[\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},-\frac{b x^3}{a},-\frac{b x^3}{10 a+6\sqrt{3}\ a}\right]-\right.$$
$$3 b x^3\left(\text{AppellF1}\left[\frac{5}{3},-\frac{1}{2},2,\frac{8}{3},-\frac{b x^3}{a},-\frac{b x^3}{10 a+6\sqrt{3}\ a}\right]-\right.$$
$$\left.\left.\left.\left.\left(5+3\sqrt{3}\right)\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{b x^3}{a},-\frac{b x^3}{10 a+6\sqrt{3}\ a}\right]\right)\right)\right)\right)$$

Problem 346: Result unnecessarily involves higher level functions.

$$\int\frac{x\sqrt{a+b x^3}}{2\left(5-3\sqrt{3}\right)a+b x^3}\,dx$$

Optimal (type 4, 738 leaves, 5 steps):

$$\frac{2\sqrt{a+b x^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)}-\frac{3^{1/4}a^{1/6}\,\text{ArcTan}\left[\frac{3^{1/4}a^{1/6}\left(\left(1-\sqrt{3}\right)a^{1/3}-2 b^{1/3}x\right)}{\sqrt{2}\ \sqrt{a+b x^3}}\right]}{\sqrt{2}\ b^{2/3}}-$$
$$\frac{3^{1/4}a^{1/6}\,\text{ArcTan}\left[\frac{3^{1/4}\left(1+\sqrt{3}\right)a^{1/6}\left(a^{1/3}+b^{1/3}x\right)}{\sqrt{2}\ \sqrt{a+b x^3}}\right]}{2\sqrt{2}\ b^{2/3}}+\frac{3^{3/4}a^{1/6}\,\text{ArcTanh}\left[\frac{3^{1/4}\left(1-\sqrt{3}\right)a^{1/6}\left(a^{1/3}+b^{1/3}x\right)}{\sqrt{2}\ \sqrt{a+b x^3}}\right]}{2\sqrt{2}\ b^{2/3}}+$$
$$\frac{a^{1/6}\,\text{ArcTanh}\left[\frac{\left(1+\sqrt{3}\right)\sqrt{a+b x^3}}{\sqrt{2}\ 3^{3/4}\sqrt{a}}\right]}{\sqrt{2}\ 3^{1/4}b^{2/3}}-\left(3^{1/4}\sqrt{2-\sqrt{3}}\ a^{1/3}\left(a^{1/3}+b^{1/3}x\right)\right.$$
$$\left.\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$
$$\left(b^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\ \sqrt{a+b x^3}\right)+\left(2\sqrt{2}\ a^{1/3}\left(a^{1/3}+b^{1/3}x\right)\right.$$
$$\left.\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$
$$\left(3^{1/4}b^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\ \sqrt{a+b x^3}\right)$$

Result (type 6, 250 leaves):

$$\left(10\left(-26+15\sqrt{3}\right)a\,x^2\sqrt{a+b\,x^3}\;\text{AppellF1}\left[\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},-\frac{b\,x^3}{a},-\frac{b\,x^3}{10\,a-6\sqrt{3}\,a}\right]\right)\Big/$$
$$\left(\left(-5+3\sqrt{3}\right)\left(2\left(-5+3\sqrt{3}\right)a-b\,x^3\right)\right.$$
$$\left(10\left(-5+3\sqrt{3}\right)a\,\text{AppellF1}\left[\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},-\frac{b\,x^3}{a},-\frac{b\,x^3}{10\,a-6\sqrt{3}\,a}\right]+\right.$$
$$3\,b\,x^3\left(\text{AppellF1}\left[\frac{5}{3},-\frac{1}{2},2,\frac{8}{3},-\frac{b\,x^3}{a},-\frac{b\,x^3}{10\,a-6\sqrt{3}\,a}\right]+\right.$$
$$\left.\left.\left.\left(-5+3\sqrt{3}\right)\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{b\,x^3}{a},-\frac{b\,x^3}{10\,a-6\sqrt{3}\,a}\right]\right)\right)\right)$$

Problem 347: Result unnecessarily involves higher level functions.

$$\int\frac{x\sqrt{a-b\,x^3}}{2\left(5-3\sqrt{3}\right)a-b\,x^3}\,dx$$

Optimal (type 4, 758 leaves, 5 steps):

$$\frac{2\sqrt{a-b\,x^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}\,x\right)}-\frac{3^{1/4}\,a^{1/6}\,\text{ArcTan}\left[\frac{3^{1/4}\left(1+\sqrt{3}\right)a^{1/6}\left(a^{1/3}-b^{1/3}\,x\right)}{\sqrt{2}\,\sqrt{a-b\,x^3}}\right]}{2\sqrt{2}\,b^{2/3}}-$$

$$\frac{3^{1/4}\,a^{1/6}\,\text{ArcTan}\left[\frac{3^{1/4}\,a^{1/6}\left(\left(1-\sqrt{3}\right)a^{1/3}+2\,b^{1/3}\,x\right)}{\sqrt{2}\,\sqrt{a-b\,x^3}}\right]}{\sqrt{2}\,b^{2/3}}+\frac{3^{3/4}\,a^{1/6}\,\text{ArcTanh}\left[\frac{3^{1/4}\left(1-\sqrt{3}\right)a^{1/6}\left(a^{1/3}-b^{1/3}\,x\right)}{\sqrt{2}\,\sqrt{a-b\,x^3}}\right]}{2\sqrt{2}\,b^{2/3}}+$$

$$\frac{a^{1/6}\,\text{ArcTanh}\left[\frac{\left(1+\sqrt{3}\right)\sqrt{a-b\,x^3}}{\sqrt{2}\,3^{3/4}\,\sqrt{a}}\right]}{\sqrt{2}\,3^{1/4}\,b^{2/3}}-\left(3^{1/4}\sqrt{2-\sqrt{3}}\;a^{1/3}\left(a^{1/3}-b^{1/3}\,x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}+a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}\,x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}\,x}\right],\,-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(b^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}-b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}\,x\right)^2}}\;\sqrt{a-b\,x^3}\right)+\left(2\sqrt{2}\;a^{1/3}\left(a^{1/3}-b^{1/3}\,x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}+a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}\,x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}\,x}\right],\,-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(3^{1/4}\,b^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}-b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}\,x\right)^2}}\;\sqrt{a-b\,x^3}\right)$$

Result (type 6, 242 leaves):

$$-\left(\left(10\left(26-15\sqrt{3}\right)a\,x^2\sqrt{a-b\,x^3}\;\text{AppellF1}\left[\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},\frac{b\,x^3}{a},\frac{b\,x^3}{10\,a-6\sqrt{3}\,a}\right]\right)\Big/\right.$$

$$\left(\left(-5+3\sqrt{3}\right)\left(2\left(-5+3\sqrt{3}\right)a+b\,x^3\right)\left(10\left(-5+3\sqrt{3}\right)a\,\text{AppellF1}\left[\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},\right.\right.\right.$$

$$\left.\frac{b\,x^3}{a},\frac{b\,x^3}{10\,a-6\sqrt{3}\,a}\right]-3\,b\,x^3\left(\text{AppellF1}\left[\frac{5}{3},-\frac{1}{2},2,\frac{8}{3},\frac{b\,x^3}{a},\frac{b\,x^3}{10\,a-6\sqrt{3}\,a}\right]+\right.$$

$$\left.\left.\left.\left.\left(-5+3\sqrt{3}\right)\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},\frac{b\,x^3}{a},\frac{b\,x^3}{10\,a-6\sqrt{3}\,a}\right]\right)\right)\right)\right)$$

Problem 348: Result unnecessarily involves higher level functions.

$$\int\frac{x\sqrt{-a+b\,x^3}}{2\left(5-3\sqrt{3}\right)a-b\,x^3}\,dx$$

Optimal (type 4, 774 leaves, 5 steps):

$$\frac{2\sqrt{-a+b x^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}x\right)}-\frac{3^{3/4}a^{1/6}\,\text{ArcTan}\left[\frac{3^{1/4}\left(1-\sqrt{3}\right)a^{1/6}\left(a^{1/3}-b^{1/3}x\right)}{\sqrt{2}\sqrt{-a+b x^3}}\right]}{2\sqrt{2}\,b^{2/3}}+$$

$$\frac{a^{1/6}\,\text{ArcTan}\left[\frac{\left(1+\sqrt{3}\right)\sqrt{-a+b x^3}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right]}{\sqrt{2}\,3^{1/4}\,b^{2/3}}+\frac{3^{1/4}a^{1/6}\,\text{ArcTanh}\left[\frac{3^{1/4}\left(1+\sqrt{3}\right)a^{1/6}\left(a^{1/3}-b^{1/3}x\right)}{\sqrt{2}\sqrt{-a+b x^3}}\right]}{2\sqrt{2}\,b^{2/3}}+$$

$$\frac{3^{1/4}a^{1/6}\,\text{ArcTanh}\left[\frac{3^{1/4}a^{1/6}\left(\left(1-\sqrt{3}\right)a^{1/3}+2b^{1/3}x\right)}{\sqrt{2}\sqrt{-a+b x^3}}\right]}{\sqrt{2}\,b^{2/3}}-$$

$$\left(3^{1/4}\sqrt{2+\sqrt{3}}\,a^{1/3}\left(a^{1/3}-b^{1/3}x\right)\sqrt{\frac{a^{2/3}+a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}x\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}x}{\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}x}\right],-7+4\sqrt{3}\right]\right)\Big/$$

$$\left(b^{2/3}\sqrt{-\frac{a^{1/3}\left(a^{1/3}-b^{1/3}x\right)}{\left(\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}x\right)^2}}\sqrt{-a+b x^3}\right)+\left(2\sqrt{2}\,a^{1/3}\left(a^{1/3}-b^{1/3}x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}+a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}x}{\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}x}\right],-7+4\sqrt{3}\right]\right)\Big/$$

$$\left(3^{1/4}b^{2/3}\sqrt{-\frac{a^{1/3}\left(a^{1/3}-b^{1/3}x\right)}{\left(\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}x\right)^2}}\sqrt{-a+b x^3}\right)$$

Result (type 6, 243 leaves):

$$-\left(\left(10\left(26-15\sqrt{3}\right)a x^2\sqrt{-a+b x^3}\,\text{AppellF1}\left[\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},\frac{b x^3}{a},\frac{b x^3}{10a-6\sqrt{3}\,a}\right]\right)\Big/\right.$$

$$\left(\left(-5+3\sqrt{3}\right)\left(2\left(-5+3\sqrt{3}\right)a+b x^3\right)\left(10\left(-5+3\sqrt{3}\right)a\,\text{AppellF1}\left[\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},\right.\right.\right.$$

$$\frac{b x^3}{a},\frac{b x^3}{10a-6\sqrt{3}\,a}\right]-3b x^3\left(\text{AppellF1}\left[\frac{5}{3},-\frac{1}{2},2,\frac{8}{3},\frac{b x^3}{a},\frac{b x^3}{10a-6\sqrt{3}\,a}\right]+\right.$$

$$\left.\left.\left.\left.\left(-5+3\sqrt{3}\right)\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},\frac{b x^3}{a},\frac{b x^3}{10a-6\sqrt{3}\,a}\right]\right)\right)\right)\right)$$

Problem 349: Result unnecessarily involves higher level functions.

$$\int \frac{x\sqrt{-a-b x^3}}{2\left(5-3\sqrt{3}\right)a+b x^3}\,dx$$

Optimal (type 4, 768 leaves, 5 steps):

$$\frac{2\sqrt{-a-b x^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)}-\frac{3^{3/4}a^{1/6}\,\text{ArcTan}\left[\frac{3^{1/4}\left(1-\sqrt{3}\right)a^{1/6}\left(a^{1/3}+b^{1/3}x\right)}{\sqrt{2}\sqrt{-a-b x^3}}\right]}{2\sqrt{2}\,b^{2/3}}+\frac{a^{1/6}\,\text{ArcTan}\left[\frac{\left(1+\sqrt{3}\right)\sqrt{-a-b x^3}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right]}{\sqrt{2}\,3^{1/4}b^{2/3}}+\frac{3^{1/4}a^{1/6}\,\text{ArcTanh}\left[\frac{3^{1/4}a^{1/6}\left(\left(1-\sqrt{3}\right)a^{1/3}-2b^{1/3}x\right)}{\sqrt{2}\sqrt{-a-b x^3}}\right]}{\sqrt{2}\,b^{2/3}}+\frac{3^{1/4}a^{1/6}\,\text{ArcTanh}\left[\frac{3^{1/4}\left(1+\sqrt{3}\right)a^{1/6}\left(a^{1/3}+b^{1/3}x\right)}{\sqrt{2}\sqrt{-a-b x^3}}\right]}{2\sqrt{2}\,b^{2/3}}-\left(3^{1/4}\sqrt{2+\sqrt{3}}\,a^{1/3}\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7+4\sqrt{3}\right]\right)\Bigg/\left(b^{2/3}\sqrt{-\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\sqrt{-a-b x^3}\right)+\left(2\sqrt{2}\,a^{1/3}\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7+4\sqrt{3}\right]\right)\Bigg/\left(3^{1/4}b^{2/3}\sqrt{-\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\sqrt{-a-b x^3}\right)$$

Result (type 6, 253 leaves):

$$\left(10\left(-26+15\sqrt{3}\right)a\,x^2\sqrt{-a-b\,x^3}\;\text{AppellF1}\left[\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},-\frac{b\,x^3}{a},-\frac{b\,x^3}{10\,a-6\sqrt{3}\,a}\right]\right)\Big/$$
$$\left(\left(-5+3\sqrt{3}\right)\left(2\left(-5+3\sqrt{3}\right)a-b\,x^3\right)\right.$$
$$\left(10\left(-5+3\sqrt{3}\right)a\,\text{AppellF1}\left[\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},-\frac{b\,x^3}{a},-\frac{b\,x^3}{10\,a-6\sqrt{3}\,a}\right]+\right.$$
$$3\,b\,x^3\left(\text{AppellF1}\left[\frac{5}{3},-\frac{1}{2},2,\frac{8}{3},-\frac{b\,x^3}{a},-\frac{b\,x^3}{10\,a-6\sqrt{3}\,a}\right]+\right.$$
$$\left.\left.\left.\left(-5+3\sqrt{3}\right)\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{b\,x^3}{a},-\frac{b\,x^3}{10\,a-6\sqrt{3}\,a}\right]\right)\right)\right)$$

Problem 350: Result unnecessarily involves higher level functions.

$$\int\frac{x}{\sqrt{a+b\,x^3}\,\left(2\left(5+3\sqrt{3}\right)a+b\,x^3\right)}\,dx$$

Optimal (type 3, 318 leaves, 1 step):

$$-\frac{\left(2-\sqrt{3}\right)\text{ArcTan}\left[\frac{3^{1/4}\left(1+\sqrt{3}\right)a^{1/6}\left(a^{1/3}+b^{1/3}x\right)}{\sqrt{2}\sqrt{a+b\,x^3}}\right]}{2\sqrt{2}\,3^{3/4}\,a^{5/6}\,b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\text{ArcTan}\left[\frac{\left(1-\sqrt{3}\right)\sqrt{a+b\,x^3}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right]}{3\sqrt{2}\,3^{3/4}\,a^{5/6}\,b^{2/3}}-$$
$$\frac{\left(2-\sqrt{3}\right)\text{ArcTanh}\left[\frac{3^{1/4}a^{1/6}\left(\left(1+\sqrt{3}\right)a^{1/3}-2\,b^{1/3}x\right)}{\sqrt{2}\sqrt{a+b\,x^3}}\right]}{3\sqrt{2}\,3^{1/4}\,a^{5/6}\,b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\text{ArcTanh}\left[\frac{3^{1/4}\left(1-\sqrt{3}\right)a^{1/6}\left(a^{1/3}+b^{1/3}x\right)}{\sqrt{2}\sqrt{a+b\,x^3}}\right]}{6\sqrt{2}\,3^{1/4}\,a^{5/6}\,b^{2/3}}$$

Result (type 6, 249 leaves):

$$\left(10\left(26+15\sqrt{3}\right)a\,x^2\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{b\,x^3}{a},-\frac{b\,x^3}{10\,a+6\sqrt{3}\,a}\right]\right)\Big/$$
$$\left(\left(5+3\sqrt{3}\right)\sqrt{a+b\,x^3}\left(2\left(5+3\sqrt{3}\right)a+b\,x^3\right)\right.$$
$$\left(10\left(5+3\sqrt{3}\right)a\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{b\,x^3}{a},-\frac{b\,x^3}{10\,a+6\sqrt{3}\,a}\right]-3\,b\,x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},\right.\right.\right.$$
$$\left.\left.\left.\left.-\frac{b\,x^3}{a},-\frac{b\,x^3}{10\,a+6\sqrt{3}\,a}\right]+\left(5+3\sqrt{3}\right)\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{b\,x^3}{a},-\frac{b\,x^3}{10\,a+6\sqrt{3}\,a}\right]\right)\right)\right)$$

Problem 351: Result unnecessarily involves higher level functions.

$$\int\frac{x}{\sqrt{a-b\,x^3}\,\left(2\left(5+3\sqrt{3}\right)a-b\,x^3\right)}\,dx$$

Optimal (type 3, 324 leaves, 1 step):

$$-\frac{\left(2-\sqrt{3}\right)\text{ArcTan}\left[\frac{3^{1/4}\left(1+\sqrt{3}\right)a^{1/6}\left(a^{1/3}-b^{1/3}x\right)}{\sqrt{2}\sqrt{a-b x^3}}\right]}{2\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\text{ArcTan}\left[\frac{\left(1-\sqrt{3}\right)\sqrt{a-b x^3}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right]}{3\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\text{ArcTanh}\left[\frac{3^{1/4}\left(1-\sqrt{3}\right)a^{1/6}\left(a^{1/3}-b^{1/3}x\right)}{\sqrt{2}\sqrt{a-b x^3}}\right]}{6\sqrt{2}\,3^{1/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\text{ArcTanh}\left[\frac{3^{1/4}a^{1/6}\left(\left(1+\sqrt{3}\right)a^{1/3}+2b^{1/3}x\right)}{\sqrt{2}\sqrt{a-b x^3}}\right]}{3\sqrt{2}\,3^{1/4}a^{5/6}b^{2/3}}$$

Result (type 6, 243 leaves):

$$\left(10\left(26+15\sqrt{3}\right)a x^2\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},\frac{b x^3}{a},\frac{b x^3}{10a+6\sqrt{3}a}\right]\right)\Big/\left(\left(5+3\sqrt{3}\right)\sqrt{a-b x^3}\left(2\left(5+3\sqrt{3}\right)a-b x^3\right)\left(10\left(5+3\sqrt{3}\right)a\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},\frac{b x^3}{a},\frac{b x^3}{10a+6\sqrt{3}a}\right]+3b x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},\frac{b x^3}{a},\frac{b x^3}{10a+6\sqrt{3}a}\right]+\left(5+3\sqrt{3}\right)\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},\frac{b x^3}{a},\frac{b x^3}{10a+6\sqrt{3}a}\right]\right)\right)\right)$$

Problem 352: Result unnecessarily involves higher level functions.

$$\int\frac{x}{\sqrt{-a+b x^3}\left(-2\left(5+3\sqrt{3}\right)a+b x^3\right)}\,dx$$

Optimal (type 3, 328 leaves, 1 step):

$$\frac{\left(2-\sqrt{3}\right)\text{ArcTan}\left[\frac{3^{1/4}\left(1-\sqrt{3}\right)a^{1/6}\left(a^{1/3}-b^{1/3}x\right)}{\sqrt{2}\sqrt{-a+b x^3}}\right]}{6\sqrt{2}\,3^{1/4}a^{5/6}b^{2/3}}+\frac{\left(2-\sqrt{3}\right)\text{ArcTan}\left[\frac{3^{1/4}a^{1/6}\left(\left(1+\sqrt{3}\right)a^{1/3}+2b^{1/3}x\right)}{\sqrt{2}\sqrt{-a+b x^3}}\right]}{3\sqrt{2}\,3^{1/4}a^{5/6}b^{2/3}}+\frac{\left(2-\sqrt{3}\right)\text{ArcTanh}\left[\frac{3^{1/4}\left(1+\sqrt{3}\right)a^{1/6}\left(a^{1/3}-b^{1/3}x\right)}{\sqrt{2}\sqrt{-a+b x^3}}\right]}{2\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\text{ArcTanh}\left[\frac{\left(1-\sqrt{3}\right)\sqrt{-a+b x^3}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right]}{3\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}$$

Result (type 6, 244 leaves):

$$-\left(\left(10\left(26+15\sqrt{3}\right)a x^2\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},\frac{b x^3}{a},\frac{b x^3}{10a+6\sqrt{3}a}\right]\right)\Big/\left(\left(5+3\sqrt{3}\right)\left(2\left(5+3\sqrt{3}\right)a-b x^3\right)\sqrt{-a+b x^3}\left(10\left(5+3\sqrt{3}\right)a\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},\frac{b x^3}{a},\frac{b x^3}{10a+6\sqrt{3}a}\right]+3b x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},\frac{b x^3}{a},\frac{b x^3}{10a+6\sqrt{3}a}\right]+\left(5+3\sqrt{3}\right)\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},\frac{b x^3}{a},\frac{b x^3}{10a+6\sqrt{3}a}\right]\right)\right)\right)\right)$$

Problem 353: Result unnecessarily involves higher level functions.

$$\int \frac{x}{\sqrt{-a-b x^3}\left(-2\left(5+3\sqrt{3}\right)a-b x^3\right)}\,dx$$

Optimal (type 3, 330 leaves, 1 step):

$$\frac{\left(2-\sqrt{3}\right)\text{ArcTan}\left[\frac{3^{1/4}a^{1/6}\left(\left(1+\sqrt{3}\right)a^{1/3}-2b^{1/3}x\right)}{\sqrt{2}\sqrt{-a-b x^3}}\right]}{3\sqrt{2}\,3^{1/4}a^{5/6}b^{2/3}}+\frac{\left(2-\sqrt{3}\right)\text{ArcTan}\left[\frac{3^{1/4}\left(1-\sqrt{3}\right)a^{1/6}\left(a^{1/3}+b^{1/3}x\right)}{\sqrt{2}\sqrt{-a-b x^3}}\right]}{6\sqrt{2}\,3^{1/4}a^{5/6}b^{2/3}}+\frac{\left(2-\sqrt{3}\right)\text{ArcTanh}\left[\frac{3^{1/4}\left(1+\sqrt{3}\right)a^{1/6}\left(a^{1/3}+b^{1/3}x\right)}{\sqrt{2}\sqrt{-a-b x^3}}\right]}{2\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\text{ArcTanh}\left[\frac{\left(1-\sqrt{3}\right)\sqrt{-a-b x^3}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right]}{3\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}$$

Result (type 6, 252 leaves):

$$-\left(\left(10\left(26+15\sqrt{3}\right)a x^2\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{b x^3}{a},-\frac{b x^3}{10a+6\sqrt{3}a}\right]\right)\Big/\left(\left(5+3\sqrt{3}\right)\sqrt{-a-b x^3}\left(2\left(5+3\sqrt{3}\right)a+b x^3\right)\left(10\left(5+3\sqrt{3}\right)a\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{b x^3}{a},-\frac{b x^3}{10a+6\sqrt{3}a}\right]-3b x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{b x^3}{a},-\frac{b x^3}{10a+6\sqrt{3}a}\right]+\left(5+3\sqrt{3}\right)\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{b x^3}{a},-\frac{b x^3}{10a+6\sqrt{3}a}\right]\right)\right)\right)\right)$$

Problem 354: Result unnecessarily involves higher level functions.

$$\int \frac{x}{\sqrt{a+b x^3}\left(2\left(5-3\sqrt{3}\right)a+b x^3\right)}\,dx$$

Optimal (type 3, 310 leaves, 1 step):

$$-\frac{\left(2+\sqrt{3}\right)\text{ArcTan}\left[\frac{3^{1/4}a^{1/6}\left(\left(1-\sqrt{3}\right)a^{1/3}-2b^{1/3}x\right)}{\sqrt{2}\sqrt{a+b x^3}}\right]}{3\sqrt{2}\,3^{1/4}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\text{ArcTan}\left[\frac{3^{1/4}\left(1+\sqrt{3}\right)a^{1/6}\left(a^{1/3}+b^{1/3}x\right)}{\sqrt{2}\sqrt{a+b x^3}}\right]}{6\sqrt{2}\,3^{1/4}a^{5/6}b^{2/3}}+\frac{\left(2+\sqrt{3}\right)\text{ArcTanh}\left[\frac{3^{1/4}\left(1-\sqrt{3}\right)a^{1/6}\left(a^{1/3}+b^{1/3}x\right)}{\sqrt{2}\sqrt{a+b x^3}}\right]}{2\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}+\frac{\left(2+\sqrt{3}\right)\text{ArcTanh}\left[\frac{\left(1+\sqrt{3}\right)\sqrt{a+b x^3}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right]}{3\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}$$

Result (type 6, 249 leaves):

$$-\left(\left(10\left(26-15\sqrt{3}\right) a x^2 \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{b x^3}{a},\frac{b x^3}{-10 a+6\sqrt{3} a}\right]\right)\Big/\right.$$
$$\left(\left(-5+3\sqrt{3}\right)\left(2\left(-5+3\sqrt{3}\right) a-b x^3\right)\sqrt{a+b x^3}\right.$$
$$\left(10\left(-5+3\sqrt{3}\right) a \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{b x^3}{a},-\frac{b x^3}{10 a-6\sqrt{3} a}\right]+\right.$$
$$3 b x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{b x^3}{a},-\frac{b x^3}{10 a-6\sqrt{3} a}\right]+\right.$$
$$\left.\left.\left.\left.\left(5-3\sqrt{3}\right) \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{b x^3}{a},-\frac{b x^3}{10 a-6\sqrt{3} a}\right]\right)\right)\right)\right)$$

Problem 355: Result unnecessarily involves higher level functions.

$$\int \frac{x}{\sqrt{a-b x^3}\left(2\left(5-3\sqrt{3}\right) a-b x^3\right)}\,dx$$

Optimal (type 3, 316 leaves, 1 step):

$$-\frac{\left(2+\sqrt{3}\right) \text{ArcTan}\left[\frac{3^{1/4}\left(1+\sqrt{3}\right) a^{1/6}\left(a^{1/3}-b^{1/3} x\right)}{\sqrt{2}\sqrt{a-b x^3}}\right]}{6\sqrt{2}\, 3^{1/4} a^{5/6} b^{2/3}}-\frac{\left(2+\sqrt{3}\right) \text{ArcTan}\left[\frac{3^{1/4} a^{1/6}\left(\left(1-\sqrt{3}\right) a^{1/3}+2 b^{1/3} x\right)}{\sqrt{2}\sqrt{a-b x^3}}\right]}{3\sqrt{2}\, 3^{1/4} a^{5/6} b^{2/3}}+$$
$$\frac{\left(2+\sqrt{3}\right) \text{ArcTanh}\left[\frac{3^{1/4}\left(1-\sqrt{3}\right) a^{1/6}\left(a^{1/3}-b^{1/3} x\right)}{\sqrt{2}\sqrt{a-b x^3}}\right]}{2\sqrt{2}\, 3^{3/4} a^{5/6} b^{2/3}}+\frac{\left(2+\sqrt{3}\right) \text{ArcTanh}\left[\frac{\left(1+\sqrt{3}\right)\sqrt{a-b x^3}}{\sqrt{2}\, 3^{3/4}\sqrt{a}}\right]}{3\sqrt{2}\, 3^{3/4} a^{5/6} b^{2/3}}$$

Result (type 6, 242 leaves):

$$-\left(\left(10\left(26-15\sqrt{3}\right) a x^2 \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},\frac{b x^3}{a},\frac{b x^3}{10 a-6\sqrt{3} a}\right]\right)\Big/\right.$$
$$\left(\left(-5+3\sqrt{3}\right)\sqrt{a-b x^3}\left(2\left(-5+3\sqrt{3}\right) a+b x^3\right)\left(10\left(-5+3\sqrt{3}\right) a\right.\right.$$
$$\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},\frac{b x^3}{a},\frac{b x^3}{10 a-6\sqrt{3} a}\right]-3 b x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},\frac{b x^3}{a},\right.\right.$$
$$\left.\left.\left.\left.\left.\frac{b x^3}{10 a-6\sqrt{3} a}\right]+\left(5-3\sqrt{3}\right) \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},\frac{b x^3}{a},\frac{b x^3}{10 a-6\sqrt{3} a}\right]\right)\right)\right)\right)$$

Problem 356: Result unnecessarily involves higher level functions.

$$\int \frac{x}{\left(2\left(5-3\sqrt{3}\right) a-b x^3\right)\sqrt{-a+b x^3}}\,dx$$

Optimal (type 3, 320 leaves, 1 step):

$$\frac{\left(2+\sqrt{3}\right)\text{ArcTan}\left[\frac{3^{1/4}\left(1-\sqrt{3}\right)a^{1/6}\left(a^{1/3}-b^{1/3}x\right)}{\sqrt{2}\sqrt{-a+b x^3}}\right]}{2\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\text{ArcTan}\left[\frac{\left(1+\sqrt{3}\right)\sqrt{-a+b x^3}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right]}{3\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\text{ArcTanh}\left[\frac{3^{1/4}\left(1+\sqrt{3}\right)a^{1/6}\left(a^{1/3}-b^{1/3}x\right)}{\sqrt{2}\sqrt{-a+b x^3}}\right]}{6\sqrt{2}\,3^{1/4}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\text{ArcTanh}\left[\frac{3^{1/4}a^{1/6}\left(\left(1-\sqrt{3}\right)a^{1/3}+2b^{1/3}x\right)}{\sqrt{2}\sqrt{-a+b x^3}}\right]}{3\sqrt{2}\,3^{1/4}a^{5/6}b^{2/3}}$$

Result (type 6, 243 leaves):

$$-\left(\left(10\left(26-15\sqrt{3}\right)a x^2\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},\frac{b x^3}{a},\frac{b x^3}{10a-6\sqrt{3}a}\right]\right)\Big/\left(\left(-5+3\sqrt{3}\right)\sqrt{-a+b x^3}\left(2\left(-5+3\sqrt{3}\right)a+b x^3\right)\left(10\left(-5+3\sqrt{3}\right)a\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},\frac{b x^3}{a},\frac{b x^3}{10a-6\sqrt{3}a}\right]-3b x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},\frac{b x^3}{a},\frac{b x^3}{10a-6\sqrt{3}a}\right]+\left(5-3\sqrt{3}\right)\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},\frac{b x^3}{a},\frac{b x^3}{10a-6\sqrt{3}a}\right]\right)\right)\right)\right)$$

Problem 357: Result unnecessarily involves higher level functions.

$$\int \frac{x}{\sqrt{-a-b x^3}\left(2\left(5-3\sqrt{3}\right)a+b x^3\right)}\,dx$$

Optimal (type 3, 322 leaves, 1 step):

$$\frac{\left(2+\sqrt{3}\right)\text{ArcTan}\left[\frac{3^{1/4}\left(1-\sqrt{3}\right)a^{1/6}\left(a^{1/3}+b^{1/3}x\right)}{\sqrt{2}\sqrt{-a-b x^3}}\right]}{2\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\text{ArcTan}\left[\frac{\left(1+\sqrt{3}\right)\sqrt{-a-b x^3}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right]}{3\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\text{ArcTanh}\left[\frac{3^{1/4}a^{1/6}\left(\left(1-\sqrt{3}\right)a^{1/3}-2b^{1/3}x\right)}{\sqrt{2}\sqrt{-a-b x^3}}\right]}{3\sqrt{2}\,3^{1/4}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\text{ArcTanh}\left[\frac{3^{1/4}\left(1+\sqrt{3}\right)a^{1/6}\left(a^{1/3}+b^{1/3}x\right)}{\sqrt{2}\sqrt{-a-b x^3}}\right]}{6\sqrt{2}\,3^{1/4}a^{5/6}b^{2/3}}$$

Result (type 6, 252 leaves):

$$-\left(\left(10\left(26-15\sqrt{3}\right)a x^2\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{b x^3}{a},\frac{b x^3}{-10a+6\sqrt{3}a}\right]\right)\Big/\left(\left(-5+3\sqrt{3}\right)\sqrt{-a-b x^3}\left(2\left(-5+3\sqrt{3}\right)a-b x^3\right)\left(10\left(-5+3\sqrt{3}\right)a\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{b x^3}{a},-\frac{b x^3}{10a-6\sqrt{3}a}\right]+3b x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{b x^3}{a},-\frac{b x^3}{10a-6\sqrt{3}a}\right]+\left(5-3\sqrt{3}\right)\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{b x^3}{a},-\frac{b x^3}{10a-6\sqrt{3}a}\right]\right)\right)\right)\right)$$

Problem 361: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{c+d x^3}}{x \left(a+b x^3\right)} \, dx$$

Optimal (type 3, 85 leaves, 6 steps):

$$-\frac{2 \sqrt{c} \operatorname{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{3 a}+\frac{2 \sqrt{b c-a d} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{c+d x^3}}{\sqrt{b c-a d}}\right]}{3 a \sqrt{b}}$$

Result (type 6, 160 leaves):

$$-\left(\left(2 b d x^3 \sqrt{c+d x^3} \operatorname{AppellF1}\left[\frac{1}{2},-\frac{1}{2},1,\frac{3}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]\right)\Big/\left(\left(a+b x^3\right)\left(3 b d x^3 \operatorname{AppellF1}\left[\frac{1}{2},-\frac{1}{2},1,\frac{3}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]-2 a d \operatorname{AppellF1}\left[\frac{3}{2},-\frac{1}{2},2,\frac{5}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]+b c \operatorname{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]\right)\right)\right)$$

Problem 362: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{c+d x^3}}{x^4 \left(a+b x^3\right)} \, dx$$

Optimal (type 3, 115 leaves, 7 steps):

$$-\frac{\sqrt{c+d x^3}}{3 a x^3}+\frac{\left(2 b c-a d\right) \operatorname{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{3 a^2 \sqrt{c}}-\frac{2 \sqrt{b} \sqrt{b c-a d} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{c+d x^3}}{\sqrt{b c-a d}}\right]}{3 a^2}$$

Result (type 6, 407 leaves):

$$\left(\left(6 b c d x^6 \operatorname{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Big/\left(-4 a c \operatorname{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+x^3\left(2 b c \operatorname{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d \operatorname{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)+\left(5 b d x^3\left(3 a c+b c x^3+4 a d x^3+3 b d x^6\right) \operatorname{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]-3\left(a+b x^3\right)\left(c+d x^3\right)\left(2 a d \operatorname{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]+b c \operatorname{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]\right)\right)\Big/\left(a\left(-5 b d x^3 \operatorname{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]+2 a d \operatorname{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]+b c \operatorname{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]\right)\right)\right)\Big/\left(9 x^3\left(a+b x^3\right)\sqrt{c+d x^3}\right)$$

Problem 363: Result more than twice size of optimal antiderivative.

$$\int \frac{x^3 \sqrt{c+d x^3}}{a+b x^3} \, dx$$

Optimal (type 6, 64 leaves, 2 steps):

$$\frac{x^4 \sqrt{c+d x^3} \, \text{AppellF1}\left[\frac{4}{3}, 1, -\frac{1}{2}, \frac{7}{3}, -\frac{b x^3}{a}, -\frac{d x^3}{c}\right]}{4 a \sqrt{1+\frac{d x^3}{c}}}$$

Result (type 6, 426 leaves):

$$\left(x \left(\left(32 a^2 c^2 \text{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right]\right) \Big/ \left(-8 a c \text{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right] + 3 x^3 \left(2 b c \text{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 2, \frac{7}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right] + a d \text{AppellF1}\left[\frac{4}{3}, \frac{3}{2}, 1, \frac{7}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right]\right)\right) + \left(-7 a c \left(8 a c + 11 b c x^3 + 3 a d x^3 + 8 b d x^6\right) \text{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right] + 12 x^3 \left(a + b x^3\right) \left(c + d x^3\right) \left(2 b c \text{AppellF1}\left[\frac{7}{3}, \frac{1}{2}, 2, \frac{10}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right] + a d \text{AppellF1}\left[\frac{7}{3}, \frac{3}{2}, 1, \frac{10}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right]\right)\right) \Big/ \left(-14 a c \text{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right] + 3 x^3 \left(2 b c \text{AppellF1}\left[\frac{7}{3}, \frac{1}{2}, 2, \frac{10}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right] + a d \text{AppellF1}\left[\frac{7}{3}, \frac{3}{2}, 1, \frac{10}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right]\right)\right)\right)\right) \Big/ \left(10 b \left(a + b x^3\right) \sqrt{c + d x^3}\right)$$

Problem 364: Result more than twice size of optimal antiderivative.

$$\int \frac{x \sqrt{c+d x^3}}{a+b x^3} \, dx$$

Optimal (type 6, 64 leaves, 2 steps):

$$\frac{x^2 \sqrt{c+d x^3} \, \text{AppellF1}\left[\frac{2}{3}, 1, -\frac{1}{2}, \frac{5}{3}, -\frac{b x^3}{a}, -\frac{d x^3}{c}\right]}{2 a \sqrt{1+\frac{d x^3}{c}}}$$

Result (type 6, 163 leaves):

$$\left(5\,a\,c\,x^2\,\sqrt{c+d\,x^3}\;\text{AppellF1}\left[\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\Big/\left(\left(a+b\,x^3\right)\left(10\,a\,c\,\text{AppellF1}\left[\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+3\,x^3\left(-2\,b\,c\;\text{AppellF1}\left[\frac{5}{3},-\frac{1}{2},2,\frac{8}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)\right)$$

Problem 365: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{c+d\,x^3}}{a+b\,x^3}\,dx$$

Optimal (type 6, 59 leaves, 2 steps):

$$\frac{x\,\sqrt{c+d\,x^3}\;\text{AppellF1}\left[\frac{1}{3},1,-\frac{1}{2},\frac{4}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]}{a\,\sqrt{1+\frac{d\,x^3}{c}}}$$

Result (type 6, 161 leaves):

$$\left(8\,a\,c\,x\,\sqrt{c+d\,x^3}\;\text{AppellF1}\left[\frac{1}{3},-\frac{1}{2},1,\frac{4}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\Big/\left(\left(a+b\,x^3\right)\left(8\,a\,c\,\text{AppellF1}\left[\frac{1}{3},-\frac{1}{2},1,\frac{4}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+3\,x^3\left(-2\,b\,c\;\text{AppellF1}\left[\frac{4}{3},-\frac{1}{2},2,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)\right)$$

Problem 366: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{c+d\,x^3}}{x^2\left(a+b\,x^3\right)}\,dx$$

Optimal (type 6, 62 leaves, 2 steps):

$$-\frac{\sqrt{c+d\,x^3}\;\text{AppellF1}\left[-\frac{1}{3},1,-\frac{1}{2},\frac{2}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]}{a\,x\,\sqrt{1+\frac{d\,x^3}{c}}}$$

Result (type 6, 344 leaves):

$$\frac{1}{10\,x\,\sqrt{c+d\,x^3}}\left(-\frac{10\,(c+d\,x^3)}{a}+\left(25\,c\,(2\,b\,c-3\,a\,d)\,x^3\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\Big/\left((a+b\,x^3)\left(-10\,a\,c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+3\,x^3\left(2\,b\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)\right)-\left(16\,b\,c\,d\,x^6\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\Big/\left((a+b\,x^3)\left(-16\,a\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+3\,x^3\left(2\,b\,c\,\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)\right)\right)$$

Problem 367: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{c+d\,x^3}}{x^3\,(a+b\,x^3)}\,dx$$

Optimal (type 6, 64 leaves, 2 steps):

$$-\frac{\sqrt{c+d\,x^3}\;\text{AppellF1}\left[-\frac{2}{3},1,-\frac{1}{2},\frac{1}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]}{2\,a\,x^2\,\sqrt{1+\frac{d\,x^3}{c}}}$$

Result (type 6, 344 leaves):

$$\frac{1}{8\,x^2\,\sqrt{c+d\,x^3}}\left(-\frac{4\,(c+d\,x^3)}{a}+\left(16\,c\,(4\,b\,c-3\,a\,d)\,x^3\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\Big/\left((a+b\,x^3)\left(-8\,a\,c\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+3\,x^3\left(2\,b\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)\right)+\left(7\,b\,c\,d\,x^6\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\Big/\left((a+b\,x^3)\left(-14\,a\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+3\,x^3\left(2\,b\,c\,\text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)\right)\right)$$

Problem 371: Result unnecessarily involves higher level functions and more

than twice size of optimal antiderivative.

$$\int \frac{\left(c+d x^3\right)^{3/2}}{x \left(a+b x^3\right)} \, dx$$

Optimal (type 3, 104 leaves, 7 steps):

$$\frac{2 d \sqrt{c+d x^3}}{3 b}-\frac{2 c^{3/2} \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{3 a}+\frac{2 (b c-a d)^{3/2} \text{ArcTanh}\left[\frac{\sqrt{b} \sqrt{c+d x^3}}{\sqrt{b c-a d}}\right]}{3 a b^{3/2}}$$

Result (type 6, 325 leaves):

$$\frac{1}{9 b \sqrt{c+d x^3}} 2 d \left(3 \left(c+d x^3\right)+\left(6 a c (-2 b c+a d) x^3 \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Big/\left(\left(a+b x^3\right) \left(-4 a c \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+x^3 \left(2 b c \text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d \text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)\right)+\left(5 b^2 c^2 x^3 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]\right)\Big/\left(\left(a+b x^3\right) \left(-5 b d x^3 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]+2 a d \text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]+b c \text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]\right)\right)\right)$$

Problem 372: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(c+d x^3\right)^{3/2}}{x^4 \left(a+b x^3\right)} \, dx$$

Optimal (type 3, 116 leaves, 7 steps):

$$-\frac{c \sqrt{c+d x^3}}{3 a x^3}+\frac{\sqrt{c} (2 b c-3 a d) \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{3 a^2}-\frac{2 (b c-a d)^{3/2} \text{ArcTanh}\left[\frac{\sqrt{b} \sqrt{c+d x^3}}{\sqrt{b c-a d}}\right]}{3 a^2 \sqrt{b}}$$

Result (type 6, 414 leaves):

$$\left(c\left(\left(6\,d\,(b\,c-2\,a\,d)\,x^6\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\Big/\left(-4\,a\,c\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+x^3\left(2\,b\,c\,\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)+\left(5\,b\,d\,x^3\,\left(3\,a\,(c+2\,d\,x^3)+b\,x^3\,(c+3\,d\,x^3)\right)\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]-3\,(a+b\,x^3)\,(c+d\,x^3)\left(2\,a\,d\,\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]\right)\right)\Big/\left(a\left(-5\,b\,d\,x^3\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]+2\,a\,d\,\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]\right)\right)\right)\right)\Big/\left(9\,x^3\,(a+b\,x^3)\,\sqrt{c+d\,x^3}\right)$$

Problem 373: Result more than twice size of optimal antiderivative.

$$\int\frac{x^3\,(c+d\,x^3)^{3/2}}{a+b\,x^3}\,dx$$

Optimal (type 6, 65 leaves, 2 steps):

$$\frac{c\,x^4\,\sqrt{c+d\,x^3}\,\text{AppellF1}\left[\frac{4}{3},1,-\frac{3}{2},\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]}{4\,a\,\sqrt{1+\frac{d\,x^3}{c}}}$$

Result (type 6, 382 leaves):

$$\frac{1}{110\,b^2\,\sqrt{c+d\,x^3}}x\left(4\,(c+d\,x^3)\,(14\,b\,c-11\,a\,d+5\,b\,d\,x^3)+\left(32\,a^2\,c^2\,(14\,b\,c-11\,a\,d)\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\Big/\left((a+b\,x^3)\left(-8\,a\,c\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+3\,x^3\left(2\,b\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)\right)-\left(7\,a\,c\,(27\,b^2\,c^2-88\,a\,b\,c\,d+55\,a^2\,d^2)\,x^3\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\Big/\left((a+b\,x^3)\left(-14\,a\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+3\,x^3\left(2\,b\,c\,\text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)\right)\right)$$

Problem 374: Result more than twice size of optimal antiderivative.

$$\int \frac{x\left(c+d x^3\right)^{3/2}}{a+b x^3}\,dx$$

Optimal (type 6, 65 leaves, 2 steps):

$$\frac{c x^2 \sqrt{c+d x^3}\ \text{AppellF1}\left[\frac{2}{3},1,-\frac{3}{2},\frac{5}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{2 a \sqrt{1+\frac{d x^3}{c}}}$$

Result (type 6, 437 leaves):

$$\left(x^2\left(\left(25 a c^2 (-7 b c+4 a d)\ \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Big/\left(-10 a c\ \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+3 x^3\left(2 b c\ \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d\ \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)+\left(2 d\left(-8 a c\left(10 a c+20 b c x^3+3 a d x^3+10 b d x^6\right)\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+15 x^3\left(a+b x^3\right)\left(c+d x^3\right)\left(2 b c\ \text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d\ \text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)\right)\Big/\left(-16 a c\ \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+3 x^3\left(2 b c\ \text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d\ \text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)\right)\right)\Big/\left(35 b\left(a+b x^3\right)\sqrt{c+d x^3}\right)$$

Problem 375: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(c+d x^3\right)^{3/2}}{a+b x^3}\,dx$$

Optimal (type 6, 60 leaves, 2 steps):

$$\frac{c x \sqrt{c+d x^3}\ \text{AppellF1}\left[\frac{1}{3},1,-\frac{3}{2},\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{a \sqrt{1+\frac{d x^3}{c}}}$$

Result (type 6, 434 leaves):

$$\left(x\left(\left(16\,a\,c^2\,(-5\,b\,c+2\,a\,d)\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\Big/\left(-8\,a\,c\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+3\,x^3\left(2\,b\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)+\left(d\left(-7\,a\,c\,(8\,a\,c+16\,b\,c\,x^3+3\,a\,d\,x^3+8\,b\,d\,x^6)\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+12\,x^3\,(a+b\,x^3)\,(c+d\,x^3)\left(2\,b\,c\,\text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)\right)\Big/\left(-14\,a\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+3\,x^3\left(2\,b\,c\,\text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)\right)\right)\Big/\left(10\,b\,(a+b\,x^3)\,\sqrt{c+d\,x^3}\right)$$

Problem 376: Result more than twice size of optimal antiderivative.

$$\int \frac{(c+d\,x^3)^{3/2}}{x^2\,(a+b\,x^3)}\,dx$$

Optimal (type 6, 63 leaves, 2 steps):

$$-\frac{c\,\sqrt{c+d\,x^3}\,\text{AppellF1}\left[-\frac{1}{3},1,-\frac{3}{2},\frac{2}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]}{a\,x\,\sqrt{1+\frac{d\,x^3}{c}}}$$

Result (type 6, 450 leaves):

$$\left(c\left(\left(25\,c\,(2\,b\,c-5\,a\,d)\,x^3\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\Big/\left(-10\,a\,c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+3\,x^3\left(2\,b\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)+\left(16\,a\left(b\,c\,x^3\,(10\,c+9\,d\,x^3)+2\,a\,(5\,c^2+5\,c\,d\,x^3-d^2\,x^6)\right)\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]-30\,x^3\,(a+b\,x^3)\,(c+d\,x^3)\left(2\,b\,c\,\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)\Big/\left(a\left(-16\,a\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+3\,x^3\left(2\,b\,c\,\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)\right)\right)\right)\Big/\left(10\,x\,(a+b\,x^3)\,\sqrt{c+d\,x^3}\right)$$

Problem 377: Result more than twice size of optimal antiderivative.

$$\int \frac{(c+d\,x^3)^{3/2}}{x^3\,(a+b\,x^3)}\,dx$$

Optimal (type 6, 65 leaves, 2 steps):

$$-\frac{c\,\sqrt{c+d\,x^3}\,\text{AppellF1}\left[-\frac{2}{3},1,-\frac{3}{2},\frac{1}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]}{2\,a\,x^2\,\sqrt{1+\frac{d\,x^3}{c}}}$$

Result (type 6, 449 leaves):

$$\left(c\left(\left(16\,c\,(4\,b\,c-7\,a\,d)\,x^3\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\Big/\left(-8\,a\,c\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+3\,x^3\left(2\,b\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)+\left(7\,a\,\left(b\,c\,x^3\,(8\,c+9\,d\,x^3)+a\,(8\,c^2+8\,c\,d\,x^3-4\,d^2\,x^6)\right)\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]-12\,x^3\,(a+b\,x^3)\,(c+d\,x^3)\left(2\,b\,c\,\text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)\Big/\left(a\left(-14\,a\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+3\,x^3\left(2\,b\,c\,\text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)\right)\right)\right)\Big/\left(8\,x^2\,(a+b\,x^3)\,\sqrt{c+d\,x^3}\right)$$

Problem 381: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x\,(a+b\,x^3)\,\sqrt{c+d\,x^3}}\,dx$$

Optimal (type 3, 85 leaves, 6 steps):

$$-\frac{2\,\text{ArcTanh}\left[\frac{\sqrt{c+d\,x^3}}{\sqrt{c}}\right]}{3\,a\,\sqrt{c}}+\frac{2\,\sqrt{b}\,\text{ArcTanh}\left[\frac{\sqrt{b}\,\sqrt{c+d\,x^3}}{\sqrt{b\,c-a\,d}}\right]}{3\,a\,\sqrt{b\,c-a\,d}}$$

Result (type 6, 162 leaves):

$$\left(10\,b\,d\,x^3\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]\right)\Big/\left(9\,(a+b\,x^3)\,\sqrt{c+d\,x^3}\left(-5\,b\,d\,x^3\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]+2\,a\,d\,\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]\right)\right)$$

Problem 382: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{x^4\,(a+b\,x^3)\,\sqrt{c+d\,x^3}}\,dx$$

Optimal (type 3, 117 leaves, 7 steps):

$$-\frac{\sqrt{c+d x^3}}{3 a c x^3}+\frac{(2 b c+a d) \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{3 a^2 c^{3/2}}-\frac{2 b^{3/2} \text{ArcTanh}\left[\frac{\sqrt{b} \sqrt{c+d x^3}}{\sqrt{b c-a d}}\right]}{3 a^2 \sqrt{b c-a d}}$$

Result (type 6, 409 leaves):

$$\left(\left(6 b d x^6 \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Big/\left(-4 a c \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+x^3\left(2 b c \text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d \text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)+\left(5 b d x^3\left(3 a c+b c x^3+2 a d x^3+3 b d x^6\right) \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]-3\left(a+b x^3\right)\left(c+d x^3\right)\left(2 a d \text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]+b c \text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]\right)\right)\Big/\left(a c\left(-5 b d x^3 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]+2 a d \text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]+b c \text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]\right)\right)\right)\Big/\left(9 x^3\left(a+b x^3\right)\sqrt{c+d x^3}\right)$$

Problem 383: Result more than twice size of optimal antiderivative.

$$\int \frac{x^3}{\left(a+b x^3\right)\sqrt{c+d x^3}}\,dx$$

Optimal (type 6, 64 leaves, 2 steps):

$$\frac{x^4\sqrt{1+\frac{d x^3}{c}}\ \text{AppellF1}\left[\frac{4}{3},1,\frac{1}{2},\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{4 a \sqrt{c+d x^3}}$$

Result (type 6, 165 leaves):

$$-\left(\left(7 a c x^4 \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Big/\left(2\left(a+b x^3\right)\sqrt{c+d x^3}\left(-14 a c \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+3 x^3\left(2 b c \text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d \text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)\right)\right)$$

Problem 384: Result more than twice size of optimal antiderivative.

$$\int \frac{x}{\left(a+b x^3\right)\sqrt{c+d x^3}}\,dx$$

Optimal (type 6, 64 leaves, 2 steps):

$$\frac{x^2 \sqrt{1+\frac{d x^3}{c}}\ \text{AppellF1}\left[\frac{2}{3},1,\frac{1}{2},\frac{5}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{2 a \sqrt{c+d x^3}}$$

Result (type 6, 163 leaves):

$$-\left(\left(5 a c x^2 \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Big/\left(\left(a+b x^3\right) \sqrt{c+d x^3} \left(-10 a c \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+3 x^3 \left(2 b c\, \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)\right)\right)$$

Problem 385: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(a+b x^3\right) \sqrt{c+d x^3}}\, dx$$

Optimal (type 6, 59 leaves, 2 steps):

$$\frac{x \sqrt{1+\frac{d x^3}{c}}\ \text{AppellF1}\left[\frac{1}{3},1,\frac{1}{2},\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{a \sqrt{c+d x^3}}$$

Result (type 6, 161 leaves):

$$-\left(\left(8 a c x \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Big/\left(\left(a+b x^3\right) \sqrt{c+d x^3} \left(-8 a c \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+3 x^3 \left(2 b c\, \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d \text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)\right)\right)$$

Problem 386: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^2 \left(a+b x^3\right) \sqrt{c+d x^3}}\, dx$$

Optimal (type 6, 62 leaves, 2 steps):

$$-\frac{\sqrt{1+\frac{d x^3}{c}}\ \text{AppellF1}\left[-\frac{1}{3},1,\frac{1}{2},\frac{2}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{a x \sqrt{c+d x^3}}$$

Result (type 6, 345 leaves):

$$\frac{1}{10\,x\,\sqrt{c+d\,x^3}}\left(-\frac{10\,(c+d\,x^3)}{a\,c}+\left(25\,(2\,b\,c-a\,d)\,x^3\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\Big/\left((a+b\,x^3)\left(-10\,a\,c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+3\,x^3\left(2\,b\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)\right)-\left(16\,b\,d\,x^6\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\Big/\left((a+b\,x^3)\left(-16\,a\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+3\,x^3\left(2\,b\,c\,\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)\right)\right)$$

Problem 387: Result more than twice size of optimal antiderivative.

$$\int\frac{1}{x^3\,(a+b\,x^3)\,\sqrt{c+d\,x^3}}\,dx$$

Optimal (type 6, 64 leaves, 2 steps):

$$-\frac{\sqrt{1+\frac{d\,x^3}{c}}\,\text{AppellF1}\left[-\frac{2}{3},1,\frac{1}{2},\frac{1}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]}{2\,a\,x^2\,\sqrt{c+d\,x^3}}$$

Result (type 6, 344 leaves):

$$\frac{1}{8\,x^2\,\sqrt{c+d\,x^3}}\left(-\frac{4\,(c+d\,x^3)}{a\,c}+\left(16\,(4\,b\,c+a\,d)\,x^3\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\Big/\left((a+b\,x^3)\left(-8\,a\,c\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+3\,x^3\left(2\,b\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)\right)+\left(7\,b\,d\,x^6\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\Big/\left((a+b\,x^3)\left(-14\,a\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+3\,x^3\left(2\,b\,c\,\text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)\right)\right)$$

Problem 391: Result unnecessarily involves higher level functions and more

than twice size of optimal antiderivative.

$$\int \frac{1}{x\left(a+b x^3\right)\left(c+d x^3\right)^{3/2}}\,dx$$

Optimal (type 3, 114 leaves, 7 steps):

$$-\frac{2 d}{3 c (b c-a d) \sqrt{c+d x^3}}-\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{3 a c^{3/2}}+\frac{2 b^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{c+d x^3}}{\sqrt{b c-a d}}\right]}{3 a (b c-a d)^{3/2}}$$

Result (type 6, 396 leaves):

$$\left(2 d \left(\left(6 a b x^3 \operatorname{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Big/\left(-4 a c \operatorname{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+x^3 \left(2 b c \operatorname{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d \operatorname{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)+\left(5 b x^3 \left(2 a d+b \left(c+3 d x^3\right)\right) \operatorname{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]-3 \left(a+b x^3\right) \left(2 a d \operatorname{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]+b c \operatorname{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]\right)\right)\Big/\left(c \left(-5 b d x^3 \operatorname{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]+2 a d \operatorname{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]+b c \operatorname{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]\right)\right)\right)\right)\Big/\left(9 (b c-a d) \left(a+b x^3\right) \sqrt{c+d x^3}\right)$$

Problem 392: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{x^4\left(a+b x^3\right)\left(c+d x^3\right)^{3/2}}\,dx$$

Optimal (type 3, 158 leaves, 8 steps):

$$-\frac{d (b c-3 a d)}{3 a c^2 (b c-a d) \sqrt{c+d x^3}}-\frac{1}{3 a c x^3 \sqrt{c+d x^3}}+\frac{(2 b c+3 a d) \operatorname{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{3 a^2 c^{5/2}}-\frac{2 b^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{c+d x^3}}{\sqrt{b c-a d}}\right]}{3 a^2 (b c-a d)^{3/2}}$$

Result (type 6, 501 leaves):

$$\left(\left(6\,b\,c\,d\,(b\,c-3\,a\,d)\,x^6\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\Big/\right.$$
$$\left((b\,c-a\,d)\left(-4\,a\,c\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+x^3\left(2\,b\,c\right.\right.\right.$$
$$\left.\left.\left.\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)\right)-$$
$$\left(5\,b\,d\,x^3\left(-3\,a^2\,d\,(c+2\,d\,x^3)+b^2\,c\,x^3\,(c+3\,d\,x^3)+a\,b\,(3\,c^2-c\,d\,x^3-9\,d^2\,x^6)\right)\text{AppellF1}\left[\frac{3}{2},\right.\right.$$
$$\left.\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]+3\left(-b^2\,c\,x^3\,(c+d\,x^3)+a^2\,d\,(c+3\,d\,x^3)-a\,b\,(c^2-3\,d^2\,x^6)\right)\left(2\,a\,d\right.$$
$$\left.\left.\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]\right)\right)\Big/$$
$$\left(a\,(-b\,c+a\,d)\left(-5\,b\,d\,x^3\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]+\right.\right.$$
$$2\,a\,d\,\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]+$$
$$\left.\left.\left.b\,c\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]\right)\right)\right)\Big/\left(9\,c^2\,x^3\,(a+b\,x^3)\,\sqrt{c+d\,x^3}\right)$$

Problem 393: Result more than twice size of optimal antiderivative.

$$\int \frac{x^3}{(a+b\,x^3)\,(c+d\,x^3)^{3/2}}\,dx$$

Optimal (type 6, 67 leaves, 2 steps):

$$\frac{x^4\,\sqrt{1+\frac{d\,x^3}{c}}\,\text{AppellF1}\left[\frac{4}{3},1,\frac{3}{2},\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]}{4\,a\,c\,\sqrt{c+d\,x^3}}$$

Result (type 6, 332 leaves):

$$\left(x\left(-4-\left(32\,a^2\,c\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\Big/\right.\right.$$
$$\left((a+b\,x^3)\left(-8\,a\,c\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+3\,x^3\left(2\,b\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},\right.\right.\right.\right.$$
$$\left.\left.\left.\left.2,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)\right)+$$
$$\left(7\,a\,b\,c\,x^3\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\Big/\left((a+b\,x^3)\left(-14\,a\,c\,\text{AppellF1}\left[\right.\right.\right.$$
$$\left.\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+3\,x^3\left(2\,b\,c\,\text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+\right.$$
$$\left.\left.\left.\left.\left.a\,d\,\text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)\right)\right)\right)\Big/\left(6\,(-b\,c+a\,d)\,\sqrt{c+d\,x^3}\right)$$

Problem 394: Result more than twice size of optimal antiderivative.

$$\int \frac{x}{\left(a+b x^3\right) \left(c+d x^3\right)^{3/2}} \, dx$$

Optimal (type 6, 67 leaves, 2 steps):

$$\frac{x^2 \sqrt{1+\frac{d x^3}{c}} \, \text{AppellF1}\left[\frac{2}{3},1,\frac{3}{2},\frac{5}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{2 a c \sqrt{c+d x^3}}$$

Result (type 6, 366 leaves):

$$\frac{1}{15 \sqrt{c+d x^3}} x^2 \left(\left(25 a \left(3 b c+a d\right) \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Big/ \left(\left(-b c+a d\right) \left(a+b x^3\right) \left(-10 a c \, \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+3 x^3 \left(2 b c \, \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d \, \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)\right)+2 d \left(-\frac{5}{b c^2-a c d}+\left(8 a b x^3 \, \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Big/ \left(\left(-b c+a d\right) \left(a+b x^3\right) \left(-16 a c \, \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+3 x^3 \left(2 b c \, \text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d \, \text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)\right)\right)\right)$$

Problem 395: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(a+b x^3\right) \left(c+d x^3\right)^{3/2}} \, dx$$

Optimal (type 6, 62 leaves, 2 steps):

$$\frac{x \sqrt{1+\frac{d x^3}{c}} \, \text{AppellF1}\left[\frac{1}{3},1,\frac{3}{2},\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{a c \sqrt{c+d x^3}}$$

Result (type 6, 362 leaves):

$$\frac{1}{6\sqrt{c+d x^3}} \times \Bigg(-\frac{4 d}{b c^2-a c d}+\left(16 a\,(-3 b c+a d)\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Big/\Bigg((b c-a d)\,(a+b x^3)\,\Bigg(-8 a c\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+3 x^3\left(2 b c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d\,\text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Bigg)\Bigg)-\left(7 a b d x^3\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Big/\Bigg((-b c+a d)\,(a+b x^3)\,\Bigg(-14 a c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+3 x^3\left(2 b c\,\text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d\,\text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Bigg)\Bigg)\Bigg)$$

Problem 396: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^2\,(a+b x^3)\,(c+d x^3)^{3/2}}\,dx$$

Optimal (type 6, 65 leaves, 2 steps):

$$-\frac{\sqrt{1+\frac{d x^3}{c}}\;\text{AppellF1}\left[-\frac{1}{3},1,\frac{3}{2},\frac{2}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{a c x\sqrt{c+d x^3}}$$

Result (type 6, 408 leaves):

$$\frac{1}{30 c^2 x\sqrt{c+d x^3}}\Bigg(\left(\left(25 c\,(6 b^2 c^2-3 a b c d+5 a^2 d^2)\,x^3\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Big/\Bigg((b c-a d)\,(a+b x^3)\,\Bigg(-10 a c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+3 x^3\left(2 b c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d\,\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Bigg)\Bigg)\right)+\frac{1}{-b c+a d}2\Bigg(\frac{15 b c\,(c+d x^3)}{a}-5 d\,(3 c+5 d x^3)+\left(8 b c d\,(3 b c-5 a d)\,x^6\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Big/\Bigg((a+b x^3)\,\Bigg(-16 a c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+3 x^3\left(2 b c\,\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d\,\text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Bigg)\Bigg)\Bigg)\Bigg)$$

Problem 397: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^3 \left(a+b x^3\right) \left(c+d x^3\right)^{3/2}} \, dx$$

Optimal (type 6, 67 leaves, 2 steps):

$$-\frac{\sqrt{1+\frac{d x^3}{c}} \text{AppellF1}\left[-\frac{2}{3},1,\frac{3}{2},\frac{1}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{2 a c x^2 \sqrt{c+d x^3}}$$

Result (type 6, 418 leaves):

$$\frac{1}{24 c^2 x^2 \sqrt{c+d x^3}} \left(\frac{12 b c \left(c+d x^3\right)-4 a d \left(3 c+7 d x^3\right)}{a \left(-b c+a d\right)}+\left(16 c \left(12 b^2 c^2+3 a b c d-7 a^2 d^2\right) x^3 \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right) \Big/ \left(\left(b c-a d\right) \left(a+b x^3\right) \left(-8 a c \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+3 x^3 \left(2 b c \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d \text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)\right)+\left(7 b c d \left(3 b c-7 a d\right) x^6 \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right) \Big/ \left(\left(b c-a d\right) \left(a+b x^3\right) \left(-14 a c \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+3 x^3 \left(2 b c \text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d \text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)\right)\right)$$

Problem 402: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{c+d x^3}}{x \left(8 c-d x^3\right)^2} \, dx$$

Optimal (type 3, 88 leaves, 7 steps):

$$\frac{\sqrt{c+d x^3}}{24 c \left(8 c-d x^3\right)}+\frac{5 \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{288 c^{3/2}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{96 c^{3/2}}$$

Result (type 6, 316 leaves):

$$\frac{1}{72\sqrt{c+d x^3}}\left(\left(24\, d x^3\, \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\left(16 c\, \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+d x^3\left(\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\, \text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)+\frac{1}{-8 c+d x^3}\left(-3-\frac{3 d x^3}{c}+\left(10\, d x^3\, \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]\right)\Big/\left(5\, d x^3\, \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]+16 c\, \text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]-c\, \text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]\right)\right)\right)$$

Problem 403: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{c+d x^3}}{x^4\left(8 c-d x^3\right)^2}\, dx$$

Optimal (type 3, 124 leaves, 8 steps):

$$\frac{d\sqrt{c+d x^3}}{96 c^2\left(8 c-d x^3\right)}-\frac{\sqrt{c+d x^3}}{24 c x^3\left(8 c-d x^3\right)}+\frac{7 d\, \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3\sqrt{c}}\right]}{1152 c^{5/2}}-\frac{d\, \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{128 c^{5/2}}$$

Result (type 6, 338 leaves):

$$\frac{1}{96 c^2 x^3\sqrt{c+d x^3}}\left(\left(8 c d^2 x^6\, \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\left(16 c\, \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+d x^3\left(\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\, \text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)+\frac{1}{-8 c+d x^3}\left(4 c^2+3 c d x^3-d^2 x^6+\left(10\, c d^2 x^6\, \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]\right)\Big/\left(5\, d x^3\, \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]+16 c\, \text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]-c\, \text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]\right)\right)\right)$$

Problem 404: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{c+d x^3}}{x^7\left(8 c-d x^3\right)^2}\, dx$$

Optimal (type 3, 164 leaves, 9 steps):

$$\frac{5 d^2 \sqrt{c+d x^3}}{1536 c^3 \left(8 c-d x^3\right)}-\frac{\sqrt{c+d x^3}}{48 c x^6 \left(8 c-d x^3\right)}-\frac{7 d \sqrt{c+d x^3}}{384 c^2 x^3 \left(8 c-d x^3\right)}+\frac{23 d^2 \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{18432 c^{7/2}}-\frac{d^2 \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{2048 c^{7/2}}$$

Result (type 6, 349 leaves):

$$\left(\left(40 c d^3 x^9 \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\left(16 c \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+d x^3\left(\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)+\frac{1}{-8 c+d x^3}\left(32 c^3+60 c^2 d x^3+23 c d^2 x^6-5 d^3 x^9+\left(10 c d^3 x^9 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]\right)\Big/\left(5 d x^3 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]+16 c \text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]-c \text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]\right)\right)\right)\Big/\left(1536 c^3 x^6 \sqrt{c+d x^3}\right)$$

Problem 405: Result unnecessarily involves higher level functions.

$$\int \frac{x^7 \sqrt{c+d x^3}}{\left(8 c-d x^3\right)^2} \, dx$$

Optimal (type 4, 663 leaves, 15 steps):

$$\frac{13 x^2 \sqrt{c+d x^3}}{21 d^2}+\frac{746 c \sqrt{c+d x^3}}{21 d^{8/3}\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)}+\frac{x^5 \sqrt{c+d x^3}}{3 d\left(8 c-d x^3\right)}+$$

$$\frac{76 c^{7/6} \operatorname{ArcTan}\left[\frac{\sqrt{3} c^{1/6}\left(c^{1/3}+d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]}{3 \sqrt{3} d^{8/3}}-\frac{76 c^{7/6} \operatorname{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3} x\right)^2}{3 c^{1/6} \sqrt{c+d x^3}}\right]}{9 d^{8/3}}+$$

$$\frac{76 c^{7/6} \operatorname{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{9 d^{8/3}}-\left(373 \sqrt{2-\sqrt{3}}\, c^{4/3}\left(c^{1/3}+d^{1/3} x\right)\right.$$

$$\left.\sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\ \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]\right) \Big/$$

$$\left(7\times 3^{3/4} d^{8/3} \sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}\right)+\left(746 \sqrt{2}\, c^{4/3}\left(c^{1/3}+d^{1/3} x\right)\right.$$

$$\left.\sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\ \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]\right) \Big/$$

$$\left(21\times 3^{1/4} d^{8/3} \sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}\right)$$

Result (type 6, 344 leaves):

$$\left(2 x^2\left(5\left(c+d x^3\right)\left(-52 c+3 d x^3\right)+\left(10400 c^3 \operatorname{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\right.\right.$$

$$\left(40 c \operatorname{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+\right.$$

$$\left.3 d x^3\left(\operatorname{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \operatorname{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)+$$

$$\left(11936 c^2 d x^3 \operatorname{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/$$

$$\left(64 c \operatorname{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\operatorname{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-\right.\right.$$

$$\left.\left.\left.\left.4 \operatorname{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)\Big/\left(105 d^2\left(-8 c+d x^3\right) \sqrt{c+d x^3}\right)$$

Problem 406: Result unnecessarily involves higher level functions.

$$\int \frac{x^4 \sqrt{c+d x^3}}{\left(8 c-d x^3\right)^2}\, dx$$

Optimal (type 4, 641 leaves, 14 steps):

$$\frac{7\sqrt{c+d x^3}}{3 d^{5/3}\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)}+\frac{x^2\sqrt{c+d x^3}}{3 d\left(8 c-d x^3\right)}+\frac{5 c^{1/6}\,\text{ArcTan}\left[\frac{\sqrt{3}\, c^{1/6}\left(c^{1/3}+d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]}{3\sqrt{3}\, d^{5/3}}-\frac{5 c^{1/6}\,\text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3} x\right)^2}{3 c^{1/6}\sqrt{c+d x^3}}\right]}{9 d^{5/3}}+$$

$$\frac{5 c^{1/6}\,\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3\sqrt{c}}\right]}{9 d^{5/3}}-\left(7\sqrt{2-\sqrt{3}}\, c^{1/3}\left(c^{1/3}+d^{1/3} x\right)\sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(2\times 3^{3/4} d^{5/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\sqrt{c+d x^3}\right)+\left(7\sqrt{2}\, c^{1/3}\left(c^{1/3}+d^{1/3} x\right)\right.$$

$$\left.\sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(3\times 3^{1/4} d^{5/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\sqrt{c+d x^3}\right)$$

Result (type 6, 357 leaves):

$$\frac{1}{15\sqrt{c+d x^3}} x^2\left(-\frac{5\left(c+d x^3\right)}{d\left(-8 c+d x^3\right)}+\left(200 c^2\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\right.$$

$$\left(d\left(-8 c+d x^3\right)\left(40 c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+\right.\right.$$

$$\left.\left.3 d x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\,\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)-$$

$$\left(224 c x^3\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/$$

$$\left(\left(8 c-d x^3\right)\left(64 c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\right.\right.$$

$$\left.\left.\left.\left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\,\text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)$$

Problem 407: Result unnecessarily involves higher level functions.

$$\int \frac{x\sqrt{c+d\,x^3}}{\left(8\,c-d\,x^3\right)^2}\,dx$$

Optimal (type 4, 644 leaves, 14 steps):

$$\frac{\sqrt{c+d\,x^3}}{24\,c\,d^{2/3}\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x\right)}+\frac{x^2\sqrt{c+d\,x^3}}{24\,c\left(8\,c-d\,x^3\right)}+\frac{\text{ArcTan}\left[\frac{\sqrt{3}\,c^{1/6}\left(c^{1/3}+d^{1/3}\,x\right)}{\sqrt{c+d\,x^3}}\right]}{48\sqrt{3}\,c^{5/6}\,d^{2/3}}-\frac{\text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3}\,x\right)^2}{3\,c^{1/6}\sqrt{c+d\,x^3}}\right]}{144\,c^{5/6}\,d^{2/3}}+\frac{\text{ArcTanh}\left[\frac{\sqrt{c+d\,x^3}}{3\sqrt{c}}\right]}{144\,c^{5/6}\,d^{2/3}}-\left(\sqrt{2-\sqrt{3}}\left(c^{1/3}+d^{1/3}\,x\right)\sqrt{\frac{c^{2/3}-c^{1/3}\,d^{1/3}\,x+d^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}\,x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x}\right],-7-4\sqrt{3}\right]\right)\Bigg/\left(16\times 3^{3/4}\,c^{2/3}\,d^{2/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x\right)^2}}\sqrt{c+d\,x^3}\right)+\left(\left(c^{1/3}+d^{1/3}\,x\right)\sqrt{\frac{c^{2/3}-c^{1/3}\,d^{1/3}\,x+d^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}\,x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x}\right],-7-4\sqrt{3}\right]\right)\Bigg/\left(12\sqrt{2}\,3^{1/4}\,c^{2/3}\,d^{2/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x\right)^2}}\sqrt{c+d\,x^3}\right)$$

Result (type 6, 353 leaves):

$$\frac{1}{120\sqrt{c+d\,x^3}}x^2\left(\frac{5\left(c+d\,x^3\right)}{c\left(8\,c-d\,x^3\right)}+\left(100\,c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\Bigg/\left(\left(8\,c-d\,x^3\right)\left(40\,c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]+3\,d\,x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]-4\,\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\right)\right)+\left(32\,d\,x^3\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\Bigg/\left(\left(-8\,c+d\,x^3\right)\left(64\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]+3\,d\,x^3\left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]-4\,\text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\right)\right)\right)$$

Problem 408: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{c+d x^3}}{x^2 \left(8 c-d x^3\right)^2} \, dx$$

Optimal (type 4, 665 leaves, 15 steps):

$$-\frac{\sqrt{c+d x^3}}{48 c^2 x}+\frac{d^{1/3} \sqrt{c+d x^3}}{48 c^2 \left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)}+\frac{\sqrt{c+d x^3}}{24 c x \left(8 c-d x^3\right)}-\frac{d^{1/3} \text{ArcTan}\left[\frac{\sqrt{3} c^{1/6} \left(c^{1/3}+d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]}{48 \sqrt{3} c^{11/6}}+\frac{d^{1/3} \text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3} x\right)^2}{3 c^{1/6} \sqrt{c+d x^3}}\right]}{144 c^{11/6}}-\frac{d^{1/3} \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{144 c^{11/6}}-\left(\sqrt{2-\sqrt{3}} d^{1/3} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]\right) \Bigg/ \left(32\times 3^{3/4} c^{5/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}\right)+\left(d^{1/3} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]\right) \Bigg/ \left(24 \sqrt{2} 3^{1/4} c^{5/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}\right)$$

Result (type 6, 372 leaves):

$$\frac{1}{30\sqrt{c+d x^3}}\left(-\frac{5\left(6 c-d x^3\right)\left(c+d x^3\right)}{8 c^2\left(8 c x-d x^4\right)}+\left(125 d x^2 \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\left(40 c \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)-\left(4 d^2 x^5 \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(c\left(8 c-d x^3\right)\left(64 c \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)$$

Problem 409: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{c+d x^3}}{x^5\left(8 c-d x^3\right)^2}\,dx$$

Optimal (type 4, 687 leaves, 16 steps):

$$-\frac{7\sqrt{c+d x^3}}{768 c^2 x^4}-\frac{d\sqrt{c+d x^3}}{96 c^3 x}+\frac{d^{4/3}\sqrt{c+d x^3}}{96 c^3\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)}+$$

$$\frac{\sqrt{c+d x^3}}{24 c x^4\left(8 c-d x^3\right)}-\frac{17 d^{4/3}\,\text{ArcTan}\left[\frac{\sqrt{3}\,c^{1/6}\left(c^{1/3}+d^{1/3}x\right)}{\sqrt{c+d x^3}}\right]}{3072\sqrt{3}\,c^{17/6}}+\frac{17 d^{4/3}\,\text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3}x\right)^2}{3 c^{1/6}\sqrt{c+d x^3}}\right]}{9216 c^{17/6}}-$$

$$\frac{17 d^{4/3}\,\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3\sqrt{c}}\right]}{9216 c^{17/6}}-\left(\sqrt{2-\sqrt{3}}\,d^{4/3}\left(c^{1/3}+d^{1/3}x\right)\right.$$

$$\left.\sqrt{\frac{c^{2/3}-c^{1/3}d^{1/3}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(64\times 3^{3/4}c^{8/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\sqrt{c+d x^3}\right)+\left(d^{4/3}\left(c^{1/3}+d^{1/3}x\right)\right.$$

$$\left.\sqrt{\frac{c^{2/3}-c^{1/3}d^{1/3}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(48\sqrt{2}\,3^{1/4}c^{8/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\sqrt{c+d x^3}\right)$$

Result (type 6, 362 leaves):

$$\left(-5\left(c+d x^3\right)\left(24 c^2+57 c d x^3-8 d^2 x^6\right)+\left(5750 c^2 d^2 x^6\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\right.$$

$$\left(40 c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+\right.$$

$$\left.3 d x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\,\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)-$$

$$\left(256 c d^3 x^9\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(64 c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+\right.$$

$$\left.\left.3 d x^3\left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\,\text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\Big/$$

$$\left(3840 c^3 x^4\left(8 c-d x^3\right)\sqrt{c+d x^3}\right)$$

Problem 410: Result unnecessarily involves higher level functions.

$$\int\frac{\sqrt{c+d x^3}}{x^8\left(8 c-d x^3\right)^2}\,dx$$

Optimal (type 4, 711 leaves, 17 steps):

$$-\frac{5\sqrt{c+d x^3}}{672 c^2 x^7}-\frac{53 d\sqrt{c+d x^3}}{21504 c^3 x^4}-\frac{d^2\sqrt{c+d x^3}}{5376 c^4 x}+\frac{d^{7/3}\sqrt{c+d x^3}}{5376 c^4\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)}+\frac{\sqrt{c+d x^3}}{24 c x^7\left(8c-d x^3\right)}-\frac{13 d^{7/3}\text{ArcTan}\left[\frac{\sqrt{3}\,c^{1/6}\left(c^{1/3}+d^{1/3}x\right)}{\sqrt{c+d x^3}}\right]}{12288\sqrt{3}\,c^{23/6}}+\frac{13 d^{7/3}\text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3}x\right)^2}{3 c^{1/6}\sqrt{c+d x^3}}\right]}{36864 c^{23/6}}-\frac{13 d^{7/3}\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3\sqrt{c}}\right]}{36864 c^{23/6}}-\left(\sqrt{2-\sqrt{3}}\,d^{7/3}\left(c^{1/3}+d^{1/3}x\right)\sqrt{\frac{c^{2/3}-c^{1/3}d^{1/3}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Bigg/\left(3584\times 3^{3/4}c^{11/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\sqrt{c+d x^3}\right)+\left(d^{7/3}\left(c^{1/3}+d^{1/3}x\right)\sqrt{\frac{c^{2/3}-c^{1/3}d^{1/3}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Bigg/\left(2688\sqrt{2}\,3^{1/4}c^{11/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\sqrt{c+d x^3}\right)$$

Result (type 6, 377 leaves):

$$\left(-5\left(384c^4+648c^3 d x^3+243c^2d^2x^6-25c d^3x^9-4d^4x^{12}\right)+\left(15250c^2d^3x^9\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8c}\right]\right)\Big/\left(40c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8c}\right]+3d x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8c}\right]-4\,\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8c}\right]\right)\right)-\left(128c d^4x^{12}\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8c}\right]\right)\Big/\left(64c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8c}\right]+3d x^3\left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8c}\right]-4\,\text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8c}\right]\right)\right)\right)\Big/\left(107520c^4x^7\left(8c-d x^3\right)\sqrt{c+d x^3}\right)$$

Problem 415: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(c+d x^3\right)^{3/2}}{x \left(8 c-d x^3\right)^2} \, dx$$

Optimal (type 3, 85 leaves, 7 steps):

$$\frac{3 \sqrt{c+d x^3}}{8 \left(8 c-d x^3\right)}-\frac{3 \,\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{32 \sqrt{c}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{96 \sqrt{c}}$$

Result (type 6, 317 leaves):

$$\frac{1}{72 \sqrt{c+d x^3}}\left(-\left(\left(168 c d x^3 \,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\left(16 c \,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+d x^3\left(\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \,\text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)+\frac{1}{-8 c+d x^3}\left(-27\left(c+d x^3\right)+\left(10 c d x^3 \,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]\right)\Big/\left(5 d x^3 \,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]+16 c \,\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]-c \,\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]\right)\right)\right)$$

Problem 416: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(c+d x^3\right)^{3/2}}{x^4 \left(8 c-d x^3\right)^2} \, dx$$

Optimal (type 3, 121 leaves, 8 steps):

$$\frac{5 d \sqrt{c+d x^3}}{96 c \left(8 c-d x^3\right)}-\frac{\sqrt{c+d x^3}}{24 x^3 \left(8 c-d x^3\right)}+\frac{3 d \,\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{128 c^{3/2}}-\frac{7 d \,\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{384 c^{3/2}}$$

Result (type 6, 333 leaves):

$$\frac{1}{144\sqrt{c+d x^3}}\left(\left(60 d^2 x^3 \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\left(16 c\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+d x^3\left(\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\,\text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)+\frac{1}{2\left(-8 c+d x^3\right)}\left(-3 d+\frac{12 c}{x^3}-\frac{15 d^2 x^3}{c}+\left(70 d^2 x^3 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]\right)\Big/\left(5 d x^3 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]+16 c\,\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]-c\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]\right)\right)\right)$$

Problem 417: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(c+d x^3\right)^{3/2}}{x^7\left(8 c-d x^3\right)^2}\,dx$$

Optimal (type 3, 161 leaves, 9 steps):

$$\frac{7 d^2\sqrt{c+d x^3}}{512 c^2\left(8 c-d x^3\right)}-\frac{\sqrt{c+d x^3}}{48 x^6\left(8 c-d x^3\right)}-\frac{23 d\sqrt{c+d x^3}}{384 c x^3\left(8 c-d x^3\right)}+\frac{15 d^2 \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3\sqrt{c}}\right]}{2048 c^{5/2}}-\frac{17 d^2 \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{2048 c^{5/2}}$$

Result (type 6, 349 leaves):

$$\left(\left(168 c d^3 x^9 \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\left(16 c\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+d x^3\left(\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\,\text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)+\frac{1}{-8 c+d x^3}\left(32 c^3+124 c^2 d x^3+71 c d^2 x^6-21 d^3 x^9+\left(170 c d^3 x^9 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]\right)\Big/\left(5 d x^3 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]+16 c\,\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]-c\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]\right)\right)\right)\Big/\left(1536 c^2 x^6\sqrt{c+d x^3}\right)$$

Problem 418: Result unnecessarily involves higher level functions.

$$\int \frac{x^7 \left(c+d x^3\right)^{3/2}}{\left(8 c-d x^3\right)^2} \, dx$$

Optimal (type 4, 681 leaves, 16 steps):

$$\frac{103 c x^2 \sqrt{c+d x^3}}{13 d^2}+\frac{19 x^5 \sqrt{c+d x^3}}{39 d}+\frac{5906 c^2 \sqrt{c+d x^3}}{13 d^{8/3} \left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)}+\frac{x^5 \left(c+d x^3\right)^{3/2}}{3 d \left(8 c-d x^3\right)}+$$
$$\frac{108 \sqrt{3} c^{13/6} \text{ArcTan}\left[\frac{\sqrt{3} c^{1/6} \left(c^{1/3}+d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]}{d^{8/3}}-\frac{108 c^{13/6} \text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3} x\right)^2}{3 c^{1/6} \sqrt{c+d x^3}}\right]}{d^{8/3}}+$$
$$\frac{108 c^{13/6} \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{d^{8/3}}-\left(2953\times 3^{1/4} \sqrt{2-\sqrt{3}}\; c^{7/3} \left(c^{1/3}+d^{1/3} x\right)\right.$$
$$\left.\sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\; \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$
$$\left(13 d^{8/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\; \sqrt{c+d x^3}\right)+\left(5906 \sqrt{2}\; c^{7/3} \left(c^{1/3}+d^{1/3} x\right)\right.$$
$$\left.\sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\; \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$
$$\left(13\times 3^{1/4} d^{8/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\; \sqrt{c+d x^3}\right)$$

Result (type 6, 357 leaves):

$$\left(2\left(5\left(c+d x^3\right)\left(-412 c^2 x^2+24 c d x^5+d^2 x^8\right)+\left(82400 c^4 x^2 \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(40 c \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)+\left(94496 c^3 d x^5 \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(64 c \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)\Big/\left(65 d^2\left(-8 c+d x^3\right)\sqrt{c+d x^3}\right)$$

Problem 419: Result unnecessarily involves higher level functions.

$$\int \frac{x^4\left(c+d x^3\right)^{3/2}}{\left(8 c-d x^3\right)^2}\,dx$$

Optimal (type 4, 657 leaves, 15 steps):

$$\frac{13 x^2\sqrt{c+d x^3}}{21 d}+\frac{265 c\sqrt{c+d x^3}}{7 d^{5/3}\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3} x\right)}+\frac{x^2\left(c+d x^3\right)^{3/2}}{3 d\left(8 c-d x^3\right)}+\frac{9\sqrt{3}\, c^{7/6} \text{ArcTan}\left[\frac{\sqrt{3}\, c^{1/6}\left(c^{1/3}+d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]}{d^{5/3}}-\frac{9 c^{7/6} \text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3} x\right)^2}{3 c^{1/6}\sqrt{c+d x^3}}\right]}{d^{5/3}}+\frac{9 c^{7/6} \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3\sqrt{c}}\right]}{d^{5/3}}-\left(265\times 3^{1/4}\sqrt{2-\sqrt{3}}\, c^{4/3}\left(c^{1/3}+d^{1/3} x\right)\sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3} x\right)^2}}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3} x}\right],-7-4\sqrt{3}\right]\right)\Big/\left(14 d^{5/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3} x\right)^2}}\sqrt{c+d x^3}\right)+\left(265\sqrt{2}\, c^{4/3}\left(c^{1/3}+d^{1/3} x\right)\sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3} x\right)^2}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3} x}\right],-7-4\sqrt{3}\right]\right)\Big/\left(7\times 3^{1/4} d^{5/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3} x\right)^2}}\sqrt{c+d x^3}\right)$$

Result (type 6, 368 leaves):

$$\frac{1}{7\sqrt{c+d x^3}} x^2 \left(\frac{\left(c+d x^3\right)\left(-37 c+2 d x^3\right)}{d\left(-8 c+d x^3\right)}+\left(1480 c^3 \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/ \left(d\left(-8 c+d x^3\right)\left(40 c \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)-\left(1696 c^2 x^3 \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/ \left(\left(8 c-d x^3\right)\left(64 c \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)$$

Problem 420: Result unnecessarily involves higher level functions.

$$\int \frac{x\left(c+d x^3\right)^{3/2}}{\left(8 c-d x^3\right)^2} \, dx$$

Optimal (type 4, 638 leaves, 14 steps):

$$\frac{19\sqrt{c+d\,x^3}}{8\,d^{2/3}\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x\right)}+\frac{3\,x^2\sqrt{c+d\,x^3}}{8\left(8\,c-d\,x^3\right)}+$$

$$\frac{9\sqrt{3}\,c^{1/6}\,\text{ArcTan}\left[\frac{\sqrt{3}\,c^{1/6}\left(c^{1/3}+d^{1/3}\,x\right)}{\sqrt{c+d\,x^3}}\right]}{16\,d^{2/3}}-\frac{9\,c^{1/6}\,\text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3}\,x\right)^2}{3\,c^{1/6}\sqrt{c+d\,x^3}}\right]}{16\,d^{2/3}}+$$

$$\frac{9\,c^{1/6}\,\text{ArcTanh}\left[\frac{\sqrt{c+d\,x^3}}{3\sqrt{c}}\right]}{16\,d^{2/3}}-\left(19\times 3^{1/4}\sqrt{2-\sqrt{3}}\;c^{1/3}\left(c^{1/3}+d^{1/3}\,x\right)\sqrt{\frac{c^{2/3}-c^{1/3}\,d^{1/3}\,x+d^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}\,x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x}\right],-7-4\sqrt{3}\right]\right)\Bigg/$$

$$\left(16\,d^{2/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x\right)^2}}\;\sqrt{c+d\,x^3}\right)+\left(19\,c^{1/3}\left(c^{1/3}+d^{1/3}\,x\right)\right.$$

$$\left.\sqrt{\frac{c^{2/3}-c^{1/3}\,d^{1/3}\,x+d^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}\,x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x}\right],-7-4\sqrt{3}\right]\right)\Bigg/$$

$$\left(4\sqrt{2}\;3^{1/4}\,d^{2/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x\right)^2}}\;\sqrt{c+d\,x^3}\right)$$

Result (type 6, 330 leaves) :

$$\left(x^2\left(15\left(c+d\,x^3\right)-\right.\right.$$

$$\left(500\,c^2\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\Bigg/\left(40\,c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]+\right.$$

$$\left.3\,d\,x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]-4\,\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\right)-$$

$$\left(608\,c\,d\,x^3\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\Bigg/$$

$$\left(64\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]+3\,d\,x^3\left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]-\right.\right.$$

$$\left.\left.\left.\left.4\,\text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\right)\right)\right)\Bigg/\left(40\left(8\,c-d\,x^3\right)\sqrt{c+d\,x^3}\right)$$

Problem 421: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(c+d\,x^3\right)^{3/2}}{x^2\left(8\,c-d\,x^3\right)^2}\,dx$$

Optimal (type 4, 522 leaves, 6 steps):

$$-\frac{\sqrt{c+d x^3}}{16 c x}+\frac{d^{1/3}\sqrt{c+d x^3}}{16 c\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)}+\frac{3\sqrt{c+d x^3}}{8 x\left(8 c-d x^3\right)}-\left(3^{1/4}\sqrt{2-\sqrt{3}}\, d^{1/3}\left(c^{1/3}+d^{1/3} x\right)\sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4\sqrt{3}\right]\right)\Big/\left(32 c^{2/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\sqrt{c+d x^3}\right)+\left(d^{1/3}\left(c^{1/3}+d^{1/3} x\right)\sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4\sqrt{3}\right]\right)\Big/\left(8\sqrt{2}\, 3^{1/4} c^{2/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\sqrt{c+d x^3}\right)$$

Result (type 4, 242 leaves):

$$\frac{\left(2 c-d x^3\right)\sqrt{c+d x^3}}{16 c x\left(-8 c+d x^3\right)}-\left((-1)^{1/6}(-d)^{1/3}\sqrt{(-1)^{5/6}\left(-1+\frac{(-d)^{1/3} x}{c^{1/3}}\right)}\sqrt{1+\frac{(-d)^{1/3} x}{c^{1/3}}+\frac{(-d)^{2/3} x^2}{c^{2/3}}}\left(-i\sqrt{3}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-d)^{1/3} x}{c^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+(-1)^{1/3}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-d)^{1/3} x}{c^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right)\Big/\left(16\times 3^{1/4} c^{1/3}\sqrt{c+d x^3}\right)$$

Problem 422: Result unnecessarily involves higher level functions.

$$\int\frac{\left(c+d x^3\right)^{3/2}}{x^5\left(8 c-d x^3\right)^2}\,dx$$

Optimal (type 4, 684 leaves, 16 steps):

$$-\frac{13\sqrt{c+d x^3}}{256 c x^4}-\frac{d\sqrt{c+d x^3}}{32 c^2 x}+\frac{d^{4/3}\sqrt{c+d x^3}}{32 c^2\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)}+\frac{3\sqrt{c+d x^3}}{8 x^4\left(8 c-d x^3\right)}-\frac{9\sqrt{3}\,d^{4/3}\,\text{ArcTan}\left[\frac{\sqrt{3}\,c^{1/6}\left(c^{1/3}+d^{1/3}x\right)}{\sqrt{c+d x^3}}\right]}{1024 c^{11/6}}+\frac{9 d^{4/3}\,\text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3}x\right)^2}{3 c^{1/6}\sqrt{c+d x^3}}\right]}{1024 c^{11/6}}-\frac{9 d^{4/3}\,\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3\sqrt{c}}\right]}{1024 c^{11/6}}-\left(3^{1/4}\sqrt{2-\sqrt{3}}\,d^{4/3}\left(c^{1/3}+d^{1/3}x\right)\sqrt{\frac{c^{2/3}-c^{1/3}d^{1/3}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/\left(64 c^{5/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\sqrt{c+d x^3}\right)+\left(d^{4/3}\left(c^{1/3}+d^{1/3}x\right)\sqrt{\frac{c^{2/3}-c^{1/3}d^{1/3}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/\left(16\sqrt{2}\,3^{1/4}c^{5/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\sqrt{c+d x^3}\right)$$

Result (type 6, 361 leaves):

$$\left(-\frac{5\left(c+d x^3\right)\left(8 c^2+51 c d x^3-8 d^2 x^6\right)}{c^2}+\left(7250 d^2 x^6\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(40 c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\,\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)-\left(256 d^3 x^9\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(c\left(64 c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\,\text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)\Big/\left(1280 x^4\left(8 c-d x^3\right)\sqrt{c+d x^3}\right)$$

Problem 423: Result unnecessarily involves higher level functions.

$$\int \frac{\left(c+d x^3\right)^{3/2}}{x^8 \left(8 c-d x^3\right)^2} \, dx$$

Optimal (type 4, 708 leaves, 17 steps):

$$-\frac{11 \sqrt{c+d x^3}}{224 c x^7}-\frac{83 d \sqrt{c+d x^3}}{7168 c^2 x^4}-\frac{19 d^2 \sqrt{c+d x^3}}{1792 c^3 x}+\frac{19 d^{7/3} \sqrt{c+d x^3}}{1792 c^3 \left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)}+$$

$$\frac{3 \sqrt{c+d x^3}}{8 x^7 \left(8 c-d x^3\right)}-\frac{9 \sqrt{3}\, d^{7/3} \text{ArcTan}\left[\frac{\sqrt{3}\, c^{1/6} \left(c^{1/3}+d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]}{4096 c^{17/6}}+\frac{9 d^{7/3} \text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3} x\right)^2}{3 c^{1/6} \sqrt{c+d x^3}}\right]}{4096 c^{17/6}}-$$

$$\frac{9 d^{7/3} \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{4096 c^{17/6}}-\left(19 \times 3^{1/4} \sqrt{2-\sqrt{3}}\, d^{7/3} \left(c^{1/3}+d^{1/3} x\right)\right.$$

$$\left.\sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$

$$\left(3584 c^{8/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\ \sqrt{c+d x^3}\right)+\left(19 d^{7/3} \left(c^{1/3}+d^{1/3} x\right)\right.$$

$$\left.\sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$

$$\left(896 \sqrt{2}\, 3^{1/4} c^{8/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\ \sqrt{c+d x^3}\right)$$

Result (type 6, 373 leaves):

$$\Bigg(-5\left(c+d x^3\right)\left(128 c^3+312 c^2 d x^3+525 c d^2 x^6-76 d^3 x^9\right)+\left(58750 c^2 d^3 x^9 \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(40 c \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)-\left(2432 c d^4 x^{12} \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(64 c \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\Bigg)\Big/\left(35840 c^3 x^7\left(8 c-d x^3\right)\sqrt{c+d x^3}\right)$$

Problem 428: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{x\left(8 c-d x^3\right)^2 \sqrt{c+d x^3}}\, dx$$

Optimal (type 3, 88 leaves, 7 steps):

$$\frac{\sqrt{c+d x^3}}{216 c^2\left(8 c-d x^3\right)}+\frac{13 \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{2592 c^{5/2}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{96 c^{5/2}}$$

Result (type 6, 329 leaves):

$$\frac{1}{216 c^2 \sqrt{c+d x^3}}\Bigg(\frac{c+d x^3}{8 c-d x^3}+\left(8 c d x^3 \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\left(16 c \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+d x^3\left(\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)+\left(30 c d x^3 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]\right)\Big/\left(\left(-8 c+d x^3\right)\left(5 d x^3 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]+16 c \text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]-c \text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]\right)\right)\Bigg)$$

Problem 429: Result unnecessarily involves higher level functions and more

than twice size of optimal antiderivative.

$$\int \frac{1}{x^4 \left(8 c - d x^3\right)^2 \sqrt{c + d x^3}} \, dx$$

Optimal (type 3, 124 leaves, 8 steps):

$$\frac{5 d \sqrt{c + d x^3}}{864 c^3 \left(8 c - d x^3\right)} - \frac{\sqrt{c + d x^3}}{24 c^2 x^3 \left(8 c - d x^3\right)} + \frac{11 d \, \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{10368 c^{7/2}} + \frac{d \, \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{384 c^{7/2}}$$

Result (type 6, 347 leaves):

$$\left(-\frac{\left(c + d x^3\right) \left(-36 c + 5 d x^3\right)}{-8 c + d x^3} + \left(40 c d^2 x^6 \, \text{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right]\right) \Big/ \left(\left(8 c - d x^3\right) \left(16 c \, \text{AppellF1}\left[1, \frac{1}{2}, 1, 2, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right] + d x^3 \left(\text{AppellF1}\left[2, \frac{1}{2}, 2, 3, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right] - 4 \, \text{AppellF1}\left[2, \frac{3}{2}, 1, 3, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right]\right)\right)\right) + \left(30 c d^2 x^6 \, \text{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, 1, \frac{5}{2}, -\frac{c}{d x^3}, \frac{8 c}{d x^3}\right]\right) \Big/ \left(\left(8 c - d x^3\right) \left(5 d x^3 \, \text{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, 1, \frac{5}{2}, -\frac{c}{d x^3}, \frac{8 c}{d x^3}\right] + 16 c \, \text{AppellF1}\left[\frac{5}{2}, \frac{1}{2}, 2, \frac{7}{2}, -\frac{c}{d x^3}, \frac{8 c}{d x^3}\right] - c \, \text{AppellF1}\left[\frac{5}{2}, \frac{3}{2}, 1, \frac{7}{2}, -\frac{c}{d x^3}, \frac{8 c}{d x^3}\right]\right)\right)\right) \Big/ \left(864 c^3 x^3 \sqrt{c + d x^3}\right)$$

Problem 430: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{x^7 \left(8 c - d x^3\right)^2 \sqrt{c + d x^3}} \, dx$$

Optimal (type 3, 164 leaves, 9 steps):

$$-\frac{35 d^2 \sqrt{c + d x^3}}{13824 c^4 \left(8 c - d x^3\right)} - \frac{\sqrt{c + d x^3}}{48 c^2 x^6 \left(8 c - d x^3\right)} + \frac{3 d \sqrt{c + d x^3}}{128 c^3 x^3 \left(8 c - d x^3\right)} + \frac{31 d^2 \, \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{165888 c^{9/2}} - \frac{19 d^2 \, \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{6144 c^{9/2}}$$

Result (type 6, 349 leaves):

$$\left(-\left(\left(280\,c\,d^3\,x^9\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\Big/\left(\left(8\,c-d\,x^3\right)\left(16\,c\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]+d\,x^3\left(\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]-4\,\text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\right)\right)\right)+\frac{1}{-8\,c+d\,x^3}\left(288\,c^3-36\,c^2\,d\,x^3-289\,c\,d^2\,x^6+35\,d^3\,x^9+\left(570\,c\,d^3\,x^9\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^3},\frac{8\,c}{d\,x^3}\right]\right)\Big/\left(5\,d\,x^3\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^3},\frac{8\,c}{d\,x^3}\right]+16\,c\,\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d\,x^3},\frac{8\,c}{d\,x^3}\right]-c\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d\,x^3},\frac{8\,c}{d\,x^3}\right]\right)\right)\right)\Big/\left(13\,824\,c^4\,x^6\,\sqrt{c+d\,x^3}\right)$$

Problem 431: Result unnecessarily involves higher level functions.

$$\int \frac{x^7}{\left(8\,c-d\,x^3\right)^2\sqrt{c+d\,x^3}}\,dx$$

Optimal (type 4, 641 leaves, 14 steps):

$$\frac{62\sqrt{c+d x^3}}{27 d^{8/3}\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)}+\frac{8 x^2 \sqrt{c+d x^3}}{27 d^2 \left(8 c-d x^3\right)}+$$

$$\frac{44 c^{1/6} \text{ArcTan}\left[\frac{\sqrt{3} c^{1/6} \left(c^{1/3}+d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]}{27 \sqrt{3} d^{8/3}}-\frac{44 c^{1/6} \text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3} x\right)^2}{3 c^{1/6} \sqrt{c+d x^3}}\right]}{81 d^{8/3}}+$$

$$\frac{44 c^{1/6} \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{81 d^{8/3}}-\left(31 \sqrt{2-\sqrt{3}} c^{1/3} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$

$$\left(9\times 3^{3/4} d^{8/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}\right)+\left(62 \sqrt{2} c^{1/3} \left(c^{1/3}+d^{1/3} x\right)\right.$$

$$\left.\sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$

$$\left(27\times 3^{1/4} d^{8/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}\right)$$

Result (type 6, 333 leaves):

$$\left(8 x^2 \left(5 \left(c+d x^3\right)-\right.\right.$$

$$\left(200 c^2 \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(40 c \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+\right.$$

$$\left.3 d x^3 \left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)-$$

$$\left(248 c d x^3 \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/$$

$$\left(64 c \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3 \left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-\right.\right.$$

$$\left.\left.\left.\left.4 \text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)\Big/\left(135 d^2 \left(8 c-d x^3\right) \sqrt{c+d x^3}\right)$$

Problem 432: Result unnecessarily involves higher level functions.

$$\int \frac{x^4}{\left(8 c-d x^3\right)^2 \sqrt{c+d x^3}} \, dx$$

Optimal (type 4, 647 leaves, 14 steps):

$$\frac{\sqrt{c+d x^3}}{27 c d^{5/3} \left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)}+\frac{x^2 \sqrt{c+d x^3}}{27 c d \left(8 c-d x^3\right)}+\frac{\text{ArcTan}\left[\frac{\sqrt{3} c^{1/6} \left(c^{1/3}+d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]}{27 \sqrt{3} c^{5/6} d^{5/3}}-\frac{\text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3} x\right)^2}{3 c^{1/6} \sqrt{c+d x^3}}\right]}{81 c^{5/6} d^{5/3}}+\frac{\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{81 c^{5/6} d^{5/3}}-\left(\sqrt{2-\sqrt{3}} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/\left(18\times 3^{3/4} c^{2/3} d^{5/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}\right)+\left(\sqrt{2} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/\left(27\times 3^{1/4} c^{2/3} d^{5/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}\right)$$

Result (type 6, 360 leaves):

$$\frac{1}{135 \sqrt{c+d x^3}} x^2 \left(\frac{5 c+5 d x^3}{8 c^2 d-c d^2 x^3}+\left(200 c \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(d \left(-8 c+d x^3\right) \left(40 c \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3 \left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)-\left(32 x^3 \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right) \left(64 c \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3 \left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)$$

Problem 433: Result unnecessarily involves higher level functions.

$$\int \frac{x}{\left(8 c-d x^3\right)^2 \sqrt{c+d x^3}} \, dx$$

Optimal (type 4, 644 leaves, 14 steps):

$$\frac{\sqrt{c+d x^3}}{216 c^2 d^{2/3}\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)}+\frac{x^2 \sqrt{c+d x^3}}{216 c^2\left(8 c-d x^3\right)}-\frac{7 \operatorname{ArcTan}\left[\frac{\sqrt{3} c^{1/6}\left(c^{1/3}+d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]}{432 \sqrt{3} c^{11/6} d^{2/3}}+\frac{7 \operatorname{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3} x\right)^2}{3 c^{1/6} \sqrt{c+d x^3}}\right]}{1296 c^{11/6} d^{2/3}}-\frac{7 \operatorname{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{1296 c^{11/6} d^{2/3}}-\left(\sqrt{2-\sqrt{3}}\left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]\right) \Big/ \left(144 \times 3^{3/4} c^{5/3} d^{2/3} \sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}\right)+\left(\left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]\right) \Big/ \left(108 \sqrt{2}\, 3^{1/4} c^{5/3} d^{2/3} \sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}\right)$$

Result (type 6, 332 leaves):

$$\left(x^2\left(\left(2500 \operatorname{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right) \Big/ \left(40 c \operatorname{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\operatorname{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \operatorname{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)+\frac{1}{c^2}\left(5\left(c+d x^3\right)-\left(32 c d x^3 \operatorname{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right) \Big/ \left(64 c \operatorname{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\operatorname{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \operatorname{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)\right) \Big/ \left(1080\left(8 c-d x^3\right) \sqrt{c+d x^3}\right)$$

Problem 434: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^2\left(8 c-d x^3\right)^2 \sqrt{c+d x^3}}\, dx$$

Optimal (type 4, 665 leaves, 15 steps):

$$-\frac{7\sqrt{c+d x^3}}{432 c^3 x}+\frac{7 d^{1/3}\sqrt{c+d x^3}}{432 c^3\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)}+\frac{\sqrt{c+d x^3}}{216 c^2 x\left(8 c-d x^3\right)}-\frac{d^{1/3}\,\text{ArcTan}\left[\frac{\sqrt{3}\,c^{1/6}\left(c^{1/3}+d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]}{216\sqrt{3}\,c^{17/6}}+\frac{d^{1/3}\,\text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3} x\right)^2}{3 c^{1/6}\sqrt{c+d x^3}}\right]}{648 c^{17/6}}-\frac{d^{1/3}\,\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3\sqrt{c}}\right]}{648 c^{17/6}}-\left(7\sqrt{2-\sqrt{3}}\, d^{1/3}\left(c^{1/3}+d^{1/3} x\right)\sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4\sqrt{3}\right]\right)\Big/\left(288\times 3^{3/4} c^{8/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\sqrt{c+d x^3}\right)+\left(7 d^{1/3}\left(c^{1/3}+d^{1/3} x\right)\sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4\sqrt{3}\right]\right)\Big/\left(216\sqrt{2}\,3^{1/4} c^{8/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}}\sqrt{c+d x^3}\right)$$

Result (type 6, 375 leaves):

$$\frac{1}{135\sqrt{c+d x^3}}\left(-\frac{5\left(54 c-7 d x^3\right)\left(c+d x^3\right)}{16 c^3\left(8 c x-d x^4\right)}+\left(250 d x^2\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(c\left(8 c-d x^3\right)\left(40 c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\,\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)-\left(14 d^2 x^5\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(c^2\left(8 c-d x^3\right)\left(64 c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\,\text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)$$

Problem 435: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^5 \left(8 c - d x^3\right)^2 \sqrt{c + d x^3}} \, dx$$

Optimal (type 4, 687 leaves, 16 steps):

$$-\frac{31 \sqrt{c+d x^3}}{6912 c^3 x^4}+\frac{5 d \sqrt{c+d x^3}}{864 c^4 x}-\frac{5 d^{4/3} \sqrt{c+d x^3}}{864 c^4 \left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)}+\frac{\sqrt{c+d x^3}}{216 c^2 x^4 \left(8 c-d x^3\right)}-\frac{25 d^{4/3} \text{ArcTan}\left[\frac{\sqrt{3} c^{1/6} \left(c^{1/3}+d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]}{27648 \sqrt{3} c^{23/6}}+\frac{25 d^{4/3} \text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3} x\right)^2}{3 c^{1/6} \sqrt{c+d x^3}}\right]}{82944 c^{23/6}}-\frac{25 d^{4/3} \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{82944 c^{23/6}}+\left(5 \sqrt{2-\sqrt{3}} d^{4/3} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/\left(576 \times 3^{3/4} c^{11/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}\right)-\left(5 d^{4/3} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/\left(432 \sqrt{2} 3^{1/4} c^{11/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}\right)$$

Result (type 6, 384 leaves):

$$\left(\frac{\left(c+d x^3\right)\left(216 c^2-351 c d x^3+40 d^2 x^6\right)}{-8 c+d x^3}-\left(2450 c^2 d^2 x^6 \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\left(40 c \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)+\left(256 c d^3 x^9 \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\left(64 c \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)\Big/\left(6912 c^4 x^4 \sqrt{c+d x^3}\right)$$

Problem 436: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^8 \left(8 c-d x^3\right)^2 \sqrt{c+d x^3}}\, dx$$

Optimal (type 4, 711 leaves, 17 steps):

$$-\frac{17 \sqrt{c+d x^3}}{6048 c^3 x^7}+\frac{391 d \sqrt{c+d x^3}}{193536 c^4 x^4}-\frac{289 d^2 \sqrt{c+d x^3}}{48384 c^5 x}+\frac{289 d^{7/3} \sqrt{c+d x^3}}{48384 c^5 \left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)}+\frac{\sqrt{c+d x^3}}{216 c^2 x^7 \left(8 c-d x^3\right)}-\frac{17 d^{7/3} \text{ArcTan}\left[\frac{\sqrt{3} c^{1/6} \left(c^{1/3}+d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]}{110592 \sqrt{3} c^{29/6}}+\frac{17 d^{7/3} \text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3} x\right)^2}{3 c^{1/6} \sqrt{c+d x^3}}\right]}{331776 c^{29/6}}-\frac{17 d^{7/3} \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{331776 c^{29/6}}-\left(289 \sqrt{2-\sqrt{3}} d^{7/3} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/\left(32256\times 3^{3/4} c^{14/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}\right)+\left(289 d^{7/3} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/\left(24192 \sqrt{2}\, 3^{1/4} c^{14/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}\right)$$

Result (type 6, 377 leaves):

$$\left(-5\left(3456\,c^4-216\,c^3\,d\,x^3+5967\,c^2\,d^2\,x^6+8483\,c\,d^3\,x^9-1156\,d^4\,x^{12}\right)+\left(480\,250\,c^2\,d^3\,x^9\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\Big/\left(40\,c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]+3\,d\,x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]-4\,\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\right)-\left(36\,992\,c\,d^4\,x^{12}\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\Big/\left(64\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]+3\,d\,x^3\left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]-4\,\text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\right)\right)\Big/\left(967\,680\,c^5\,x^7\left(8\,c-d\,x^3\right)\sqrt{c+d\,x^3}\right)$$

Problem 437: Result more than twice size of optimal antiderivative.

$$\int\frac{x^6}{\left(8\,c-d\,x^3\right)^2\sqrt{c+d\,x^3}}\,dx$$

Optimal (type 6, 66 leaves, 2 steps):

$$\frac{x^7\sqrt{1+\frac{d\,x^3}{c}}\,\text{AppellF1}\left[\frac{7}{3},2,\frac{1}{2},\frac{10}{3},\frac{d\,x^3}{8\,c},-\frac{d\,x^3}{c}\right]}{448\,c^2\sqrt{c+d\,x^3}}$$

Result (type 6, 331 leaves):

$$\left(2\,x\left(4\left(c+d\,x^3\right)-\left(128\,c^2\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\Big/\left(32\,c\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]+3\,d\,x^3\left(\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]-4\,\text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\right)-\left(161\,c\,d\,x^3\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\Big/\left(56\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]+3\,d\,x^3\left(\text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]-4\,\text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\right)\right)\right)\Big/\left(27\,d^2\left(8\,c-d\,x^3\right)\sqrt{c+d\,x^3}\right)$$

Problem 438: Result more than twice size of optimal antiderivative.

$$\int \frac{x^3}{\left(8 c - d x^3\right)^2 \sqrt{c + d x^3}} \, dx$$

Optimal (type 6, 66 leaves, 2 steps):

$$\frac{x^4 \sqrt{1 + \frac{d x^3}{c}} \, \text{AppellF1}\left[\frac{4}{3}, 2, \frac{1}{2}, \frac{7}{3}, \frac{d x^3}{8 c}, -\frac{d x^3}{c}\right]}{256 c^2 \sqrt{c + d x^3}}$$

Result (type 6, 355 leaves):

$$\frac{1}{27 \sqrt{c + d x^3}} x \left(\frac{c + d x^3}{8 c^2 d - c d^2 x^3} + \left(32 c \, \text{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right]\right) \Big/ \left(d \left(-8 c + d x^3\right) \left(32 c \, \text{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right] + 3 d x^3 \left(\text{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 2, \frac{7}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right] - 4 \, \text{AppellF1}\left[\frac{4}{3}, \frac{3}{2}, 1, \frac{7}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right]\right)\right)\right) + \left(7 x^3 \, \text{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right]\right) \Big/ \left(\left(8 c - d x^3\right) \left(56 c \, \text{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right] + 3 d x^3 \left(\text{AppellF1}\left[\frac{7}{3}, \frac{1}{2}, 2, \frac{10}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right] - 4 \, \text{AppellF1}\left[\frac{7}{3}, \frac{3}{2}, 1, \frac{10}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right]\right)\right)\right)\right)$$

Problem 439: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(8 c - d x^3\right)^2 \sqrt{c + d x^3}} \, dx$$

Optimal (type 6, 64 leaves, 2 steps):

$$\frac{x \sqrt{1 + \frac{d x^3}{c}} \, \text{AppellF1}\left[\frac{1}{3}, 2, \frac{1}{2}, \frac{4}{3}, \frac{d x^3}{8 c}, -\frac{d x^3}{c}\right]}{64 c^2 \sqrt{c + d x^3}}$$

Result (type 6, 327 leaves):

$$\left(x\left(\left(832\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\Big/\left(32\,c\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]+3\,d\,x^3\left(\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]-4\,\text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\right)+\frac{1}{c^2}\left(c+d\,x^3+\left(7\,c\,d\,x^3\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\Big/\left(56\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]+3\,d\,x^3\left(\text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]-4\,\text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\right)\right)\right)\right)\Big/\left(216\,\left(8\,c-d\,x^3\right)\sqrt{c+d\,x^3}\right)$$

Problem 440: Result more than twice size of optimal antiderivative.

$$\int\frac{1}{x^3\,\left(8\,c-d\,x^3\right)^2\sqrt{c+d\,x^3}}\,dx$$

Optimal (type 6, 66 leaves, 2 steps):

$$-\frac{\sqrt{1+\frac{d\,x^3}{c}}\;\text{AppellF1}\left[-\frac{2}{3},2,\frac{1}{2},\frac{1}{3},\frac{d\,x^3}{8\,c},-\frac{d\,x^3}{c}\right]}{128\,c^2\,x^2\,\sqrt{c+d\,x^3}}$$

Result (type 6, 372 leaves):

$$\left(-\frac{\left(c+d\,x^3\right)\left(-216\,c+29\,d\,x^3\right)}{-8\,c+d\,x^3}-\left(64\,c^2\,d\,x^3\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\Big/\left(\left(8\,c-d\,x^3\right)\left(32\,c\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]+3\,d\,x^3\left(\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]-4\,\text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\right)\right)+\left(203\,c\,d^2\,x^6\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\Big/\left(\left(8\,c-d\,x^3\right)\left(56\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]+3\,d\,x^3\left(\text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]-4\,\text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\right)\right)\right)\Big/\left(3456\,c^3\,x^2\,\sqrt{c+d\,x^3}\right)$$

Problem 441: Result more than twice size of optimal antiderivative.

$$\int\frac{1}{x^6\,\left(8\,c-d\,x^3\right)^2\sqrt{c+d\,x^3}}\,dx$$

Optimal (type 6, 66 leaves, 2 steps):

$$-\frac{\sqrt{1+\frac{d x^3}{c}}\ \text{AppellF1}\left[-\frac{5}{3},2,\frac{1}{2},-\frac{2}{3},\frac{d x^3}{8 c},-\frac{d x^3}{c}\right]}{320\ c^2\ x^5\ \sqrt{c+d x^3}}$$

Result (type 6, 384 leaves):

$$\left(\frac{\left(c+d x^3\right)\left(864 c^2-1080 c d x^3+119 d^2 x^6\right)}{-8 c+d x^3}+\left(21952 c^2 d^2 x^6\ \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\left(32 c\ \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\ \text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)-\left(833 c d^3 x^9\ \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\left(56 c\ \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\ \text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)\Big/\left(34560 c^4 x^5 \sqrt{c+d x^3}\right)$$

Problem 446: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{x\left(8 c-d x^3\right)^2\left(c+d x^3\right)^{3/2}}\,dx$$

Optimal (type 3, 106 leaves, 8 steps):

$$\frac{5}{648 c^3 \sqrt{c+d x^3}}+\frac{1}{216 c^2\left(8 c-d x^3\right)\sqrt{c+d x^3}}+\frac{7\ \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3\sqrt{c}}\right]}{7776\ c^{7/2}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{96\ c^{7/2}}$$

Result (type 6, 338 leaves):

$$\frac{1}{324\sqrt{c+d x^3}}\left(\frac{43 c-5 d x^3}{16 c^4-2 c^3 d x^3}-\left(20 d x^3\ \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(c^2\left(8 c-d x^3\right)\left(16 c\ \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+d x^3\left(\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\ \text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)+\left(45 d x^3\ \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]\right)\Big/\left(c^2\left(-8 c+d x^3\right)\left(5 d x^3\ \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]+16 c\ \text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]-c\ \text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]\right)\right)\right)$$

Problem 447: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{x^4 \left(8 c-d x^3\right)^2 \left(c+d x^3\right)^{3/2}} \, dx$$

Optimal (type 3, 143 leaves, 9 steps):

$$-\frac{35 d}{2592 c^4 \sqrt{c+d x^3}}+\frac{5 d}{864 c^3 \left(8 c-d x^3\right) \sqrt{c+d x^3}}-\frac{1}{24 c^2 x^3 \left(8 c-d x^3\right) \sqrt{c+d x^3}}+\frac{5 d \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{31104 c^{9/2}}+\frac{5 d \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{384 c^{9/2}}$$

Result (type 6, 350 leaves):

$$\left(\frac{108 c^2+265 c d x^3-35 d^2 x^6}{-8 c+d x^3}+\left(280 c d^2 x^6 \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\left(16 c \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+d x^3 \left(\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)+\left(450 c d^2 x^6 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]\right)\Big/\left(\left(8 c-d x^3\right)\left(5 d x^3 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]+16 c \text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]-c \text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^3},\frac{8 c}{d x^3}\right]\right)\right)\right)\Big/\left(2592 c^4 x^3 \sqrt{c+d x^3}\right)$$

Problem 448: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^7 \left(8 c-d x^3\right)^2 \left(c+d x^3\right)^{3/2}} \, dx$$

Optimal (type 3, 185 leaves, 10 steps):

$$\frac{665 d^2}{41472 c^5 \sqrt{c+d x^3}}-\frac{71 d^2}{13824 c^4 \left(8 c-d x^3\right) \sqrt{c+d x^3}}-\frac{1}{48 c^2 x^6 \left(8 c-d x^3\right) \sqrt{c+d x^3}}+\frac{17 d}{384 c^3 x^3 \left(8 c-d x^3\right) \sqrt{c+d x^3}}+\frac{13 d^2 \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{497664 c^{11/2}}-\frac{33 d^2 \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{2048 c^{11/2}}$$

Result (type 6, 349 leaves):

$$\left(-\left(\left(5320\, c\, d^3\, x^9\, \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\Big/\left(\left(8\,c-d\,x^3\right)\left(16\,c\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]+d\,x^3\left(\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]-4\,\text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\right)\right)\right)+\frac{1}{-8\,c+d\,x^3}\left(864\,c^3-1836\,c^2\,d\,x^3-5107\,c\,d^2\,x^6+665\,d^3\,x^9+\left(8910\,c\,d^3\,x^9\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^3},\frac{8\,c}{d\,x^3}\right]\right)\Big/\left(5\,d\,x^3\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^3},\frac{8\,c}{d\,x^3}\right]+16\,c\,\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d\,x^3},\frac{8\,c}{d\,x^3}\right]-c\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d\,x^3},\frac{8\,c}{d\,x^3}\right]\right)\right)\right)\Big/\left(41\,472\,c^5\,x^6\,\sqrt{c+d\,x^3}\right)$$

Problem 449: Result unnecessarily involves higher level functions.

$$\int \frac{x^7}{\left(8\,c-d\,x^3\right)^2\left(c+d\,x^3\right)^{3/2}}\,dx$$

Optimal (type 4, 668 leaves, 15 steps):

$$-\frac{2 x^2}{81 c d^2 \sqrt{c+d x^3}}+\frac{8 x^2}{27 d^2 \left(8 c-d x^3\right) \sqrt{c+d x^3}}+\frac{2 \sqrt{c+d x^3}}{81 c d^{8/3} \left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)}+\frac{4 \text{ArcTan}\left[\frac{\sqrt{3} c^{1/6} \left(c^{1/3}+d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]}{81 \sqrt{3} c^{5/6} d^{8/3}}-$$

$$\frac{4 \text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3} x\right)^2}{3 c^{1/6} \sqrt{c+d x^3}}\right]}{243 c^{5/6} d^{8/3}}+\frac{4 \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{243 c^{5/6} d^{8/3}}-\left(\sqrt{2-\sqrt{3}} \left(c^{1/3}+d^{1/3} x\right)\right.$$

$$\left.\sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$

$$\left(27\times 3^{3/4} c^{2/3} d^{8/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}\right)+\left(2 \sqrt{2} \left(c^{1/3}+d^{1/3} x\right)\right.$$

$$\left.\sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$

$$\left(81\times 3^{1/4} c^{2/3} d^{8/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}\right)$$

Result (type 6, 357 leaves):

$$\frac{1}{405 d^2 \sqrt{c+d x^3}} 2 x^2 \left(\frac{20 c+5 d x^3}{8 c^2-c d x^3}+\left(800 c \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\right.$$

$$\left(\left(-8 c+d x^3\right) \left(40 c \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+\right.\right.$$

$$\left.\left.3 d x^3 \left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)+$$

$$\left(32 d x^3 \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/$$

$$\left(\left(-8 c+d x^3\right) \left(64 c \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\right.\right.$$

$$\left.\left.\left.\left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)$$

Problem 450: Result unnecessarily involves higher level functions.

$$\int \frac{x^4}{\left(8 c-d x^3\right)^2 \left(c+d x^3\right)^{3/2}} \, dx$$

Optimal (type 4, 671 leaves, 15 steps):

$$-\frac{x^2}{81 c^2 d \sqrt{c+d x^3}}+\frac{x^2}{27 c d \left(8 c-d x^3\right) \sqrt{c+d x^3}}+\frac{\sqrt{c+d x^3}}{81 c^2 d^{5/3} \left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)}-\frac{\text{ArcTan}\left[\frac{\sqrt{3} c^{1/6} \left(c^{1/3}+d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]}{81 \sqrt{3} c^{11/6} d^{5/3}}+\frac{\text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3} x\right)^2}{3 c^{1/6} \sqrt{c+d x^3}}\right]}{243 c^{11/6} d^{5/3}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{243 c^{11/6} d^{5/3}}-\left(\sqrt{2-\sqrt{3}} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]\right) \Bigg/ \left(54 \times 3^{3/4} c^{5/3} d^{5/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}\right)+\left(\sqrt{2} \left(c^{1/3}+d^{1/3} x\right) \sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]\right) \Bigg/ \left(81 \times 3^{1/4} c^{5/3} d^{5/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}\right)$$

Result (type 6, 337 leaves):

$$\left(x^2\left(\left(1000\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(d\left(40\,c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3\,d\,x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\,\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)+\frac{1}{c^2}\left(5\left(-\frac{5 c}{d}+x^3\right)-\left(32\,c\,x^3\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(64\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3\,d\,x^3\left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4\,\text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)\right)\Big/\left(405\left(8 c-d x^3\right)\sqrt{c+d x^3}\right)$$

Problem 451: Result unnecessarily involves higher level functions.

$$\int\frac{x}{\left(8 c-d x^3\right)^2\left(c+d x^3\right)^{3/2}}\,dx$$

Optimal (type 4, 665 leaves, 15 steps):

$$\frac{5 x^2}{648 c^3\sqrt{c+d x^3}}+\frac{x^2}{216 c^2\left(8 c-d x^3\right)\sqrt{c+d x^3}}-\frac{5\sqrt{c+d x^3}}{648 c^3 d^{2/3}\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)}-\frac{5\,\text{ArcTan}\left[\frac{\sqrt{3}\,c^{1/6}\left(c^{1/3}+d^{1/3}x\right)}{\sqrt{c+d x^3}}\right]}{1296\sqrt{3}\,c^{17/6}d^{2/3}}+\frac{5\,\text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3}x\right)^2}{3 c^{1/6}\sqrt{c+d x^3}}\right]}{3888\,c^{17/6}d^{2/3}}-\frac{5\,\text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3\sqrt{c}}\right]}{3888\,c^{17/6}d^{2/3}}+\left(5\sqrt{2-\sqrt{3}}\left(c^{1/3}+d^{1/3}x\right)\sqrt{\frac{c^{2/3}-c^{1/3}d^{1/3}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/\left(432\times 3^{3/4}c^{8/3}d^{2/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\sqrt{c+d x^3}\right)-\left(5\left(c^{1/3}+d^{1/3}x\right)\sqrt{\frac{c^{2/3}-c^{1/3}d^{1/3}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/\left(324\sqrt{2}\,3^{1/4}c^{8/3}d^{2/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}x\right)^2}}\sqrt{c+d x^3}\right)$$

Result (type 6, 366 leaves):

$$\frac{1}{162\sqrt{c+d x^3}}\left(\frac{43 c x^2-5 d x^5}{32 c^4-4 c^3 d x^3}-\left(25 x^2 \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\right.$$
$$\left(c\left(8 c-d x^3\right)\left(40 c \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+\right.\right.$$
$$\left.\left.3 d x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)+$$
$$\left(8 d x^5 \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/$$
$$\left(c^2\left(8 c-d x^3\right)\left(64 c \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+\right.\right.$$
$$\left.\left.\left.3 d x^3\left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)$$

Problem 452: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^2\left(8 c-d x^3\right)^2\left(c+d x^3\right)^{3/2}}\,dx$$

Optimal (type 4, 686 leaves, 16 steps):

$$\frac{5}{648\, c^3\, x\, \sqrt{c+d\, x^3}}+\frac{1}{216\, c^2\, x\, \left(8\, c-d\, x^3\right)\, \sqrt{c+d\, x^3}}-\frac{31\, \sqrt{c+d\, x^3}}{1296\, c^4\, x}+$$

$$\frac{31\, d^{1/3}\, \sqrt{c+d\, x^3}}{1296\, c^4\, \left(\left(1+\sqrt{3}\right)\, c^{1/3}+d^{1/3}\, x\right)}-\frac{d^{1/3}\, \text{ArcTan}\left[\frac{\sqrt{3}\, c^{1/6}\, \left(c^{1/3}+d^{1/3}\, x\right)}{\sqrt{c+d\, x^3}}\right]}{1296\, \sqrt{3}\, c^{23/6}}+$$

$$\frac{d^{1/3}\, \text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3}\, x\right)^2}{3\, c^{1/6}\, \sqrt{c+d\, x^3}}\right]}{3888\, c^{23/6}}-\frac{d^{1/3}\, \text{ArcTanh}\left[\frac{\sqrt{c+d\, x^3}}{3\, \sqrt{c}}\right]}{3888\, c^{23/6}}-\left(31\, \sqrt{2-\sqrt{3}}\, d^{1/3}\, \left(c^{1/3}+d^{1/3}\, x\right)\right.$$

$$\left.\sqrt{\frac{c^{2/3}-c^{1/3}\, d^{1/3}\, x+d^{2/3}\, x^2}{\left(\left(1+\sqrt{3}\right)\, c^{1/3}+d^{1/3}\, x\right)^2}}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\, c^{1/3}+d^{1/3}\, x}{\left(1+\sqrt{3}\right)\, c^{1/3}+d^{1/3}\, x}\right],\, -7-4\, \sqrt{3}\right]\right)\Big/$$

$$\left(864\times 3^{3/4}\, c^{11/3}\, \sqrt{\frac{c^{1/3}\, \left(c^{1/3}+d^{1/3}\, x\right)}{\left(\left(1+\sqrt{3}\right)\, c^{1/3}+d^{1/3}\, x\right)^2}}\, \sqrt{c+d\, x^3}\right)+\left(31\, d^{1/3}\, \left(c^{1/3}+d^{1/3}\, x\right)\right.$$

$$\left.\sqrt{\frac{c^{2/3}-c^{1/3}\, d^{1/3}\, x+d^{2/3}\, x^2}{\left(\left(1+\sqrt{3}\right)\, c^{1/3}+d^{1/3}\, x\right)^2}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\, c^{1/3}+d^{1/3}\, x}{\left(1+\sqrt{3}\right)\, c^{1/3}+d^{1/3}\, x}\right],\, -7-4\, \sqrt{3}\right]\right)\Big/$$

$$\left(648\, \sqrt{2}\, 3^{1/4}\, c^{11/3}\, \sqrt{\frac{c^{1/3}\, \left(c^{1/3}+d^{1/3}\, x\right)}{\left(\left(1+\sqrt{3}\right)\, c^{1/3}+d^{1/3}\, x\right)^2}}\, \sqrt{c+d\, x^3}\right)$$

Result (type 6, 374 leaves):

$$\frac{1}{6480\, c^4\, \sqrt{c+d\, x^3}}$$

$$\left(\frac{5\, \left(162\, c^2+227\, c\, d\, x^3-31\, d^2\, x^6\right)}{-8\, c\, x+d\, x^4}+\left(13\,000\, c^2\, d\, x^2\, \text{AppellF1}\left[\frac{2}{3},\, \frac{1}{2},\, 1,\, \frac{5}{3},\, -\frac{d\, x^3}{c},\, \frac{d\, x^3}{8\, c}\right]\right)\Big/\right.$$

$$\left(\left(8\, c-d\, x^3\right)\, \left(40\, c\, \text{AppellF1}\left[\frac{2}{3},\, \frac{1}{2},\, 1,\, \frac{5}{3},\, -\frac{d\, x^3}{c},\, \frac{d\, x^3}{8\, c}\right]+\right.\right.$$

$$\left.\left.3\, d\, x^3\, \left(\text{AppellF1}\left[\frac{5}{3},\, \frac{1}{2},\, 2,\, \frac{8}{3},\, -\frac{d\, x^3}{c},\, \frac{d\, x^3}{8\, c}\right]-4\, \text{AppellF1}\left[\frac{5}{3},\, \frac{3}{2},\, 1,\, \frac{8}{3},\, -\frac{d\, x^3}{c},\, \frac{d\, x^3}{8\, c}\right]\right)\right)\right)-$$

$$\left(992\, c\, d^2\, x^5\, \text{AppellF1}\left[\frac{5}{3},\, \frac{1}{2},\, 1,\, \frac{8}{3},\, -\frac{d\, x^3}{c},\, \frac{d\, x^3}{8\, c}\right]\right)\Big/$$

$$\left(\left(8\, c-d\, x^3\right)\, \left(64\, c\, \text{AppellF1}\left[\frac{5}{3},\, \frac{1}{2},\, 1,\, \frac{8}{3},\, -\frac{d\, x^3}{c},\, \frac{d\, x^3}{8\, c}\right]+\right.\right.$$

$$\left.\left.\left.3\, d\, x^3\, \left(\text{AppellF1}\left[\frac{8}{3},\, \frac{1}{2},\, 2,\, \frac{11}{3},\, -\frac{d\, x^3}{c},\, \frac{d\, x^3}{8\, c}\right]-4\, \text{AppellF1}\left[\frac{8}{3},\, \frac{3}{2},\, 1,\, \frac{11}{3},\, -\frac{d\, x^3}{c},\, \frac{d\, x^3}{8\, c}\right]\right)\right)\right)\right)$$

Problem 453: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^5 \left(8 c-d x^3\right)^2 \left(c+d x^3\right)^{3/2}} \, dx$$

Optimal (type 4, 708 leaves, 17 steps):

$$\frac{5}{648 c^3 x^4 \sqrt{c+d x^3}}+\frac{1}{216 c^2 x^4 \left(8 c-d x^3\right) \sqrt{c+d x^3}}-\frac{253 \sqrt{c+d x^3}}{20736 c^4 x^4}+$$

$$\frac{77 d \sqrt{c+d x^3}}{2592 c^5 x}-\frac{77 d^{4/3} \sqrt{c+d x^3}}{2592 c^5 \left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)}-\frac{11 d^{4/3} \text{ArcTan}\left[\frac{\sqrt{3} c^{1/6} \left(c^{1/3}+d^{1/3} x\right)}{\sqrt{c+d x^3}}\right]}{82944 \sqrt{3} c^{29/6}}+$$

$$\frac{11 d^{4/3} \text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3} x\right)^2}{3 c^{1/6} \sqrt{c+d x^3}}\right]}{248832 c^{29/6}}-\frac{11 d^{4/3} \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{3 \sqrt{c}}\right]}{248832 c^{29/6}}+\left(77 \sqrt{2-\sqrt{3}} d^{4/3} \left(c^{1/3}+d^{1/3} x\right)\right.$$

$$\left.\sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]\right) \Bigg/$$

$$\left(1728 \times 3^{3/4} c^{14/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}\right)-\left(77 d^{4/3} \left(c^{1/3}+d^{1/3} x\right)\right.$$

$$\left.\sqrt{\frac{c^{2/3}-c^{1/3} d^{1/3} x+d^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) c^{1/3}+d^{1/3} x}{\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x}\right],-7-4 \sqrt{3}\right]\right) \Bigg/$$

$$\left(1296 \sqrt{2} 3^{1/4} c^{14/3} \sqrt{\frac{c^{1/3} \left(c^{1/3}+d^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) c^{1/3}+d^{1/3} x\right)^2}} \sqrt{c+d x^3}\right)$$

Result (type 6, 389 leaves):

$$\left(\frac{5\left(648 c^3-2997 c^2 d x^3-4565 c d^2 x^6+616 d^3 x^9\right)}{-8 c+d x^3}-\left(244750 c^2 d^2 x^6 \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\bigg/\left(\left(8 c-d x^3\right)\left(40 c \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)+\left(19712 c d^3 x^9 \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\bigg/\left(\left(8 c-d x^3\right)\left(64 c \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)\bigg/\left(103680 c^5 x^4 \sqrt{c+d x^3}\right)$$

Problem 454: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^8\left(8 c-d x^3\right)^2\left(c+d x^3\right)^{3/2}}\,dx$$

Optimal (type 4, 732 leaves, 18 steps):

$$\frac{5}{648\,c^3\,x^7\,\sqrt{c+d\,x^3}}+\frac{1}{216\,c^2\,x^7\,\left(8\,c-d\,x^3\right)\,\sqrt{c+d\,x^3}}-\frac{191\,\sqrt{c+d\,x^3}}{18\,144\,c^4\,x^7}+$$

$$\frac{8257\,d\,\sqrt{c+d\,x^3}}{580\,608\,c^5\,x^4}-\frac{5179\,d^2\,\sqrt{c+d\,x^3}}{145\,152\,c^6\,x}+\frac{5179\,d^{7/3}\,\sqrt{c+d\,x^3}}{145\,152\,c^6\,\left(\left(1+\sqrt{3}\right)\,c^{1/3}+d^{1/3}\,x\right)}-$$

$$\frac{7\,d^{7/3}\,\text{ArcTan}\left[\frac{\sqrt{3}\,c^{1/6}\,\left(c^{1/3}+d^{1/3}\,x\right)}{\sqrt{c+d\,x^3}}\right]}{331\,776\,\sqrt{3}\,c^{35/6}}+\frac{7\,d^{7/3}\,\text{ArcTanh}\left[\frac{\left(c^{1/3}+d^{1/3}\,x\right)^2}{3\,c^{1/6}\,\sqrt{c+d\,x^3}}\right]}{995\,328\,c^{35/6}}-$$

$$\frac{7\,d^{7/3}\,\text{ArcTanh}\left[\frac{\sqrt{c+d\,x^3}}{3\,\sqrt{c}}\right]}{995\,328\,c^{35/6}}-\left(5179\,\sqrt{2-\sqrt{3}}\,d^{7/3}\,\left(c^{1/3}+d^{1/3}\,x\right)\,\sqrt{\frac{c^{2/3}-c^{1/3}\,d^{1/3}\,x+d^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,c^{1/3}+d^{1/3}\,x\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,c^{1/3}+d^{1/3}\,x}{\left(1+\sqrt{3}\right)\,c^{1/3}+d^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\,\right]\right)\Big/$$

$$\left(96\,768\times 3^{3/4}\,c^{17/3}\,\sqrt{\frac{c^{1/3}\,\left(c^{1/3}+d^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,c^{1/3}+d^{1/3}\,x\right)^2}}\,\sqrt{c+d\,x^3}\right)+\left(5179\,d^{7/3}\,\left(c^{1/3}+d^{1/3}\,x\right)\right.$$

$$\left.\sqrt{\frac{c^{2/3}-c^{1/3}\,d^{1/3}\,x+d^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,c^{1/3}+d^{1/3}\,x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,c^{1/3}+d^{1/3}\,x}{\left(1+\sqrt{3}\right)\,c^{1/3}+d^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\,\right]\right)\Big/$$

$$\left(72\,576\,\sqrt{2}\,3^{1/4}\,c^{17/3}\,\sqrt{\frac{c^{1/3}\,\left(c^{1/3}+d^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,c^{1/3}+d^{1/3}\,x\right)^2}}\,\sqrt{c+d\,x^3}\right)$$

Result (type 6, 374 leaves) :

$$\left(-51840\,c^4+93960\,c^3\,d\,x^3-509085\,c^2\,d^2\,x^6-766345\,c\,d^3\,x^9+103580\,d^4\,x^{12}+\left(8293750\,c^2\,d^3\,x^9\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\Big/\left(40\,c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]+3\,d\,x^3\left(\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]-4\,\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\right)-\left(662912\,c\,d^4\,x^{12}\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\Big/\left(64\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]+3\,d\,x^3\left(\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]-4\,\text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d\,x^3}{c},\frac{d\,x^3}{8\,c}\right]\right)\right)\right)\Big/\left(2903040\,c^6\,x^7\left(8\,c-d\,x^3\right)\sqrt{c+d\,x^3}\right)$$

Problem 455: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^6}{\left(8\,c-d\,x^3\right)^2\left(c+d\,x^3\right)^{3/2}}\,dx$$

Optimal (type 4, 256 leaves, ? steps):

$$\frac{2\,x\left(4\,c+d\,x^3\right)}{81\,c\,d^2\left(8\,c-d\,x^3\right)\sqrt{c+d\,x^3}}-\left(2\sqrt{2+\sqrt{3}}\left(c^{1/3}+d^{1/3}\,x\right)\sqrt{\frac{c^{2/3}-c^{1/3}\,d^{1/3}\,x+d^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)c^{1/3}+d^{1/3}\,x}{\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x}\right],-7-4\sqrt{3}\right]\right)\Big/\left(81\times3^{1/4}\,c\,d^{7/3}\sqrt{\frac{c^{1/3}\left(c^{1/3}+d^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)c^{1/3}+d^{1/3}\,x\right)^2}}\sqrt{c+d\,x^3}\right)$$

Result (type 4, 189 leaves):

$$-\left(\left(\left(6\,(-d)^{1/3}\,x\,\left(4\,c+d\,x^3\right)+2\,i\,3^{3/4}\,c^{1/3}\,\sqrt{\frac{(-1)^{5/6}\left(-c^{1/3}+(-d)^{1/3}\,x\right)}{c^{1/3}}}\,\sqrt{1+\frac{(-d)^{1/3}\,x}{c^{1/3}}+\frac{(-d)^{2/3}\,x^2}{c^{2/3}}}\right.\right.\right.$$
$$\left.\left(-8\,c+d\,x^3\right)\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-d)^{1/3}\,x}{c^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\Big/$$
$$\left.\left(243\,c\,(-d)^{7/3}\,\left(-8\,c+d\,x^3\right)\,\sqrt{c+d\,x^3}\right)\right)$$

Problem 456: Result more than twice size of optimal antiderivative.

$$\int \frac{x^3}{\left(8\,c-d\,x^3\right)^2\,\left(c+d\,x^3\right)^{3/2}}\,dx$$

Optimal (type 6, 66 leaves, 2 steps):

$$\frac{x^4\,\sqrt{1+\frac{d\,x^3}{c}}\,\text{AppellF1}\left[\frac{4}{3},\,2,\,\frac{3}{2},\,\frac{7}{3},\,\frac{d\,x^3}{8\,c},\,-\frac{d\,x^3}{c}\right]}{256\,c^3\,\sqrt{c+d\,x^3}}$$

Result (type 6, 333 leaves):

$$\left(\left(160\,x\,\text{AppellF1}\left[\frac{1}{3},\,\frac{1}{2},\,1,\,\frac{4}{3},\,-\frac{d\,x^3}{c},\,\frac{d\,x^3}{8\,c}\right]\right)\Big/\left(d\,\left(32\,c\,\text{AppellF1}\left[\frac{1}{3},\,\frac{1}{2},\,1,\,\frac{4}{3},\,-\frac{d\,x^3}{c},\,\frac{d\,x^3}{8\,c}\right]+\right.\right.\right.$$
$$\left.\left.\left.3\,d\,x^3\,\left(\text{AppellF1}\left[\frac{4}{3},\,\frac{1}{2},\,2,\,\frac{7}{3},\,-\frac{d\,x^3}{c},\,\frac{d\,x^3}{8\,c}\right]-4\,\text{AppellF1}\left[\frac{4}{3},\,\frac{3}{2},\,1,\,\frac{7}{3},\,-\frac{d\,x^3}{c},\,\frac{d\,x^3}{8\,c}\right]\right)\right)\right)+$$
$$\frac{1}{c^2}x\,\left(-\frac{5\,c}{d}+x^3+\left(7\,c\,x^3\,\text{AppellF1}\left[\frac{4}{3},\,\frac{1}{2},\,1,\,\frac{7}{3},\,-\frac{d\,x^3}{c},\,\frac{d\,x^3}{8\,c}\right]\right)\Big/\right.$$
$$\left(56\,c\,\text{AppellF1}\left[\frac{4}{3},\,\frac{1}{2},\,1,\,\frac{7}{3},\,-\frac{d\,x^3}{c},\,\frac{d\,x^3}{8\,c}\right]+3\,d\,x^3\,\left(\text{AppellF1}\left[\frac{7}{3},\,\frac{1}{2},\,2,\,\frac{10}{3},\,-\frac{d\,x^3}{c},\,\frac{d\,x^3}{8\,c}\right]-\right.\right.$$
$$\left.\left.\left.\left.4\,\text{AppellF1}\left[\frac{7}{3},\,\frac{3}{2},\,1,\,\frac{10}{3},\,-\frac{d\,x^3}{c},\,\frac{d\,x^3}{8\,c}\right]\right)\right)\right)\right)\Big/\left(81\,\left(8\,c-d\,x^3\right)\,\sqrt{c+d\,x^3}\right)$$

Problem 457: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(8\,c-d\,x^3\right)^2\,\left(c+d\,x^3\right)^{3/2}}\,dx$$

Optimal (type 6, 64 leaves, 2 steps):

$$\frac{x\sqrt{1+\frac{d x^3}{c}}\ \text{AppellF1}\left[\frac{1}{3},2,\frac{3}{2},\frac{4}{3},\frac{d x^3}{8 c},-\frac{d x^3}{c}\right]}{64 c^3\sqrt{c+d x^3}}$$

Result (type 6, 331 leaves):

$$\left(x\left(43 c-5 d x^3+\left(1216 c^2 \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(32 c \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)-\left(35 c d x^3 \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(56 c \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)\Big/\left(648 c^3\left(8 c-d x^3\right)\sqrt{c+d x^3}\right)$$

Problem 458: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^3\left(8 c-d x^3\right)^2\left(c+d x^3\right)^{3/2}}\,dx$$

Optimal (type 6, 66 leaves, 2 steps):

$$-\frac{\sqrt{1+\frac{d x^3}{c}}\ \text{AppellF1}\left[-\frac{2}{3},2,\frac{3}{2},\frac{1}{3},\frac{d x^3}{8 c},-\frac{d x^3}{c}\right]}{128 c^3 x^2\sqrt{c+d x^3}}$$

Result (type 6, 375 leaves):

$$\left(\frac{648 c^2+1249 c d x^3-167 d^2 x^6}{-8 c+d x^3}-\left(19648 c^2 d x^3 \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\left(32 c \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)+\left(1169 c d^2 x^6 \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\Big/\left(\left(8 c-d x^3\right)\left(56 c \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]+3 d x^3\left(\text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]-4 \text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d x^3}{c},\frac{d x^3}{8 c}\right]\right)\right)\right)\right)\Big/\left(10368 c^4 x^2\sqrt{c+d x^3}\right)$$

Problem 459: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^6 \left(8 c - d x^3\right)^2 \left(c + d x^3\right)^{3/2}} \, dx$$

Optimal (type 6, 66 leaves, 2 steps):

$$-\frac{\sqrt{1+\frac{d x^3}{c}} \text{ AppellF1}\left[-\frac{5}{3}, 2, \frac{3}{2}, -\frac{2}{3}, \frac{d x^3}{8 c}, -\frac{d x^3}{c}\right]}{320 c^3 x^5 \sqrt{c+d x^3}}$$

Result (type 6, 388 leaves):

$$\left(\frac{2592 c^3 - 7128 c^2 d x^3 - 15373 c d^2 x^6 + 2027 d^3 x^9}{-8 c + d x^3} + \left(262336 c^2 d^2 x^6 \text{ AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right]\right) \Big/ \left(\left(8 c - d x^3\right) \left(32 c \text{ AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right] + 3 d x^3 \left(\text{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 2, \frac{7}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right] - 4 \text{ AppellF1}\left[\frac{4}{3}, \frac{3}{2}, 1, \frac{7}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right]\right)\right)\right) - \left(14189 c d^3 x^9 \text{ AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right]\right) \Big/ \left(\left(8 c - d x^3\right) \left(56 c \text{ AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right] + 3 d x^3 \left(\text{AppellF1}\left[\frac{7}{3}, \frac{1}{2}, 2, \frac{10}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right] - 4 \text{ AppellF1}\left[\frac{7}{3}, \frac{3}{2}, 1, \frac{10}{3}, -\frac{d x^3}{c}, \frac{d x^3}{8 c}\right]\right)\right)\right)\right) \Big/ \left(103680 c^5 x^5 \sqrt{c + d x^3}\right)$$

Problem 463: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{c + d x^3}}{x \left(a + b x^3\right)^2} \, dx$$

Optimal (type 3, 121 leaves, 7 steps):

$$\frac{\sqrt{c+d x^3}}{3 a \left(a+b x^3\right)} - \frac{2 \sqrt{c} \text{ ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{3 a^2} + \frac{\left(2 b c - a d\right) \text{ ArcTanh}\left[\frac{\sqrt{b} \sqrt{c+d x^3}}{\sqrt{b c-a d}}\right]}{3 a^2 \sqrt{b} \sqrt{b c - a d}}$$

Result (type 6, 306 leaves):

$$\left(-\left(\left(6\,c\,d\,x^3\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\Big/\left(-4\,a\,c\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+x^3\left(2\,b\,c\,\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)\right)+\frac{1}{a}\left(3\left(c+d\,x^3\right)+\left(10\,b\,c\,d\,x^3\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]\right)\Big/\left(-5\,b\,d\,x^3\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]+2\,a\,d\,\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]\right)\right)\right)\Big/\left(9\left(a+b\,x^3\right)\sqrt{c+d\,x^3}\right)$$

Problem 464: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{c+d\,x^3}}{x^4\left(a+b\,x^3\right)^2}\,dx$$

Optimal (type 3, 161 leaves, 8 steps):

$$-\frac{2\,b\,\sqrt{c+d\,x^3}}{3\,a^2\left(a+b\,x^3\right)}-\frac{\sqrt{c+d\,x^3}}{3\,a\,x^3\left(a+b\,x^3\right)}+\frac{\left(4\,b\,c-a\,d\right)\text{ArcTanh}\left[\frac{\sqrt{c+d\,x^3}}{\sqrt{c}}\right]}{3\,a^3\sqrt{c}}-\frac{\sqrt{b}\left(4\,b\,c-3\,a\,d\right)\text{ArcTanh}\left[\frac{\sqrt{b}\,\sqrt{c+d\,x^3}}{\sqrt{b\,c-a\,d}}\right]}{3\,a^3\sqrt{b\,c-a\,d}}$$

Result (type 6, 410 leaves):

$$\left(\left(12\,a\,b\,c\,d\,x^6\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\Big/\left(-4\,a\,c\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+x^3\left(2\,b\,c\,\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)+\left(5\,b\,d\,x^3\left(3\,a\,c+2\,b\,c\,x^3+4\,a\,d\,x^3+6\,b\,d\,x^6\right)\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]-3\left(a+2\,b\,x^3\right)\left(c+d\,x^3\right)\left(2\,a\,d\,\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]\right)\right)\Big/\left(-5\,b\,d\,x^3\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]+2\,a\,d\,\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]\right)\right)\Big/\left(9\,a^2\,x^3\left(a+b\,x^3\right)\sqrt{c+d\,x^3}\right)$$

Problem 465: Result more than twice size of optimal antiderivative.

$$\int \frac{x^3 \sqrt{c+d x^3}}{\left(a+b x^3\right)^2} \, dx$$

Optimal (type 6, 64 leaves, 2 steps):

$$\frac{x^4 \sqrt{c+d x^3} \text{AppellF1}\left[\frac{4}{3}, 2, -\frac{1}{2}, \frac{7}{3}, -\frac{b x^3}{a}, -\frac{d x^3}{c}\right]}{4 a^2 \sqrt{1+\frac{d x^3}{c}}}$$

Result (type 6, 324 leaves):

$$\left(x \left(-4 \left(c+d x^3\right) + \left(32 a c^2 \text{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right]\right) \Big/ \left(8 a c \text{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right] - 3 x^3 \left(2 b c \text{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 2, \frac{7}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right] + a d \text{AppellF1}\left[\frac{4}{3}, \frac{3}{2}, 1, \frac{7}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right]\right)\right) - \left(35 a c d x^3 \text{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right]\right) \Big/ \left(-14 a c \text{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right] + 3 x^3 \left(2 b c \text{AppellF1}\left[\frac{7}{3}, \frac{1}{2}, 2, \frac{10}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right] + a d \text{AppellF1}\left[\frac{7}{3}, \frac{3}{2}, 1, \frac{10}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right]\right)\right)\right)\right) \Big/ \left(12 b \left(a+b x^3\right) \sqrt{c+d x^3}\right)$$

Problem 466: Result more than twice size of optimal antiderivative.

$$\int \frac{x \sqrt{c+d x^3}}{\left(a+b x^3\right)^2} \, dx$$

Optimal (type 6, 64 leaves, 2 steps):

$$\frac{x^2 \sqrt{c+d x^3} \text{AppellF1}\left[\frac{2}{3}, 2, -\frac{1}{2}, \frac{5}{3}, -\frac{b x^3}{a}, -\frac{d x^3}{c}\right]}{2 a^2 \sqrt{1+\frac{d x^3}{c}}}$$

Result (type 6, 324 leaves):

$$\Big(x^2 \Big(\frac{5 (c+d x^3)}{a} + \Big(25 c^2 \text{AppellF1}\Big[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\Big]\Big) \Big/ \Big(10 a c \text{AppellF1}\Big[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\Big] - 3 x^3 \Big(2 b c \text{AppellF1}\Big[\frac{5}{3}, \frac{1}{2}, 2, \frac{8}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\Big] + a d \text{AppellF1}\Big[\frac{5}{3}, \frac{3}{2}, 1, \frac{8}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\Big]\Big)\Big) + \Big(8 c d x^3 \text{AppellF1}\Big[\frac{5}{3}, \frac{1}{2}, 1, \frac{8}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\Big]\Big) \Big/ \Big(-16 a c \text{AppellF1}\Big[\frac{5}{3}, \frac{1}{2}, 1, \frac{8}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\Big] + 3 x^3 \Big(2 b c \text{AppellF1}\Big[\frac{8}{3}, \frac{1}{2}, 2, \frac{11}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\Big] + a d \text{AppellF1}\Big[\frac{8}{3}, \frac{3}{2}, 1, \frac{11}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\Big]\Big)\Big)\Big)\Big) \Big/ \Big(15 (a+b x^3) \sqrt{c+d x^3}\Big)$$

Problem 467: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{c+d x^3}}{(a+b x^3)^2} \, dx$$

Optimal (type 6, 59 leaves, 2 steps):

$$\frac{x \sqrt{c+d x^3} \text{AppellF1}\Big[\frac{1}{3}, 2, -\frac{1}{2}, \frac{4}{3}, -\frac{b x^3}{a}, -\frac{d x^3}{c}\Big]}{a^2 \sqrt{1+\frac{d x^3}{c}}}$$

Result (type 6, 322 leaves):

$$\Big(x \Big(\frac{4 (c+d x^3)}{a} + \Big(64 c^2 \text{AppellF1}\Big[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\Big]\Big) \Big/ \Big(8 a c \text{AppellF1}\Big[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\Big] - 3 x^3 \Big(2 b c \text{AppellF1}\Big[\frac{4}{3}, \frac{1}{2}, 2, \frac{7}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\Big] + a d \text{AppellF1}\Big[\frac{4}{3}, \frac{3}{2}, 1, \frac{7}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\Big]\Big)\Big) - \Big(7 c d x^3 \text{AppellF1}\Big[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\Big]\Big) \Big/ \Big(-14 a c \text{AppellF1}\Big[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\Big] + 3 x^3 \Big(2 b c \text{AppellF1}\Big[\frac{7}{3}, \frac{1}{2}, 2, \frac{10}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\Big] + a d \text{AppellF1}\Big[\frac{7}{3}, \frac{3}{2}, 1, \frac{10}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\Big]\Big)\Big)\Big)\Big) \Big/ \Big(12 (a+b x^3) \sqrt{c+d x^3}\Big)$$

Problem 468: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{c+d x^3}}{x^2 (a+b x^3)^2} \, dx$$

Optimal (type 6, 62 leaves, 2 steps):

$$-\frac{\sqrt{c+d x^3}\ \text{AppellF1}\left[-\frac{1}{3},2,-\frac{1}{2},\frac{2}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{a^2 x \sqrt{1+\frac{d x^3}{c}}}$$

Result (type 6, 347 leaves):

$$\left(-10\left(3 a+4 b x^3\right)\left(c+d x^3\right)+\left(25 a c\left(-8 b c+9 a d\right) x^3\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Big/\left(10 a c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]-3 x^3\left(2 b c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d\,\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)+\left(64 a b c d x^6\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Big/\left(16 a c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]-3 x^3\left(2 b c\,\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d\,\text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)\right)\Big/\left(30 a^2 x\left(a+b x^3\right)\sqrt{c+d x^3}\right)$$

Problem 469: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{c+d x^3}}{x^3\left(a+b x^3\right)^2}\,dx$$

Optimal (type 6, 64 leaves, 2 steps):

$$-\frac{\sqrt{c+d x^3}\ \text{AppellF1}\left[-\frac{2}{3},2,-\frac{1}{2},\frac{1}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{2 a^2 x^2 \sqrt{1+\frac{d x^3}{c}}}$$

Result (type 6, 347 leaves):

$$\left(-4\left(3a+5bx^3\right)\left(c+dx^3\right)+\left(16ac\left(-20bc+9ad\right)x^3\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right]\right)\Big/\left(8ac\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right]-3x^3\left(2bc\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right]+ad\,\text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right]\right)\right)+\left(35abcdx^6\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right]\right)\Big/\left(-14ac\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right]+3x^3\left(2bc\,\text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right]+ad\,\text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right]\right)\right)\right)\Big/\left(24a^2x^2\left(a+bx^3\right)\sqrt{c+dx^3}\right)$$

Problem 473: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\frac{\left(c+dx^3\right)^{3/2}}{x\left(a+bx^3\right)^2}\,dx$$

Optimal (type 3, 131 leaves, 7 steps):

$$\frac{\left(bc-ad\right)\sqrt{c+dx^3}}{3ab\left(a+bx^3\right)}-\frac{2c^{3/2}\,\text{ArcTanh}\left[\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right]}{3a^2}+\frac{\sqrt{bc-ad}\left(2bc+ad\right)\text{ArcTanh}\left[\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right]}{3a^2b^{3/2}}$$

Result (type 6, 328 leaves):

$$\left(-\left(\left(6cd\left(bc+ad\right)x^3\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{dx^3}{c},-\frac{bx^3}{a}\right]\right)\Big/\left(-4ac\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{dx^3}{c},-\frac{bx^3}{a}\right]+x^3\left(2bc\,\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{dx^3}{c},-\frac{bx^3}{a}\right]+ad\,\text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{dx^3}{c},-\frac{bx^3}{a}\right]\right)\right)\right)+\frac{1}{a}\left(3\left(bc-ad\right)\left(c+dx^3\right)+\left(10b^2c^2dx^3\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{dx^3},-\frac{a}{bx^3}\right]\right)\Big/\left(-5bdx^3\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{dx^3},-\frac{a}{bx^3}\right]+2ad\,\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{dx^3},-\frac{a}{bx^3}\right]+bc\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{dx^3},-\frac{a}{bx^3}\right]\right)\right)\right)\Big/\left(9b\left(a+bx^3\right)\sqrt{c+dx^3}\right)$$

Problem 474: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\left(c+d x^3\right)^{3/2}}{x^4 \left(a+b x^3\right)^2} \, dx$$

Optimal (type 3, 170 leaves, 8 steps):

$$-\frac{(2 b c-a d) \sqrt{c+d x^3}}{3 a^2 \left(a+b x^3\right)}-\frac{c \sqrt{c+d x^3}}{3 a x^3 \left(a+b x^3\right)}+\frac{\sqrt{c} (4 b c-3 a d) \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{3 a^3}-\frac{\sqrt{b c-a d} (4 b c-a d) \text{ArcTanh}\left[\frac{\sqrt{b} \sqrt{c+d x^3}}{\sqrt{b c-a d}}\right]}{3 a^3 \sqrt{b}}$$

Result (type 6, 439 leaves):

$$\left(\left(6 a c d (-2 b c+a d) x^6 \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Big/\left(4 a c \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},-\frac{b x^3}{a}\right]-x^3 \left(2 b c \text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d \text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)+\left(5 b d x^3 \left(2 b c x^3 \left(c+3 d x^3\right)+3 a \left(c^2+c d x^3-d^2 x^6\right)\right) \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]-3 \left(c+d x^3\right) \left(2 b c x^3+a \left(c-d x^3\right)\right) \left(2 a d \text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]+b c \text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]\right)\right)\Big/\left(-5 b d x^3 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]+2 a d \text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]+b c \text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]\right)\right)\Big/\left(9 a^2 x^3 \left(a+b x^3\right) \sqrt{c+d x^3}\right)$$

Problem 475: Result more than twice size of optimal antiderivative.

$$\int \frac{x^3 \left(c+d x^3\right)^{3/2}}{\left(a+b x^3\right)^2} \, dx$$

Optimal (type 6, 65 leaves, 2 steps):

$$\frac{c x^4 \sqrt{c+d x^3} \text{AppellF1}\left[\frac{4}{3},2,-\frac{3}{2},\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{4 a^2 \sqrt{1+\frac{d x^3}{c}}}$$

Result (type 6, 358 leaves):

$$\left(x\left(-4\left(c+d x^3\right)\left(5 b c-11 a d-6 b d x^3\right)-\left(32 a c^2(-5 b c+11 a d) \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Big/\left(8 a c \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]-3 x^3\left(2 b c \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d \text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)+\left(7 a c d(-43 b c+55 a d) x^3 \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Big/\left(-14 a c \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+3 x^3\left(2 b c \text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d \text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)\right)\right)\Big/\left(60 b^2\left(a+b x^3\right)\sqrt{c+d x^3}\right)$$

Problem 476: Result more than twice size of optimal antiderivative.

$$\int \frac{x\left(c+d x^3\right)^{3/2}}{\left(a+b x^3\right)^2} \, dx$$

Optimal (type 6, 65 leaves, 2 steps):

$$\frac{c x^2 \sqrt{c+d x^3} \text{ AppellF1}\left[\frac{2}{3},2,-\frac{3}{2},\frac{5}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{2 a^2 \sqrt{1+\frac{d x^3}{c}}}$$

Result (type 6, 439 leaves):

$$\left(x^2 \left(-\left(\left(25\, c^2\, (b\, c+2\, a\, d)\, \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d\, x^3}{c},-\frac{b\, x^3}{a}\right]\right)\right/\right.\right.$$
$$\left(-10\, a\, c\, \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d\, x^3}{c},-\frac{b\, x^3}{a}\right]+3\, x^3 \left(2\, b\, c\, \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\right.\right.\right.$$
$$\left.\left.\left.\left.\frac{8}{3},-\frac{d\, x^3}{c},-\frac{b\, x^3}{a}\right]+a\, d\, \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d\, x^3}{c},-\frac{b\, x^3}{a}\right]\right)\right)\right)+$$
$$\left(-8\, a\, c\, \left(a\, d\, \left(10\, c+3\, d\, x^3\right)-b\, c\, \left(10\, c+9\, d\, x^3\right)\right)\, \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d\, x^3}{c},-\frac{b\, x^3}{a}\right]-\right.$$
$$15\, (b\, c-a\, d)\, x^3\, \left(c+d\, x^3\right) \left(2\, b\, c\, \text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d\, x^3}{c},-\frac{b\, x^3}{a}\right]+\right.$$
$$\left.\left.a\, d\, \text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d\, x^3}{c},-\frac{b\, x^3}{a}\right]\right)\right)\Big/$$
$$\left(a \left(16\, a\, c\, \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d\, x^3}{c},-\frac{b\, x^3}{a}\right]-\right.\right.$$
$$3\, x^3 \left(2\, b\, c\, \text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d\, x^3}{c},-\frac{b\, x^3}{a}\right]+\right.$$
$$\left.\left.\left.\left.\left.a\, d\, \text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d\, x^3}{c},-\frac{b\, x^3}{a}\right]\right)\right)\right)\right)\right)\Big/\left(15\, b\, \left(a+b\, x^3\right) \sqrt{c+d\, x^3}\right)$$

Problem 477: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(c+d\, x^3\right)^{3/2}}{\left(a+b\, x^3\right)^2}\, dx$$

Optimal (type 6, 60 leaves, 2 steps):

$$\frac{c\, x\, \sqrt{c+d\, x^3}\, \text{AppellF1}\left[\frac{1}{3},2,-\frac{3}{2},\frac{4}{3},-\frac{b\, x^3}{a},-\frac{d\, x^3}{c}\right]}{a^2 \sqrt{1+\frac{d\, x^3}{c}}}$$

Result (type 6, 437 leaves):

$$\left(x\left(-\left(\left(32 c^2 (2 b c+a d) \operatorname{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right]\right)\right/\right.\right.$$

$$\left(-8 a c \operatorname{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right]+3 x^3\left(2 b c \operatorname{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 2,\right.\right.\right.$$

$$\left.\left.\left.\left.\frac{7}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right]+a d \operatorname{AppellF1}\left[\frac{4}{3}, \frac{3}{2}, 1, \frac{7}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right]\right)\right)\right)+$$

$$\left(-7 a c\left(a d\left(8 c+3 d x^3\right)-b c\left(8 c+9 d x^3\right)\right) \operatorname{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right]-\right.$$

$$12(b c-a d) x^3\left(c+d x^3\right)\left(2 b c \operatorname{AppellF1}\left[\frac{7}{3}, \frac{1}{2}, 2, \frac{10}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right]+\right.$$

$$\left.\left.a d \operatorname{AppellF1}\left[\frac{7}{3}, \frac{3}{2}, 1, \frac{10}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right]\right)\right)\Big/$$

$$\left(a\left(14 a c \operatorname{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right]-\right.\right.$$

$$3 x^3\left(2 b c \operatorname{AppellF1}\left[\frac{7}{3}, \frac{1}{2}, 2, \frac{10}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right]+\right.$$

$$\left.\left.\left.\left.\left.a d \operatorname{AppellF1}\left[\frac{7}{3}, \frac{3}{2}, 1, \frac{10}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right]\right)\right)\right)\right)\right)\Big/\left(12 b\left(a+b x^3\right) \sqrt{c+d x^3}\right)$$

Problem 478: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(c+d x^3\right)^{3/2}}{x^2\left(a+b x^3\right)^2} \, dx$$

Optimal (type 6, 63 leaves, 2 steps):

$$-\frac{c \sqrt{c+d x^3} \operatorname{AppellF1}\left[-\frac{1}{3}, 2, -\frac{3}{2}, \frac{2}{3}, -\frac{b x^3}{a}, -\frac{d x^3}{c}\right]}{a^2 x \sqrt{1+\frac{d x^3}{c}}}$$

Result (type 6, 365 leaves):

$$\left(-10\left(c+d x^3\right)\left(3 a c+4 b c x^3-a d x^3\right)+\left(25 a c^2\left(-8 b c+11 a d\right) x^3 \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Big/\left(10 a c \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]-3 x^3\left(2 b c \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)+\left(16 a c d\left(-4 b c+a d\right) x^6 \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Big/\left(-16 a c \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+3 x^3\left(2 b c \text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d \text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)\right)\Big/\left(30 a^2 x\left(a+b x^3\right)\sqrt{c+d x^3}\right)$$

Problem 479: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(c+d x^3\right)^{3/2}}{x^3\left(a+b x^3\right)^2}\, dx$$

Optimal (type 6, 65 leaves, 2 steps):

$$-\frac{c\sqrt{c+d x^3}\ \text{AppellF1}\left[-\frac{2}{3},2,-\frac{3}{2},\frac{1}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{2 a^2 x^2\sqrt{1+\frac{d x^3}{c}}}$$

Result (type 6, 366 leaves):

$$\left(-4 \left(c+d x^3\right) \left(3 a c+5 b c x^3-2 a d x^3\right)+\left(16 a c^2 \left(-20 b c+17 a d\right) x^3 \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Big/\left(8 a c \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]-3 x^3 \left(2 b c \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d \text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)-\left(7 a c d \left(-5 b c+2 a d\right) x^6 \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Big/\left(-14 a c \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+3 x^3 \left(2 b c \text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d \text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)\right)\Big/\left(24 a^2 x^2 \left(a+b x^3\right) \sqrt{c+d x^3}\right)$$

Problem 483: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{x \left(a+b x^3\right)^2 \sqrt{c+d x^3}} \, dx$$

Optimal (type 3, 132 leaves, 7 steps):

$$\frac{b \sqrt{c+d x^3}}{3 a \left(b c-a d\right) \left(a+b x^3\right)}-\frac{2 \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{3 a^2 \sqrt{c}}+\frac{\sqrt{b} \left(2 b c-3 a d\right) \text{ArcTanh}\left[\frac{\sqrt{b} \sqrt{c+d x^3}}{\sqrt{b c-a d}}\right]}{3 a^2 \left(b c-a d\right)^{3/2}}$$

Result (type 6, 396 leaves):

$$\left(b\left(\left(6\,c\,d\,x^3\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right/$$
$$\left(-4\,a\,c\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+x^3\right.$$
$$\left.\left(2\,b\,c\,\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)+$$
$$\left(5\,d\,x^3\left(2\,a\,d+b\left(c+3\,d\,x^3\right)\right)\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]-\right.$$
$$3\left(c+d\,x^3\right)\left(2\,a\,d\,\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]+\right.$$
$$\left.\left.b\,c\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]\right)\right)\Big/\left(a\left(-5\,b\,d\,x^3\right.\right.$$
$$\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]+2\,a\,d\,\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]+$$
$$\left.\left.\left.\left.b\,c\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]\right)\right)\right)\right)\Big/\left(9\left(-b\,c+a\,d\right)\left(a+b\,x^3\right)\sqrt{c+d\,x^3}\right)$$

Problem 484: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\frac{1}{x^4\left(a+b\,x^3\right)^2\sqrt{c+d\,x^3}}\,dx$$

Optimal (type 3, 185 leaves, 8 steps):

$$-\frac{b\left(2\,b\,c-a\,d\right)\sqrt{c+d\,x^3}}{3\,a^2\,c\left(b\,c-a\,d\right)\left(a+b\,x^3\right)}-\frac{\sqrt{c+d\,x^3}}{3\,a\,c\,x^3\left(a+b\,x^3\right)}+$$
$$\frac{\left(4\,b\,c+a\,d\right)\text{ArcTanh}\left[\frac{\sqrt{c+d\,x^3}}{\sqrt{c}}\right]}{3\,a^3\,c^{3/2}}-\frac{b^{3/2}\left(4\,b\,c-5\,a\,d\right)\text{ArcTanh}\left[\frac{\sqrt{b}\sqrt{c+d\,x^3}}{\sqrt{b\,c-a\,d}}\right]}{3\,a^3\left(b\,c-a\,d\right)^{3/2}}$$

Result (type 6, 489 leaves):

$$\left(\left(6\,a\,b\,d\,(-2\,b\,c+a\,d)\,x^6\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\Big/\left((-b\,c+a\,d)\left(-4\,a\,c\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+x^3\left(2\,b\,c\,\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)\right)+\left(5\,b\,d\,x^3\left(-a^2\,d\,(3\,c+2\,d\,x^3)+2\,b^2\,c\,x^3\,(c+3\,d\,x^3)+3\,a\,b\,(c^2+c\,d\,x^3-d^2\,x^6)\right)\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]+3\,(c+d\,x^3)\,(a^2\,d-2\,b^2\,c\,x^3+a\,b\,(-c+d\,x^3))\left(2\,a\,d\,\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]\right)\right)\Big/\left(c\,(b\,c-a\,d)\left(-5\,b\,d\,x^3\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]+2\,a\,d\,\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]\right)\right)\right)\Big/\left(9\,a^2\,x^3\,(a+b\,x^3)\,\sqrt{c+d\,x^3}\right)$$

Problem 485: Result more than twice size of optimal antiderivative.

$$\int \frac{x^3}{(a+b\,x^3)^2\,\sqrt{c+d\,x^3}}\,dx$$

Optimal (type 6, 64 leaves, 2 steps):

$$\frac{x^4\,\sqrt{1+\frac{d\,x^3}{c}}\,\text{AppellF1}\left[\frac{4}{3},2,\frac{1}{2},\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]}{4\,a^2\,\sqrt{c+d\,x^3}}$$

Result (type 6, 331 leaves):

$$\left(x\left(4\,(c+d\,x^3)+\left(32\,a\,c^2\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\Big/\left(-8\,a\,c\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+3\,x^3\left(2\,b\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)-\left(7\,a\,c\,d\,x^3\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\Big/\left(-14\,a\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+3\,x^3\left(2\,b\,c\,\text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)\right)\right)\Big/\left(12\,(-b\,c+a\,d)\,(a+b\,x^3)\,\sqrt{c+d\,x^3}\right)$$

Problem 486: Result more than twice size of optimal antiderivative.

$$\int \frac{x}{\left(a+b x^3\right)^2 \sqrt{c+d x^3}} \, dx$$

Optimal (type 6, 64 leaves, 2 steps):

$$\frac{x^2 \sqrt{1+\frac{d x^3}{c}} \, \text{AppellF1}\left[\frac{2}{3}, 2, \frac{1}{2}, \frac{5}{3}, -\frac{b x^3}{a}, -\frac{d x^3}{c}\right]}{2 a^2 \sqrt{c+d x^3}}$$

Result (type 6, 342 leaves):

$$\left(x^2 \left(-\frac{5 b \left(c+d x^3\right)}{a} + \left(25 c \left(b c-3 a d\right) \text{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right]\right) \Big/ \left(-10 a c \, \text{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right] + 3 x^3 \left(2 b c \, \text{AppellF1}\left[\frac{5}{3}, \frac{1}{2}, 2, \frac{8}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right] + a d \, \text{AppellF1}\left[\frac{5}{3}, \frac{3}{2}, 1, \frac{8}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right]\right)\right) - \left(8 b c d x^3 \, \text{AppellF1}\left[\frac{5}{3}, \frac{1}{2}, 1, \frac{8}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right]\right) \Big/ \left(-16 a c \, \text{AppellF1}\left[\frac{5}{3}, \frac{1}{2}, 1, \frac{8}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right] + 3 x^3 \left(2 b c \, \text{AppellF1}\left[\frac{8}{3}, \frac{1}{2}, 2, \frac{11}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right] + a d \, \text{AppellF1}\left[\frac{8}{3}, \frac{3}{2}, 1, \frac{11}{3}, -\frac{d x^3}{c}, -\frac{b x^3}{a}\right]\right)\right)\right)\right) \Big/ \left(15 \left(-b c+a d\right) \left(a+b x^3\right) \sqrt{c+d x^3}\right)$$

Problem 487: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(a+b x^3\right)^2 \sqrt{c+d x^3}} \, dx$$

Optimal (type 6, 59 leaves, 2 steps):

$$\frac{x \sqrt{1+\frac{d x^3}{c}} \, \text{AppellF1}\left[\frac{1}{3}, 2, \frac{1}{2}, \frac{4}{3}, -\frac{b x^3}{a}, -\frac{d x^3}{c}\right]}{a^2 \sqrt{c+d x^3}}$$

Result (type 6, 341 leaves):

$$\left(x\left(-\frac{4\,b\,\left(c+d\,x^3\right)}{a}+\left(32\,c\,\left(2\,b\,c-3\,a\,d\right)\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\Big/\left(-8\,a\,c\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+3\,x^3\,\left(2\,b\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)+\left(7\,b\,c\,d\,x^3\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\Big/\left(-14\,a\,c\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+3\,x^3\,\left(2\,b\,c\,\text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)\right)\right)\Big/\left(12\,\left(-b\,c+a\,d\right)\,\left(a+b\,x^3\right)\,\sqrt{c+d\,x^3}\right)$$

Problem 488: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^2\,\left(a+b\,x^3\right)^2\,\sqrt{c+d\,x^3}}\,dx$$

Optimal (type 6, 62 leaves, 2 steps):

$$-\frac{\sqrt{1+\frac{d\,x^3}{c}}\,\text{AppellF1}\left[-\frac{1}{3},2,\frac{1}{2},\frac{2}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]}{a^2\,x\,\sqrt{c+d\,x^3}}$$

Result (type 6, 399 leaves):

$$\left(\frac{10\,\left(c+d\,x^3\right)\,\left(-3\,a^2\,d+4\,b^2\,c\,x^3+3\,a\,b\,\left(c-d\,x^3\right)\right)}{c}-\left(25\,a\,\left(8\,b^2\,c^2-15\,a\,b\,c\,d+3\,a^2\,d^2\right)\,x^3\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\Big/\left(-10\,a\,c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+3\,x^3\,\left(2\,b\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)+\left(16\,a\,b\,d\,\left(4\,b\,c-3\,a\,d\right)\,x^6\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\Big/\left(-16\,a\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+3\,x^3\,\left(2\,b\,c\,\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)\right)\Big/\left(30\,a^2\,\left(-b\,c+a\,d\right)\,x\,\left(a+b\,x^3\right)\,\sqrt{c+d\,x^3}\right)$$

Problem 489: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^3 \left(a+b x^3\right)^2 \sqrt{c+d x^3}} \, dx$$

Optimal (type 6, 64 leaves, 2 steps):

$$-\frac{\sqrt{1+\frac{d x^3}{c}} \text{AppellF1}\left[-\frac{2}{3},2,\frac{1}{2},\frac{1}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{2 a^2 x^2 \sqrt{c+d x^3}}$$

Result (type 6, 399 leaves):

$$\left(\frac{4 \left(c+d x^3\right) \left(-3 a^2 d+5 b^2 c x^3+3 a b \left(c-d x^3\right)\right)}{c}+\left(16 a \left(-20 b^2 c^2+21 a b c d+3 a^2 d^2\right) x^3 \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right) \Big/ \left(-8 a c \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+3 x^3 \left(2 b c \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d \text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)+\left(7 a b d \left(-5 b c+3 a d\right) x^6 \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right) \Big/ \left(-14 a c \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+3 x^3 \left(2 b c \text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d \text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)\right) \Big/ \left(24 a^2 \left(-b c+a d\right) x^2 \left(a+b x^3\right) \sqrt{c+d x^3}\right)$$

Problem 493: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{x \left(a+b x^3\right)^2 \left(c+d x^3\right)^{3/2}} \, dx$$

Optimal (type 3, 172 leaves, 8 steps):

$$\frac{d \left(b c+2 a d\right)}{3 a c \left(b c-a d\right)^2 \sqrt{c+d x^3}}+\frac{b}{3 a \left(b c-a d\right) \left(a+b x^3\right) \sqrt{c+d x^3}}-\frac{2 \text{ArcTanh}\left[\frac{\sqrt{c+d x^3}}{\sqrt{c}}\right]}{3 a^2 c^{3/2}}+\frac{b^{3/2} \left(2 b c-5 a d\right) \text{ArcTanh}\left[\frac{\sqrt{b} \sqrt{c+d x^3}}{\sqrt{b c-a d}}\right]}{3 a^2 \left(b c-a d\right)^{5/2}}$$

Result (type 6, 453 leaves):

$$\left(-\left(\left(6\,b\,d\,(b\,c+2\,a\,d)\,x^3\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right/\right.$$
$$\left(-4\,a\,c\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+x^3\left(2\,b\,c\right.\right.$$
$$\left.\left.\left.\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)\right)+$$
$$\left(-5\,b\,d\,x^3\left(4\,a^2\,d^2+b^2\,c\,(c+3\,d\,x^3)+2\,a\,b\,d\,(2\,c+3\,d\,x^3)\right)\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]+\right.$$
$$3\left(2\,a^2\,d^2+2\,a\,b\,d^2\,x^3+b^2\,c\,(c+d\,x^3)\right)\left(2\,a\,d\,\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]+\right.$$
$$\left.\left.b\,c\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]\right)\right)\Big/$$
$$\left(a\,c\left(-5\,b\,d\,x^3\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]+\right.\right.$$
$$2\,a\,d\,\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]+$$
$$\left.\left.\left.b\,c\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d\,x^3},-\frac{a}{b\,x^3}\right]\right)\right)\right)\Big/\left(9\,(b\,c-a\,d)^2\,(a+b\,x^3)\,\sqrt{c+d\,x^3}\right)$$

Problem 494: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\frac{1}{x^4\,(a+b\,x^3)^2\,(c+d\,x^3)^{3/2}}\,dx$$

Optimal (type 3, 241 leaves, 9 steps):

$$-\frac{d\,(2\,b^2\,c^2-2\,a\,b\,c\,d+3\,a^2\,d^2)}{3\,a^2\,c^2\,(b\,c-a\,d)^2\,\sqrt{c+d\,x^3}}-\frac{b\,(2\,b\,c-a\,d)}{3\,a^2\,c\,(b\,c-a\,d)\,(a+b\,x^3)\,\sqrt{c+d\,x^3}}-\frac{1}{3\,a\,c\,x^3\,(a+b\,x^3)\,\sqrt{c+d\,x^3}}+$$
$$\frac{(4\,b\,c+3\,a\,d)\,\text{ArcTanh}\left[\frac{\sqrt{c+d\,x^3}}{\sqrt{c}}\right]}{3\,a^3\,c^{5/2}}-\frac{b^{5/2}\,(4\,b\,c-7\,a\,d)\,\text{ArcTanh}\left[\frac{\sqrt{b}\,\sqrt{c+d\,x^3}}{\sqrt{b\,c-a\,d}}\right]}{3\,a^3\,(b\,c-a\,d)^{5/2}}$$

Result (type 6, 582 leaves):

$$\frac{1}{9 a^2 c^2 (b c-a d)^2 x^3 (a+b x^3) \sqrt{c+d x^3}}$$
$$\left(\left(6 a b c d (2 b^2 c^2-2 a b c d+3 a^2 d^2) x^6 \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Big/\right.$$
$$\left(-4 a c \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+\right.$$
$$\left.\left.x^3 \left(2 b c \text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d \text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)-\right.$$
$$\left(-5 b d x^3 \left(3 a^3 d^2 (c+2 d x^3)+2 b^3 c^2 x^3 (c+3 d x^3)+a b^2 c (3 c^2+2 c d x^3-6 d^2 x^6)+\right.\right.$$
$$\left.a^2 b d (-6 c^2-c d x^3+9 d^2 x^6)\right) \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]+$$
$$3 \left(2 b^3 c^2 x^3 (c+d x^3)+a^3 d^2 (c+3 d x^3)+a b^2 c (c^2-c d x^3-2 d^2 x^6)+\right.$$
$$\left.a^2 b d (-2 c^2-c d x^3+3 d^2 x^6)\right) \left(2 a d \text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]+\right.$$
$$\left.\left.\left. b c \text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]\right)\right)\Big/\right.$$
$$\left(-5 b d x^3 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]+2 a d \text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]+\right.$$
$$\left.\left. b c \text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^3},-\frac{a}{b x^3}\right]\right)\right)$$

Problem 495: Result more than twice size of optimal antiderivative.

$$\int \frac{x^3}{(a+b x^3)^2 (c+d x^3)^{3/2}} \, dx$$

Optimal (type 6, 67 leaves, 2 steps):

$$\frac{x^4 \sqrt{1+\frac{d x^3}{c}} \text{AppellF1}\left[\frac{4}{3},2,\frac{3}{2},\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{4 a^2 c \sqrt{c+d x^3}}$$

Result (type 6, 346 leaves):

$$\left(x\left(-4\left(b c+2 a d+3 b d x^3\right)+\left(32 a c\left(b c+2 a d\right) \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Big/\left(8 a c \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]-3 x^3\left(2 b c \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d \text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)+\left(21 a b c d x^3 \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Big/\left(-14 a c \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+3 x^3\left(2 b c \text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d \text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)\right)\right)\Big/\left(12\left(b c-a d\right)^2\left(a+b x^3\right)\sqrt{c+d x^3}\right)$$

Problem 496: Result more than twice size of optimal antiderivative.

$$\int \frac{x}{\left(a+b x^3\right)^2\left(c+d x^3\right)^{3/2}}\,dx$$

Optimal (type 6, 67 leaves, 2 steps):

$$\frac{x^2\sqrt{1+\frac{d x^3}{c}}\ \text{AppellF1}\left[\frac{2}{3},2,\frac{3}{2},\frac{5}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{2 a^2 c\sqrt{c+d x^3}}$$

Result (type 6, 482 leaves):

$$\left(x^2\left(-\left(\left(25\left(b^2 c^2-6 a b c d-a^2 d^2\right) \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Big/\left(-10 a c \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+3 x^3\left(2 b c \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)\right)+\left(8 a c\left(20 a^2 d^2+18 a b d^2 x^3+b^2 c\left(10 c+9 d x^3\right)\right) \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]-15 x^3\left(2 a^2 d^2+2 a b d^2 x^3+b^2 c\left(c+d x^3\right)\right)\left(2 b c \text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d \text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)\Big/\left(a c\left(16 a c \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]-3 x^3\left(2 b c \text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d \text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)\right)\right)\right)\Big/\left(15\left(b c-a d\right)^2\left(a+b x^3\right)\sqrt{c+d x^3}\right)$$

Problem 497: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(a+b x^3\right)^2 \left(c+d x^3\right)^{3/2}} \, dx$$

Optimal (type 6, 62 leaves, 2 steps):

$$\frac{x \sqrt{1+\frac{d x^3}{c}} \,\text{AppellF1}\left[\frac{1}{3},2,\frac{3}{2},\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{a^2 c \sqrt{c+d x^3}}$$

Result (type 6, 480 leaves):

$$\left(x \left(-\left(\left(32 \left(2 b^2 c^2-6 a b c d+a^2 d^2\right) \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right) \Big/ \left(-8 a c \,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+3 x^3 \left(2 b c \,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d \,\text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)\right)+\left(7 a c \left(16 a^2 d^2+18 a b d^2 x^3+b^2 c \left(8 c+9 d x^3\right)\right) \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]-12 x^3 \left(2 a^2 d^2+2 a b d^2 x^3+b^2 c \left(c+d x^3\right)\right) \left(2 b c \,\text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d \,\text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right) \Big/ \left(a c \left(14 a c \,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]-3 x^3 \left(2 b c \,\text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d \,\text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)\right)\right)\right) \Big/ \left(12 (b c-a d)^2 \left(a+b x^3\right) \sqrt{c+d x^3}\right)$$

Problem 498: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^2 \left(a+b x^3\right)^2 \left(c+d x^3\right)^{3/2}} \, dx$$

Optimal (type 6, 65 leaves, 2 steps):

$$-\frac{\sqrt{1+\frac{d x^3}{c}} \,\text{AppellF1}\left[-\frac{1}{3},2,\frac{3}{2},\frac{2}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{a^2 c x \sqrt{c+d x^3}}$$

Result (type 6, 483 leaves):

$$\frac{1}{30\,a^2\,c^2\,\left(b\,c-a\,d\right)^2\,x\,\left(a+b\,x^3\right)\,\sqrt{c+d\,x^3}}\left(-10\,\left(4\,b^3\,c^2\,x^3\,\left(c+d\,x^3\right)+a^3\,d^2\,\left(3\,c+5\,d\,x^3\right)+3\,a\,b^2\,c\,\left(c^2-c\,d\,x^3-2\,d^2\,x^6\right)+a^2\,b\,d\,\left(-6\,c^2-3\,c\,d\,x^3+5\,d^2\,x^6\right)\right)+\left(25\,a\,c\,\left(-8\,b^3\,c^3+21\,a\,b^2\,c^2\,d-6\,a^2\,b\,c\,d^2+5\,a^3\,d^3\right)\,x^3\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\Big/\left(10\,a\,c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]-3\,x^3\left(2\,b\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)-\left(16\,a\,b\,c\,d\,\left(4\,b^2\,c^2-6\,a\,b\,c\,d+5\,a^2\,d^2\right)\,x^6\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\Big/\left(-16\,a\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+3\,x^3\left(2\,b\,c\,\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d\,x^3}{c},-\frac{b\,x^3}{a}\right]\right)\right)\right)$$

Problem 499: Result more than twice size of optimal antiderivative.

$$\int\frac{1}{x^3\,\left(a+b\,x^3\right)^2\,\left(c+d\,x^3\right)^{3/2}}\,dx$$

Optimal (type 6, 67 leaves, 2 steps):

$$-\frac{\sqrt{1+\frac{d\,x^3}{c}}\;\text{AppellF1}\left[-\frac{2}{3},2,\frac{3}{2},\frac{1}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]}{2\,a^2\,c\,x^2\,\sqrt{c+d\,x^3}}$$

Result (type 6, 483 leaves):

$$\frac{1}{24 a^2 c^2 (b c-a d)^2 x^2 (a+b x^3) \sqrt{c+d x^3}}\Bigg(-4 \left(5 b^3 c^2 x^3 (c+d x^3)+a^3 d^2 (3 c+7 d x^3)+3 a b^2 c (c^2-c d x^3-2 d^2 x^6)+a^2 b d (-6 c^2-3 c d x^3+7 d^2 x^6)\right)+$$
$$\left(16 a c (20 b^3 c^3-33 a b^2 c^2 d-6 a^2 b c d^2+7 a^3 d^3) x^3 \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Big/$$
$$\left(-8 a c \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+3 x^3 \left(2 b c \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d \text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)+$$
$$\left(7 a b c d (5 b^2 c^2-6 a b c d+7 a^2 d^2) x^6 \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\Big/$$
$$\left(-14 a c \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+3 x^3 \left(2 b c \text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+a d \text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)\Bigg)$$

Problem 508: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{x \sqrt{a+b x^3} \sqrt{c+d x^3}} \, dx$$

Optimal (type 3, 48 leaves, 3 steps):

$$-\frac{2 \text{ArcTanh}\left[\frac{\sqrt{c} \sqrt{a+b x^3}}{\sqrt{a} \sqrt{c+d x^3}}\right]}{3 \sqrt{a} \sqrt{c}}$$

Result (type 6, 155 leaves):

$$\left(4 b d x^3 \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-\frac{a}{b x^3},-\frac{c}{d x^3}\right]\right)\Big/$$
$$\left(3 \sqrt{a+b x^3} \sqrt{c+d x^3} \left(-4 b d x^3 \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-\frac{a}{b x^3},-\frac{c}{d x^3}\right]+b c \text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-\frac{a}{b x^3},-\frac{c}{d x^3}\right]+a d \text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-\frac{a}{b x^3},-\frac{c}{d x^3}\right]\right)\right)$$

Problem 509: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{x^4 \sqrt{a+b x^3} \sqrt{c+d x^3}} \, dx$$

Optimal (type 3, 91 leaves, 4 steps):

$$-\frac{\sqrt{a+b x^3}\,\sqrt{c+d x^3}}{3 a c x^3}+\frac{(b c+a d)\,\text{ArcTanh}\left[\frac{\sqrt{c}\,\sqrt{a+b x^3}}{\sqrt{a}\,\sqrt{c+d x^3}}\right]}{3 a^{3/2} c^{3/2}}$$

Result (type 6, 192 leaves):

$$\left(-\left(a+b x^3\right)\left(c+d x^3\right)+\left(2 b d (b c+a d) x^6\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-\frac{a}{b x^3},-\frac{c}{d x^3}\right]\right)\Big/\left(4 b d x^3\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-\frac{a}{b x^3},-\frac{c}{d x^3}\right]-b c\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-\frac{a}{b x^3},-\frac{c}{d x^3}\right]-a d\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-\frac{a}{b x^3},-\frac{c}{d x^3}\right]\right)\right)\Big/\left(3 a c x^3\sqrt{a+b x^3}\,\sqrt{c+d x^3}\right)$$

Problem 513: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{a+b x^3}\,\sqrt{c+d x^3}}\,dx$$

Optimal (type 6, 83 leaves, 3 steps):

$$\frac{x\sqrt{1+\frac{b x^3}{a}}\,\sqrt{1+\frac{d x^3}{c}}\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{\sqrt{a+b x^3}\,\sqrt{c+d x^3}}$$

Result (type 6, 170 leaves):

$$-\left(\left(8 a c x\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\Big/\left(\sqrt{a+b x^3}\,\sqrt{c+d x^3}\left(-8 a c\,\text{AppellF1}\left[\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+3 x^3\left(a d\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{2},\frac{3}{2},\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+b c\,\text{AppellF1}\left[\frac{4}{3},\frac{3}{2},\frac{1}{2},\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\right)\right)\right)$$

Problem 514: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^2\sqrt{a+b x^3}\,\sqrt{c+d x^3}}\,dx$$

Optimal (type 6, 86 leaves, 3 steps):

$$-\frac{\sqrt{1+\frac{b x^3}{a}}\,\sqrt{1+\frac{d x^3}{c}}\,\text{AppellF1}\left[-\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{2}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{x\sqrt{a+b x^3}\,\sqrt{c+d x^3}}$$

Result (type 6, 357 leaves):

$$\left(-\frac{10\left(a+b x^3\right)\left(c+d x^3\right)}{a c}-\left(25\left(b c+a d\right) x^3 \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\Big/\left(-10 a c \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+3 x^3\left(a d \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},\frac{3}{2},\frac{8}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+b c \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},\frac{1}{2},\frac{8}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\right)-\left(64 b d x^6 \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},\frac{1}{2},\frac{8}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\Big/\left(-16 a c \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},\frac{1}{2},\frac{8}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+3 x^3\left(a d \text{AppellF1}\left[\frac{8}{3},\frac{1}{2},\frac{3}{2},\frac{11}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+b c \text{AppellF1}\left[\frac{8}{3},\frac{3}{2},\frac{1}{2},\frac{11}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\right)\right)\Big/\left(10 x \sqrt{a+b x^3}\sqrt{c+d x^3}\right)$$

Problem 515: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^3\sqrt{a+b x^3}\sqrt{c+d x^3}}\,dx$$

Optimal (type 6, 88 leaves, 3 steps):

$$-\frac{\sqrt{1+\frac{b x^3}{a}}\sqrt{1+\frac{d x^3}{c}}\text{AppellF1}\left[-\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{1}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{2 x^2\sqrt{a+b x^3}\sqrt{c+d x^3}}$$

Result (type 6, 357 leaves):

$$\left(-\frac{\left(a+b x^3\right)\left(c+d x^3\right)}{a c}+\left(4\left(b c+a d\right) x^3 \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\Big/\left(-8 a c \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+3 x^3\left(a d \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},\frac{3}{2},\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+b c \text{AppellF1}\left[\frac{4}{3},\frac{3}{2},\frac{1}{2},\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\right)+\left(7 b d x^6 \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},\frac{1}{2},\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\Big/\left(28 a c \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},\frac{1}{2},\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]-6 x^3\left(a d \text{AppellF1}\left[\frac{7}{3},\frac{1}{2},\frac{3}{2},\frac{10}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+b c \text{AppellF1}\left[\frac{7}{3},\frac{3}{2},\frac{1}{2},\frac{10}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\right)\right)\Big/\left(2 x^2\sqrt{a+b x^3}\sqrt{c+d x^3}\right)$$

Problem 517: Result unnecessarily involves imaginary or complex numbers.

$$\int (e x)^{5/2} \sqrt{a+b x^3} \left(A+B x^3\right) dx$$

Optimal (type 4, 324 leaves, 5 steps):

$$\frac{3 a (16 A b-7 a B) e^2 \sqrt{e x} \sqrt{a+b x^3}}{320 b^2}+\frac{(16 A b-7 a B) (e x)^{7/2} \sqrt{a+b x^3}}{80 b e}+\frac{B (e x)^{7/2} \left(a+b x^3\right)^{3/2}}{8 b e}-\left(3^{3/4} a^{5/3} (16 A b-7 a B) e^2 \sqrt{e x} \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)^2}} \text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right) b^{1/3} x}{a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x}\right], \frac{1}{4} \left(2+\sqrt{3}\right)\right]\right) \Big/ \left(640 b^2 \sqrt{\frac{b^{1/3} x \left(a^{1/3}+b^{1/3} x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 234 leaves):

$$\left(e^2 \sqrt{e x} \left(-(-a)^{1/3} \left(a+b x^3\right) \left(21 a^2 B-12 a b \left(4 A+B x^3\right)-8 b^2 x^3 \left(8 A+5 B x^3\right)\right)+i\, 3^{3/4} a^2 b^{1/3} (16 A b-7 a B) x \sqrt{\frac{(-1)^{5/6} \left((-a)^{1/3}-b^{1/3} x\right)}{b^{1/3} x}} \sqrt{\frac{\frac{(-a)^{2/3}}{b^{2/3}}+\frac{(-a)^{1/3} x}{b^{1/3}}+x^2}{x^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-a)^{1/3}}{b^{1/3} x}}}{3^{1/4}}\right], (-1)^{1/3}\right]\right)\right) \Big/ \left(320 (-a)^{1/3} b^2 \sqrt{a+b x^3}\right)$$

Problem 518: Result unnecessarily involves imaginary or complex numbers.

$$\int (e x)^{3/2} \sqrt{a+b x^3} \left(A+B x^3\right) dx$$

Optimal (type 4, 581 leaves, 6 steps):

$$\frac{(14 A b-5 a B)\ (e x)^{5/2}\sqrt{a+b x^3}}{56 b e}+$$

$$\frac{3\left(1+\sqrt{3}\right) a\ (14 A b-5 a B)\ e\sqrt{e x}\ \sqrt{a+b x^3}}{112 b^{5/3}\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)}+\frac{B\ (e x)^{5/2}\left(a+b x^3\right)^{3/2}}{7 b e}-$$

$$\left(3\times 3^{1/4} a^{4/3}\ (14 A b-5 a B)\ e\sqrt{e x}\ \left(a^{1/3}+b^{1/3} x\right)\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right) b^{1/3} x}{a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x}\right],\frac{1}{4}\left(2+\sqrt{3}\right)\right]\right)\Big/$$

$$\left(112 b^{5/3}\sqrt{\frac{b^{1/3} x\left(a^{1/3}+b^{1/3} x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)^2}}\ \sqrt{a+b x^3}\right)-$$

$$\left(3^{3/4}\left(1-\sqrt{3}\right) a^{4/3}\ (14 A b-5 a B)\ e\sqrt{e x}\ \left(a^{1/3}+b^{1/3} x\right)\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right) b^{1/3} x}{a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x}\right],\frac{1}{4}\left(2+\sqrt{3}\right)\right]\right)\Big/$$

$$\left(224 b^{5/3}\sqrt{\frac{b^{1/3} x\left(a^{1/3}+b^{1/3} x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)^2}}\ \sqrt{a+b x^3}\right)$$

Result (type 4, 279 leaves) :

$$\frac{1}{112\, b^2 \sqrt{a+b x^3}} x\, (e x)^{3/2} \left(2 b \left(a+b x^3\right) \left(14 A b+3 a B+8 b B x^3\right) - a \left(14 A b-5 a B\right) \left(-3 \left(b+\frac{a}{x^3}\right)+\frac{1}{(-a)^{2/3} x} (-1)^{1/6}\, 3^{3/4}\, a\, b^{2/3} \sqrt{\frac{(-1)^{5/6} \left((-a)^{1/3}-b^{1/3} x\right)}{b^{1/3} x}} \sqrt{\frac{\frac{(-a)^{2/3}}{b^{2/3}}+\frac{(-a)^{1/3} x}{b^{1/3}}+x^2}{x^2}} \left(-i \sqrt{3}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-a)^{1/3}}{b^{1/3} x}}}{3^{1/4}}\right], (-1)^{1/3}\right] + (-1)^{1/3}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-a)^{1/3}}{b^{1/3} x}}}{3^{1/4}}\right], (-1)^{1/3}\right]\right)\right)\right)$$

Problem 520: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a+b x^3}\, \left(A+B x^3\right)}{\sqrt{e x}}\, dx$$

Optimal (type 4, 286 leaves, 4 steps):

$$\frac{(10 A b-a B) \sqrt{e x} \sqrt{a+b x^3}}{20 b e}+\frac{B \sqrt{e x} \left(a+b x^3\right)^{3/2}}{5 b e}+\left(3^{3/4} a^{2/3} (10 A b-a B) \sqrt{e x} \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)^2}}\, \text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right) b^{1/3} x}{a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x}\right], \frac{1}{4} \left(2+\sqrt{3}\right)\right]\right) \Big/ \left(40 b e \sqrt{\frac{b^{1/3} x \left(a^{1/3}+b^{1/3} x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 209 leaves):

$$\left((-a)^{1/3} x \left(a+b x^3\right) \left(10 A b+3 a B+4 b B x^3\right)-\right.$$

$$i\, 3^{3/4} a b^{1/3} (10 A b-a B) x^2 \sqrt{\frac{(-1)^{5/6} \left((-a)^{1/3}-b^{1/3} x\right)}{b^{1/3} x}} \sqrt{\frac{\frac{(-a)^{2/3}}{b^{2/3}}+\frac{(-a)^{1/3} x}{b^{1/3}}+x^2}{x^2}}$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-a)^{1/3}}{b^{1/3} x}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\Big/\left(20 (-a)^{1/3} b \sqrt{e x} \sqrt{a+b x^3}\right)$$

Problem 521: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a+b x^3} \left(A+B x^3\right)}{(e x)^{3/2}} \, dx$$

Optimal (type 4, 580 leaves, 6 steps):

$$\frac{(8 A b+a B) (e x)^{5/2} \sqrt{a+b x^3}}{4 a e^4}+\frac{3 \left(1+\sqrt{3}\right) (8 A b+a B) \sqrt{e x} \sqrt{a+b x^3}}{8 b^{2/3} e^2 \left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)}-$$

$$\frac{2 A \left(a+b x^3\right)^{3/2}}{a e \sqrt{e x}}-\left(3 \times 3^{1/4} a^{1/3} (8 A b+a B) \sqrt{e x} \left(a^{1/3}+b^{1/3} x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)^2}} \text{EllipticE}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right) b^{1/3} x}{a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x}\right],\frac{1}{4} \left(2+\sqrt{3}\right)\right]\right)\Big/$$

$$\left(8 b^{2/3} e^2 \sqrt{\frac{b^{1/3} x \left(a^{1/3}+b^{1/3} x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)-$$

$$\left(3^{3/4} \left(1-\sqrt{3}\right) a^{1/3} (8 A b+a B) \sqrt{e x} \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right) b^{1/3} x}{a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x}\right],\frac{1}{4} \left(2+\sqrt{3}\right)\right]\right)\Big/$$

$$\left(16 b^{2/3} e^2 \sqrt{\frac{b^{1/3} x \left(a^{1/3}+b^{1/3} x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 283 leaves):

$$\left(x^{3/2}\left(\frac{2\left(a+b x^3\right)\left(-8 A+B x^3\right)}{\sqrt{x}}-\frac{1}{b}\left(8 A b+a B\right) x^{5/2}\left(-3\left(b+\frac{a}{x^3}\right)+\frac{1}{(-a)^{2/3} x}\right.\right.\right.$$
$$(-1)^{1/6} 3^{3/4} a b^{2/3}\sqrt{\frac{(-1)^{5/6}\left((-a)^{1/3}-b^{1/3} x\right)}{b^{1/3} x}}\sqrt{\frac{\frac{(-a)^{2/3}}{b^{2/3}}+\frac{(-a)^{1/3} x}{b^{1/3}}+x^2}{x^2}}$$
$$\left(-i\sqrt{3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-a)^{1/3}}{b^{1/3} x}}}{3^{1/4}}\right],(-1)^{1/3}\right]+(-1)^{1/3}\right.$$
$$\left.\left.\left.\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-a)^{1/3}}{b^{1/3} x}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right)\right)\right)\Bigg/\left(8\,(e x)^{3/2}\sqrt{a+b x^3}\right)$$

Problem 523: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\sqrt{a+b x^3}\left(A+B x^3\right)}{(e x)^{7/2}}\,dx$$

Optimal (type 4, 283 leaves, 4 steps):

$$\frac{\left(4 A b+5 a B\right)\sqrt{e x}\sqrt{a+b x^3}}{10 a e^4}-\frac{2 A\left(a+b x^3\right)^{3/2}}{5 a e\,(e x)^{5/2}}+$$
$$\left(3^{3/4}\left(4 A b+5 a B\right)\sqrt{e x}\left(a^{1/3}+b^{1/3} x\right)\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)^2}}\right.$$
$$\left.\text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right) b^{1/3} x}{a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x}\right],\frac{1}{4}\left(2+\sqrt{3}\right)\right]\right)\Bigg/$$
$$\left(20 a^{1/3} e^4\sqrt{\frac{b^{1/3} x\left(a^{1/3}+b^{1/3} x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)^2}}\sqrt{a+b x^3}\right)$$

Result (type 4, 199 leaves):

$$\left(x\left((-a)^{1/3}\left(a+b x^3\right)\left(-4 A+5 B x^3\right)-\right.\right.$$
$$i\, 3^{3/4} b^{1/3}\left(4 A b+5 a B\right) x^4 \sqrt{\frac{(-1)^{5/6}\left((-a)^{1/3}-b^{1/3} x\right)}{b^{1/3} x}} \sqrt{\frac{\frac{(-a)^{2/3}}{b^{2/3}}+\frac{(-a)^{1/3} x}{b^{1/3}}+x^2}{x^2}}$$
$$\left.\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-a)^{1/3}}{b^{1/3} x}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right)\Big/\left(10\,(-a)^{1/3}(e x)^{7/2}\sqrt{a+b x^3}\right)$$

Problem 524: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a+b x^3}\left(A+B x^3\right)}{x^{9/2}}\, dx$$

Optimal (type 4, 564 leaves, 6 steps):

$$-\frac{2\left(2 A b+7 a B\right)\sqrt{a+b x^3}}{7 a \sqrt{x}}+\frac{3\left(1+\sqrt{3}\right) b^{1/3}\left(2 A b+7 a B\right)\sqrt{x}\sqrt{a+b x^3}}{7 a\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)}-$$
$$\frac{2 A\left(a+b x^3\right)^{3/2}}{7 a x^{7/2}}-\left(3 \times 3^{1/4} b^{1/3}\left(2 A b+7 a B\right)\sqrt{x}\left(a^{1/3}+b^{1/3} x\right)\right.$$
$$\left.\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)^2}}\,\text{EllipticE}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right) b^{1/3} x}{a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x}\right],\frac{1}{4}\left(2+\sqrt{3}\right)\right]\right)\Big/$$
$$\left(7 a^{2/3}\sqrt{\frac{b^{1/3} x\left(a^{1/3}+b^{1/3} x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)^2}}\sqrt{a+b x^3}\right)-$$
$$\left(3^{3/4}\left(1-\sqrt{3}\right) b^{1/3}\left(2 A b+7 a B\right)\sqrt{x}\left(a^{1/3}+b^{1/3} x\right)\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)^2}}\right.$$
$$\left.\text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right) b^{1/3} x}{a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x}\right],\frac{1}{4}\left(2+\sqrt{3}\right)\right]\right)\Big/$$
$$\left(14 a^{2/3}\sqrt{\frac{b^{1/3} x\left(a^{1/3}+b^{1/3} x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)^2}}\sqrt{a+b x^3}\right)$$

Result (type 4, 285 leaves) :

$$-\left(\left(-2\,(-a)^{2/3}\left(a+b\,x^3\right)\left(a\,A+\left(3\,A\,b+7\,a\,B\right)x^3\right)+\left(2\,A\,b+7\,a\,B\right)x^3\left(3\,(-a)^{2/3}\left(a+b\,x^3\right)+\right.\right.\right.$$
$$(-1)^{2/3}\,3^{3/4}\,a\,b^{2/3}\,x^2\,\sqrt{\frac{(-1)^{5/6}\left((-a)^{1/3}-b^{1/3}\,x\right)}{b^{1/3}\,x}}\,\sqrt{\frac{\frac{(-a)^{2/3}}{b^{2/3}}+\frac{(-a)^{1/3}\,x}{b^{1/3}}+x^2}{x^2}}$$
$$\left(\sqrt{3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-a)^{1/3}}{b^{1/3}\,x}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+(-1)^{5/6}\,\text{EllipticF}\left[\right.\right.$$
$$\left.\left.\left.\left.\left.\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-a)^{1/3}}{b^{1/3}\,x}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\right)\right)\Big/\left(7\,(-a)^{5/3}\,x^{7/2}\,\sqrt{a+b\,x^3}\right)\right)$$

Problem 526: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\sqrt{a+b\,x^3}\,\left(A+B\,x^3\right)}{x^{13/2}}\,dx$$

Optimal (type 4, 269 leaves, 4 steps) :

$$\frac{2\,\left(2\,A\,b-11\,a\,B\right)\sqrt{a+b\,x^3}}{55\,a\,x^{5/2}}-\frac{2\,A\,\left(a+b\,x^3\right)^{3/2}}{11\,a\,x^{11/2}}-$$
$$\left(3^{3/4}\,b\,\left(2\,A\,b-11\,a\,B\right)\sqrt{x}\,\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}\,x\right)^2}}\right.$$
$$\left.\text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right)b^{1/3}\,x}{a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}\,x}\right],\,\frac{1}{4}\left(2+\sqrt{3}\right)\right]\right)\Big/$$
$$\left(55\,a^{4/3}\,\sqrt{\frac{b^{1/3}\,x\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 206 leaves) :

$$\left(-\frac{2 A}{11 x^{11/2}}-\frac{2 (3 A b+11 a B)}{55 a x^{5/2}}\right) \sqrt{a+b x^3} -$$

$$\left(2 i 3^{3/4} b^{4/3} (-2 A b+11 a B) \sqrt{(-1)^{5/6} \left(-1+\frac{(-a)^{1/3}}{b^{1/3} x}\right)} \sqrt{1+\frac{(-a)^{2/3}}{b^{2/3} x^2}+\frac{(-a)^{1/3}}{b^{1/3} x}}\, x^{3/2}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-a)^{1/3}}{b^{1/3} x}}}{3^{1/4}}\right], (-1)^{1/3}\right]\right) \Big/ \left(55 (-a)^{1/3} a \sqrt{a+b x^3}\right)$$

Problem 528: Result unnecessarily involves imaginary or complex numbers.

$$\int (e x)^{5/2} \left(a+b x^3\right)^{3/2} \left(A+B x^3\right) dx$$

Optimal (type 4, 364 leaves, 6 steps):

$$\frac{27 a^2 (22 A b-7 a B) e^2 \sqrt{e x} \sqrt{a+b x^3}}{7040 b^2}+$$

$$\frac{9 a (22 A b-7 a B) (e x)^{7/2} \sqrt{a+b x^3}}{1760 b e}+\frac{(22 A b-7 a B) (e x)^{7/2} \left(a+b x^3\right)^{3/2}}{176 b e}+$$

$$\frac{B (e x)^{7/2} \left(a+b x^3\right)^{5/2}}{11 b e}-\left(9 \times 3^{3/4} a^{8/3} (22 A b-7 a B) e^2 \sqrt{e x} \left(a^{1/3}+b^{1/3} x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)^2}}\ \text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right) b^{1/3} x}{a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x}\right], \frac{1}{4} \left(2+\sqrt{3}\right)\right]\right) \Big/$$

$$\left(14080 b^2 \sqrt{\frac{b^{1/3} x \left(a^{1/3}+b^{1/3} x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 256 leaves):

$$\left(e^2 \sqrt{e\,x} \left(-(-a)^{1/3} \left(a+b\,x^3\right) \left(189\,a^3\,B-54\,a^2\,b\left(11\,A+2\,B\,x^3\right)-80\,b^3\,x^6\left(11\,A+8\,B\,x^3\right)-8\,a\,b^2\,x^3\left(209\,A+125\,B\,x^3\right)\right)+9\,\mathrm{i}\,3^{3/4}\,a^3\,b^{1/3}\left(22\,A\,b-7\,a\,B\right)x\sqrt{\frac{(-1)^{5/6}\left((-a)^{1/3}-b^{1/3}\,x\right)}{b^{1/3}\,x}}\sqrt{\frac{\frac{(-a)^{2/3}}{b^{2/3}}+\frac{(-a)^{1/3}\,x}{b^{1/3}}+x^2}{x^2}}\ \mathrm{EllipticF}\left[\mathrm{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{\mathrm{i}\,(-a)^{1/3}}{b^{1/3}\,x}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right)\Bigg/\left(7040\,(-a)^{1/3}\,b^2\sqrt{a+b\,x^3}\right)$$

Problem 529: Result unnecessarily involves imaginary or complex numbers.

$$\int (e\,x)^{3/2}\left(a+b\,x^3\right)^{3/2}\left(A+B\,x^3\right)dx$$

Optimal (type 4, 621 leaves, 7 steps):

$$\frac{9\,a\,(4\,A\,b-a\,B)\,(e\,x)^{5/2}\,\sqrt{a+b\,x^3}}{224\,b\,e}+\frac{27\,\left(1+\sqrt{3}\right)\,a^2\,(4\,A\,b-a\,B)\,e\,\sqrt{e\,x}\,\sqrt{a+b\,x^3}}{448\,b^{5/3}\,\left(a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x\right)}+$$

$$\frac{(4\,A\,b-a\,B)\,(e\,x)^{5/2}\,\left(a+b\,x^3\right)^{3/2}}{28\,b\,e}+\frac{B\,(e\,x)^{5/2}\,\left(a+b\,x^3\right)^{5/2}}{10\,b\,e}-$$

$$\left(27\times 3^{1/4}\,a^{7/3}\,(4\,A\,b-a\,B)\,e\,\sqrt{e\,x}\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right)\,b^{1/3}\,x}{a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x}\right],\,\frac{1}{4}\,\left(2+\sqrt{3}\right)\right]\right)\Bigg/$$

$$\left(448\,b^{5/3}\,\sqrt{\frac{b^{1/3}\,x\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)-$$

$$\left(9\times 3^{3/4}\,\left(1-\sqrt{3}\right)\,a^{7/3}\,(4\,A\,b-a\,B)\,e\,\sqrt{e\,x}\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right)\,b^{1/3}\,x}{a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x}\right],\,\frac{1}{4}\,\left(2+\sqrt{3}\right)\right]\right)\Bigg/$$

$$\left(896\,b^{5/3}\,\sqrt{\frac{b^{1/3}\,x\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 303 leaves) :

$$\frac{1}{2240\, b^2 \sqrt{a+b x^3}} x\, (e x)^{3/2} \left(2 b \left(a+b x^3\right) \left(27 a^2 B+16 b^2 x^3 \left(10 A+7 B x^3\right)+4 a b \left(85 A+46 B x^3\right)\right)+45 a^2 (-4 A b+a B)\right.$$
$$\left(-3 \left(b+\frac{a}{x^3}\right)+\frac{1}{(-a)^{2/3} x} (-1)^{1/6} 3^{3/4} a\, b^{2/3} \sqrt{\frac{(-1)^{5/6} \left((-a)^{1/3}-b^{1/3} x\right)}{b^{1/3} x}} \sqrt{\frac{\frac{(-a)^{2/3}}{b^{2/3}}+\frac{(-a)^{1/3} x}{b^{1/3}}+x^2}{x^2}}\right.$$
$$\left(-i \sqrt{3}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-a)^{1/3}}{b^{1/3} x}}}{3^{1/4}}\right], (-1)^{1/3}\right]+\right.$$
$$\left.\left.\left.(-1)^{1/3}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-a)^{1/3}}{b^{1/3} x}}}{3^{1/4}}\right], (-1)^{1/3}\right]\right)\right)\right)$$

Problem 531: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+b x^3\right)^{3/2} \left(A+B x^3\right)}{\sqrt{e x}}\, dx$$

Optimal (type 4, 324 leaves, 5 steps):

$$\frac{9 a (16 A b-a B) \sqrt{e x} \sqrt{a+b x^3}}{320 b e}+\frac{(16 A b-a B) \sqrt{e x} \left(a+b x^3\right)^{3/2}}{80 b e}+$$
$$\frac{B \sqrt{e x} \left(a+b x^3\right)^{5/2}}{8 b e}+\left(9\times 3^{3/4} a^{5/3} (16 A b-a B) \sqrt{e x} \left(a^{1/3}+b^{1/3} x\right)\right.$$
$$\left.\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)^2}}\, \text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right) b^{1/3} x}{a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x}\right], \frac{1}{4} \left(2+\sqrt{3}\right)\right]\right) \Big/$$
$$\left(640\, b\, e \sqrt{\frac{b^{1/3} x \left(a^{1/3}+b^{1/3} x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 234 leaves):

$$\left((-a)^{1/3}\,x\,\left(a+b\,x^3\right)\,\left(27\,a^2\,B+8\,b^2\,x^3\,\left(8\,A+5\,B\,x^3\right)+4\,a\,b\,\left(52\,A+19\,B\,x^3\right)\right)-\right.$$

$$9\,\mathrm{i}\,3^{3/4}\,a^2\,b^{1/3}\,\left(16\,A\,b-a\,B\right)\,x^2\,\sqrt{\frac{(-1)^{5/6}\,\left((-a)^{1/3}-b^{1/3}\,x\right)}{b^{1/3}\,x}}\,\sqrt{\frac{\frac{(-a)^{2/3}}{b^{2/3}}+\frac{(-a)^{1/3}\,x}{b^{1/3}}+x^2}{x^2}}$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{\mathrm{i}\,(-a)^{1/3}}{b^{1/3}\,x}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\Big/\left(320\,(-a)^{1/3}\,b\,\sqrt{e\,x}\,\sqrt{a+b\,x^3}\right)$$

Problem 532: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(a+b\,x^3\right)^{3/2}\,\left(A+B\,x^3\right)}{(e\,x)^{3/2}}\,dx$$

Optimal (type 4, 614 leaves, 7 steps):

$$\frac{9\,(14\,A\,b+a\,B)\,(e\,x)^{5/2}\,\sqrt{a+b\,x^3}}{56\,e^4}+\frac{27\,\left(1+\sqrt{3}\right)\,a\,(14\,A\,b+a\,B)\,\sqrt{e\,x}\,\sqrt{a+b\,x^3}}{112\,b^{2/3}\,e^2\,\left(a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x\right)}+$$

$$\frac{(14\,A\,b+a\,B)\,(e\,x)^{5/2}\,\left(a+b\,x^3\right)^{3/2}}{7\,a\,e^4}-\frac{2\,A\,\left(a+b\,x^3\right)^{5/2}}{a\,e\,\sqrt{e\,x}}-$$

$$\left(27\times 3^{1/4}\,a^{4/3}\,(14\,A\,b+a\,B)\,\sqrt{e\,x}\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right)\,b^{1/3}\,x}{a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x}\right],\,\frac{1}{4}\,\left(2+\sqrt{3}\right)\right]\right)\Bigg/$$

$$\left(112\,b^{2/3}\,e^2\,\sqrt{\frac{b^{1/3}\,x\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)-$$

$$\left(9\times 3^{3/4}\,\left(1-\sqrt{3}\right)\,a^{4/3}\,(14\,A\,b+a\,B)\,\sqrt{e\,x}\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right)\,b^{1/3}\,x}{a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x}\right],\,\frac{1}{4}\,\left(2+\sqrt{3}\right)\right]\right)\Bigg/$$

$$\left(224\,b^{2/3}\,e^2\,\sqrt{\frac{b^{1/3}\,x\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 301 leaves):

$$\left(x^{3/2}\left(\frac{2\left(a+b x^3\right)\left(-112 a A+14 A b x^3+17 a B x^3+8 b B x^6\right)}{\sqrt{x}}-\frac{1}{b}9 a\left(14 A b+a B\right) x^{5/2}\left(-3\left(b+\frac{a}{x^3}\right)+\right.\right.\right.$$

$$\frac{1}{(-a)^{2/3} x}\left(-1\right)^{1/6} 3^{3/4} a b^{2/3}\sqrt{\frac{\left(-1\right)^{5/6}\left((-a)^{1/3}-b^{1/3} x\right)}{b^{1/3} x}}\sqrt{\frac{\frac{(-a)^{2/3}}{b^{2/3}}+\frac{(-a)^{1/3} x}{b^{1/3}}+x^2}{x^2}}$$

$$\left(-i\sqrt{3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-\left(-1\right)^{5/6}-\frac{i (-a)^{1/3}}{b^{1/3} x}}}{3^{1/4}}\right],\left(-1\right)^{1/3}\right]+\left(-1\right)^{1/3}\right.$$

$$\left.\left.\left.\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-\left(-1\right)^{5/6}-\frac{i (-a)^{1/3}}{b^{1/3} x}}}{3^{1/4}}\right],\left(-1\right)^{1/3}\right]\right)\right)\right)\right)\Bigg/\left(112\,(e x)^{3/2}\sqrt{a+b x^3}\right)$$

Problem 534: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(a+b x^3\right)^{3/2}\left(A+B x^3\right)}{(e x)^{7/2}}\,dx$$

Optimal (type 4, 314 leaves, 5 steps):

$$\frac{9\left(2 A b+a B\right)\sqrt{e x}\sqrt{a+b x^3}}{20 e^4}+\frac{\left(2 A b+a B\right)\sqrt{e x}\left(a+b x^3\right)^{3/2}}{5 a e^4}-$$

$$\frac{2 A\left(a+b x^3\right)^{5/2}}{5 a e\,(e x)^{5/2}}+\left(9\times 3^{3/4} a^{2/3}\left(2 A b+a B\right)\sqrt{e x}\left(a^{1/3}+b^{1/3} x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)^2}}\,\text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right) b^{1/3} x}{a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x}\right],\frac{1}{4}\left(2+\sqrt{3}\right)\right]\right)\Bigg/$$

$$\left(40 e^4\sqrt{\frac{b^{1/3} x\left(a^{1/3}+b^{1/3} x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)^2}}\sqrt{a+b x^3}\right)$$

Result (type 4, 215 leaves):

$$\left(x\left((-a)^{1/3}\left(a+b x^3\right)\left(-8 a A+10 A b x^3+13 a B x^3+4 b B x^6\right)-\right.\right.$$

$$9\,\mathrm{i}\,3^{3/4} a b^{1/3}(2 A b+a B)\, x^4 \sqrt{\frac{(-1)^{5/6}\left((-a)^{1/3}-b^{1/3} x\right)}{b^{1/3} x}} \sqrt{\frac{\frac{(-a)^{2/3}}{b^{2/3}}+\frac{(-a)^{1/3} x}{b^{1/3}}+x^2}{x^2}}$$

$$\left.\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{\mathrm{i}(-a)^{1/3}}{b^{1/3} x}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right) \Big/ \left(20\,(-a)^{1/3}(e x)^{7/2} \sqrt{a+b x^3}\right)$$

Problem 536: Result unnecessarily involves imaginary or complex numbers.

$$\int (e x)^{5/2}\left(a+b x^3\right)^{5/2}\left(A+B x^3\right) dx$$

Optimal (type 4, 404 leaves, 7 steps):

$$\frac{81 a^3(4 A b-a B) e^2 \sqrt{e x} \sqrt{a+b x^3}}{5632 b^2}+\frac{27 a^2(4 A b-a B)(e x)^{7/2} \sqrt{a+b x^3}}{1408 b e}+$$
$$\frac{15 a(4 A b-a B)(e x)^{7/2}\left(a+b x^3\right)^{3/2}}{704 b e}+\frac{(4 A b-a B)(e x)^{7/2}\left(a+b x^3\right)^{5/2}}{44 b e}+$$
$$\frac{B(e x)^{7/2}\left(a+b x^3\right)^{7/2}}{14 b e}-\left(27 \times 3^{3/4} a^{11/3}(4 A b-a B) e^2 \sqrt{e x}\left(a^{1/3}+b^{1/3} x\right)\right.$$
$$\left.\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)^2}}\ \text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right) b^{1/3} x}{a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x}\right], \frac{1}{4}\left(2+\sqrt{3}\right)\right]\right) \Big/$$
$$\left(11264\, b^2 \sqrt{\frac{b^{1/3} x\left(a^{1/3}+b^{1/3} x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 276 leaves):

$$\left(e^2 \sqrt{e\,x} \left(-(-a)^{1/3} \left(a+b\,x^3\right) \left(567\,a^4\,B-324\,a^3\,b\left(7\,A+B\,x^3\right)-256\,b^4\,x^9\left(14\,A+11\,B\,x^3\right)-32\,a\,b^3\,x^6\left(329\,A+236\,B\,x^3\right)-8\,a^2\,b^2\,x^3\left(1246\,A+727\,B\,x^3\right)\right)+189\,i\,3^{3/4}\,a^4\,b^{1/3}\left(4\,A\,b-a\,B\right)\,x\,\sqrt{\frac{(-1)^{5/6}\left((-a)^{1/3}-b^{1/3}\,x\right)}{b^{1/3}\,x}}\,\sqrt{\frac{\frac{(-a)^{2/3}}{b^{2/3}}+\frac{(-a)^{1/3}\,x}{b^{1/3}}+x^2}{x^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-a)^{1/3}}{b^{1/3}\,x}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\right)\Big/\left(39424\,(-a)^{1/3}\,b^2\,\sqrt{a+b\,x^3}\right)$$

Problem 537: Result unnecessarily involves imaginary or complex numbers.

$$\int (e\,x)^{3/2}\left(a+b\,x^3\right)^{5/2}\left(A+B\,x^3\right)\,dx$$

Optimal (type 4, 661 leaves, 8 steps):

$$\frac{27 a^2 (26 A b - 5 a B) (e x)^{5/2} \sqrt{a + b x^3}}{5824 b e} + \frac{81 \left(1+\sqrt{3}\right) a^3 (26 A b - 5 a B) e \sqrt{e x} \sqrt{a + b x^3}}{11648 b^{5/3} \left(a^{1/3} + \left(1+\sqrt{3}\right) b^{1/3} x\right)} + \frac{3 a (26 A b - 5 a B) (e x)^{5/2} \left(a + b x^3\right)^{3/2}}{728 b e} +$$

$$\frac{(26 A b - 5 a B) (e x)^{5/2} \left(a + b x^3\right)^{5/2}}{260 b e} + \frac{B (e x)^{5/2} \left(a + b x^3\right)^{7/2}}{13 b e} -$$

$$\left(81 \times 3^{1/4} a^{10/3} (26 A b - 5 a B) e \sqrt{e x} \left(a^{1/3} + b^{1/3} x\right) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left(a^{1/3} + \left(1+\sqrt{3}\right) b^{1/3} x\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcCos}\left[\frac{a^{1/3} + \left(1-\sqrt{3}\right) b^{1/3} x}{a^{1/3} + \left(1+\sqrt{3}\right) b^{1/3} x}\right], \frac{1}{4}\left(2+\sqrt{3}\right)\right]\right) \Bigg/$$

$$\left(11648 b^{5/3} \sqrt{\frac{b^{1/3} x \left(a^{1/3} + b^{1/3} x\right)}{\left(a^{1/3} + \left(1+\sqrt{3}\right) b^{1/3} x\right)^2}} \sqrt{a + b x^3}\right) -$$

$$\left(27 \times 3^{3/4} \left(1-\sqrt{3}\right) a^{10/3} (26 A b - 5 a B) e \sqrt{e x} \left(a^{1/3} + b^{1/3} x\right) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left(a^{1/3} + \left(1+\sqrt{3}\right) b^{1/3} x\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3} + \left(1-\sqrt{3}\right) b^{1/3} x}{a^{1/3} + \left(1+\sqrt{3}\right) b^{1/3} x}\right], \frac{1}{4}\left(2+\sqrt{3}\right)\right]\right) \Bigg/$$

$$\left(23296 b^{5/3} \sqrt{\frac{b^{1/3} x \left(a^{1/3} + b^{1/3} x\right)}{\left(a^{1/3} + \left(1+\sqrt{3}\right) b^{1/3} x\right)^2}} \sqrt{a + b x^3}\right)$$

Result (type 4, 337 leaves) :

$$\frac{1}{58240\,(-a)^{2/3}\,b^2\,\sqrt{e\,x}\,\sqrt{a+b\,x^3}}\,e^2\,\Bigg(2\,(-a)^{2/3}\,b\,x^3\,\left(a+b\,x^3\right)\left(a^2\,(9542\,A\,b+405\,a\,B)+8\,a\,b\,(1118\,A\,b+625\,a\,B)\,x^3+112\,b^2\,(26\,A\,b+55\,a\,B)\,x^6+2240\,b^3\,B\,x^9\right)+135\,a^3\,(26\,A\,b-5\,a\,B)\left(3\,(-a)^{2/3}\,\left(a+b\,x^3\right)+(-1)^{2/3}\,3^{3/4}\,a\,b^{2/3}\,x^2\,\sqrt{\frac{(-1)^{5/6}\left((-a)^{1/3}-b^{1/3}\,x\right)}{b^{1/3}\,x}}\sqrt{\frac{\frac{(-a)^{2/3}}{b^{2/3}}+\frac{(-a)^{1/3}\,x}{b^{1/3}}+x^2}{x^2}}\left(\sqrt{3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-a)^{1/3}}{b^{1/3}\,x}}}{3^{1/4}}\right],(-1)^{1/3}\right]+(-1)^{5/6}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-a)^{1/3}}{b^{1/3}\,x}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right)\Bigg)$$

Problem 539: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(a+b\,x^3\right)^{5/2}\,\left(A+B\,x^3\right)}{\sqrt{e\,x}}\,dx$$

Optimal (type 4, 364 leaves, 6 steps):

$$\frac{27\,a^2\,(22\,A\,b-a\,B)\,\sqrt{e\,x}\,\sqrt{a+b\,x^3}}{1408\,b\,e}+\frac{3\,a\,(22\,A\,b-a\,B)\,\sqrt{e\,x}\,\left(a+b\,x^3\right)^{3/2}}{352\,b\,e}+\frac{(22\,A\,b-a\,B)\,\sqrt{e\,x}\,\left(a+b\,x^3\right)^{5/2}}{176\,b\,e}+\frac{B\,\sqrt{e\,x}\,\left(a+b\,x^3\right)^{7/2}}{11\,b\,e}+\left(27\times 3^{3/4}\,a^{8/3}\,(22\,A\,b-a\,B)\,\sqrt{e\,x}\,\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}\,x\right)^2}}\,\text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right)b^{1/3}\,x}{a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}\,x}\right],\frac{1}{4}\left(2+\sqrt{3}\right)\right]\right)\Bigg/\left(2816\,b\,e\,\sqrt{\frac{b^{1/3}\,x\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 255 leaves):

$$\left((-a)^{1/3}\,x\,\left(a+b\,x^3\right)\left(81\,a^3\,B+16\,b^3\,x^6\,\left(11\,A+8\,B\,x^3\right)+8\,a\,b^2\,x^3\,\left(77\,A+47\,B\,x^3\right)+2\,a^2\,b\,\left(517\,A+178\,B\,x^3\right)\right)-27\,i\,3^{3/4}\,a^3\,b^{1/3}\,\left(22\,A\,b-a\,B\right)\,x^2\,\sqrt{\frac{(-1)^{5/6}\,\left((-a)^{1/3}-b^{1/3}\,x\right)}{b^{1/3}\,x}}\,\sqrt{\frac{\frac{(-a)^{2/3}}{b^{2/3}}+\frac{(-a)^{1/3}\,x}{b^{1/3}}+x^2}{x^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-a)^{1/3}}{b^{1/3}\,x}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\Bigg/\left(1408\,(-a)^{1/3}\,b\,\sqrt{e\,x}\,\sqrt{a+b\,x^3}\right)$$

Problem 540: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(a+b\,x^3\right)^{5/2}\,\left(A+B\,x^3\right)}{(e\,x)^{3/2}}\,dx$$

Optimal (type 4, 650 leaves, 8 steps):

$$\frac{27\,a\,(20\,A\,b+a\,B)\,(e\,x)^{5/2}\,\sqrt{a+b\,x^3}}{224\,e^4}+\frac{81\left(1+\sqrt{3}\right)a^2\,(20\,A\,b+a\,B)\,\sqrt{e\,x}\,\sqrt{a+b\,x^3}}{448\,b^{2/3}\,e^2\left(a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}\,x\right)}+$$

$$\frac{3\,(20\,A\,b+a\,B)\,(e\,x)^{5/2}\,\left(a+b\,x^3\right)^{3/2}}{28\,e^4}+\frac{(20\,A\,b+a\,B)\,(e\,x)^{5/2}\,\left(a+b\,x^3\right)^{5/2}}{10\,a\,e^4}-$$

$$\frac{2\,A\,\left(a+b\,x^3\right)^{7/2}}{a\,e\,\sqrt{e\,x}}-\left(81\times 3^{1/4}\,a^{7/3}\,(20\,A\,b+a\,B)\,\sqrt{e\,x}\,\left(a^{1/3}+b^{1/3}\,x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}\,x\right)^2}}\;\text{EllipticE}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right)b^{1/3}\,x}{a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}\,x}\right],\frac{1}{4}\left(2+\sqrt{3}\right)\right]\right)\Bigg/$$

$$\left(448\,b^{2/3}\,e^2\,\sqrt{\frac{b^{1/3}\,x\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)-$$

$$\left(27\times 3^{3/4}\left(1-\sqrt{3}\right)a^{7/3}\,(20\,A\,b+a\,B)\,\sqrt{e\,x}\,\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}\,x\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right)b^{1/3}\,x}{a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}\,x}\right],\frac{1}{4}\left(2+\sqrt{3}\right)\right]\right)\Bigg/$$

$$\left(896\,b^{2/3}\,e^2\,\sqrt{\frac{b^{1/3}\,x\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 329 leaves):

$$\left(x^{3/2}\left(\frac{1}{5\sqrt{x}}2\left(a+b x^3\right)\left(16 b^2 x^6\left(10 A+7 B x^3\right)+4 a b x^3\left(155 A+86 B x^3\right)+a^2\left(-2240 A+367 B x^3\right)\right)-\right.\right.$$

$$\frac{1}{b}27 a^2\left(20 A b+a B\right) x^{5/2}\left(-3\left(b+\frac{a}{x^3}\right)+\frac{1}{(-a)^{2/3} x}\right.$$

$$(-1)^{1/6} 3^{3/4} a b^{2/3}\sqrt{\frac{(-1)^{5/6}\left((-a)^{1/3}-b^{1/3} x\right)}{b^{1/3} x}}\sqrt{\frac{\frac{(-a)^{2/3}}{b^{2/3}}+\frac{(-a)^{1/3} x}{b^{1/3}}+x^2}{x^2}}$$

$$\left(-i\sqrt{3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-a)^{1/3}}{b^{1/3} x}}}{3^{1/4}}\right],(-1)^{1/3}\right]+(-1)^{1/3}\right.$$

$$\left.\left.\left.\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-a)^{1/3}}{b^{1/3} x}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right)\right)\right)\Big/\left(448\,(e x)^{3/2}\sqrt{a+b x^3}\right)$$

Problem 542: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(a+b x^3\right)^{5/2}\left(A+B x^3\right)}{(e x)^{7/2}}\,dx$$

Optimal (type 4, 352 leaves, 6 steps):

$$\frac{27\,a\,(16\,A\,b+5\,a\,B)\,\sqrt{e\,x}\,\sqrt{a+b\,x^3}}{320\,e^4}+$$

$$\frac{3\,(16\,A\,b+5\,a\,B)\,\sqrt{e\,x}\,\left(a+b\,x^3\right)^{3/2}}{80\,e^4}+\frac{(16\,A\,b+5\,a\,B)\,\sqrt{e\,x}\,\left(a+b\,x^3\right)^{5/2}}{40\,a\,e^4}-$$

$$\frac{2\,A\,\left(a+b\,x^3\right)^{7/2}}{5\,a\,e\,(e\,x)^{5/2}}+\left(27\times 3^{3/4}\,a^{5/3}\,(16\,A\,b+5\,a\,B)\,\sqrt{e\,x}\,\left(a^{1/3}+b^{1/3}\,x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x\right)^2}}\;\text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right)\,b^{1/3}\,x}{a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x}\right],\,\frac{1}{4}\,\left(2+\sqrt{3}\right)\right]\right)\Big/$$

$$\left(640\,e^4\,\sqrt{\frac{b^{1/3}\,x\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 242 leaves) :

$$\left(x\left((-a)^{1/3}\,\left(a+b\,x^3\right)\,\left(8\,b^2\,x^6\,\left(8\,A+5\,B\,x^3\right)+4\,a\,b\,x^3\,\left(92\,A+35\,B\,x^3\right)+a^2\,\left(-128\,A+235\,B\,x^3\right)\right)-\right.\right.$$

$$27\,i\,3^{3/4}\,a^2\,b^{1/3}\,(16\,A\,b+5\,a\,B)\,x^4\,\sqrt{\frac{(-1)^{5/6}\,\left((-a)^{1/3}-b^{1/3}\,x\right)}{b^{1/3}\,x}}\,\sqrt{\frac{\frac{(-a)^{2/3}}{b^{2/3}}+\frac{(-a)^{1/3}\,x}{b^{1/3}}+x^2}{x^2}}$$

$$\left.\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-a)^{1/3}}{b^{1/3}\,x}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\right)\Big/\left(320\,(-a)^{1/3}\,(e\,x)^{7/2}\,\sqrt{a+b\,x^3}\right)$$

Problem 544: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{(e\,x)^{5/2}\,\left(A+B\,x^3\right)}{\sqrt{a+b\,x^3}}\,dx$$

Optimal (type 4, 286 leaves, 4 steps) :

$$\frac{(10 A b - 7 a B) e^2 \sqrt{e x} \sqrt{a + b x^3}}{20 b^2} + \frac{B (e x)^{7/2} \sqrt{a + b x^3}}{5 b e} - \left(a^{2/3} (10 A b - 7 a B) e^2 \sqrt{e x} \left(a^{1/3} + b^{1/3} x\right) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left(a^{1/3} + \left(1 + \sqrt{3}\right) b^{1/3} x\right)^2}} \text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3} + \left(1 - \sqrt{3}\right) b^{1/3} x}{a^{1/3} + \left(1 + \sqrt{3}\right) b^{1/3} x}\right], \frac{1}{4} \left(2 + \sqrt{3}\right)\right]\right) \Big/ \left(40 \times 3^{1/4} b^2 \sqrt{\frac{b^{1/3} x \left(a^{1/3} + b^{1/3} x\right)}{\left(a^{1/3} + \left(1 + \sqrt{3}\right) b^{1/3} x\right)^2}} \sqrt{a + b x^3}\right)$$

Result (type 4, 210 leaves):

$$\left(e^2 \sqrt{e x} \left(-3 (-a)^{1/3} \left(a + b x^3\right) \left(-10 A b + 7 a B - 4 b B x^3\right) + i\, 3^{3/4} a b^{1/3} (10 A b - 7 a B) x \sqrt{\frac{(-1)^{5/6} \left((-a)^{1/3} - b^{1/3} x\right)}{b^{1/3} x}} \sqrt{\frac{\frac{(-a)^{2/3}}{b^{2/3}} + \frac{(-a)^{1/3} x}{b^{1/3}} + x^2}{x^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{i (-a)^{1/3}}{b^{1/3} x}}}{3^{1/4}}\right], (-1)^{1/3}\right]\right)\right) \Big/ \left(60 (-a)^{1/3} b^2 \sqrt{a + b x^3}\right)$$

Problem 545: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(e x)^{3/2} \left(A + B x^3\right)}{\sqrt{a + b x^3}} \, dx$$

Optimal (type 4, 543 leaves, 5 steps):

$$\frac{B\,(e\,x)^{5/2}\,\sqrt{a+b\,x^3}}{4\,b\,e}+\frac{\left(1+\sqrt{3}\right)\,(8\,A\,b-5\,a\,B)\,e\,\sqrt{e\,x}\,\sqrt{a+b\,x^3}}{8\,b^{5/3}\,\left(a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x\right)}-$$

$$\left(3^{1/4}\,a^{1/3}\,(8\,A\,b-5\,a\,B)\,e\,\sqrt{e\,x}\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right)\,b^{1/3}\,x}{a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x}\right],\,\frac{1}{4}\,\left(2+\sqrt{3}\right)\right]\right)\Bigg/$$

$$\left(8\,b^{5/3}\,\sqrt{\frac{b^{1/3}\,x\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)-$$

$$\left(\left(1-\sqrt{3}\right)\,a^{1/3}\,(8\,A\,b-5\,a\,B)\,e\,\sqrt{e\,x}\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right)\,b^{1/3}\,x}{a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x}\right],\,\frac{1}{4}\,\left(2+\sqrt{3}\right)\right]\right)\Bigg/$$

$$\left(16\times 3^{1/4}\,b^{5/3}\,\sqrt{\frac{b^{1/3}\,x\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 263 leaves) :

$$\frac{1}{24 b^2 \sqrt{a+b x^3}} x (e x)^{3/2} \left(6 b B \left(a+b x^3\right) - \right.$$

$$(8 A b - 5 a B) \left(-3 \left(b+\frac{a}{x^3}\right) + \frac{1}{(-a)^{2/3} x} (-1)^{1/6} 3^{3/4} a b^{2/3} \sqrt{\frac{(-1)^{5/6} \left((-a)^{1/3} - b^{1/3} x\right)}{b^{1/3} x}}\right.$$

$$\sqrt{\frac{\frac{(-a)^{2/3}}{b^{2/3}} + \frac{(-a)^{1/3} x}{b^{1/3}} + x^2}{x^2}} \left(-i \sqrt{3}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{i (-a)^{1/3}}{b^{1/3} x}}}{3^{1/4}}\right], (-1)^{1/3}\right] + \right.$$

$$\left.\left.\left.(-1)^{1/3}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{i (-a)^{1/3}}{b^{1/3} x}}}{3^{1/4}}\right], (-1)^{1/3}\right]\right)\right)\right)$$

Problem 547: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{A + B x^3}{\sqrt{e x} \sqrt{a + b x^3}} \, dx$$

Optimal (type 4, 249 leaves, 3 steps):

$$\frac{B \sqrt{e x} \sqrt{a+b x^3}}{2 b e} + \left((4 A b - a B) \sqrt{e x} \left(a^{1/3} + b^{1/3} x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left(a^{1/3} + \left(1+\sqrt{3}\right) b^{1/3} x\right)^2}}\, \text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3} + \left(1-\sqrt{3}\right) b^{1/3} x}{a^{1/3} + \left(1+\sqrt{3}\right) b^{1/3} x}\right], \frac{1}{4}\left(2+\sqrt{3}\right)\right]\right) \Big/$$

$$\left(4 \times 3^{1/4} a^{1/3} b e \sqrt{\frac{b^{1/3} x \left(a^{1/3} + b^{1/3} x\right)}{\left(a^{1/3} + \left(1+\sqrt{3}\right) b^{1/3} x\right)^2}} \sqrt{a + b x^3}\right)$$

Result (type 4, 184 leaves):

$$\left(x\left(3\,B\,\left(a+b\,x^3\right)+\frac{1}{(-a)^{1/3}}\right.\right.$$

$$\mathbb{i}\,3^{3/4}\,b^{1/3}\,\left(-4\,A\,b+a\,B\right)\,x\,\sqrt{\frac{(-1)^{5/6}\,\left((-a)^{1/3}-b^{1/3}\,x\right)}{b^{1/3}\,x}}\,\sqrt{\frac{\frac{(-a)^{2/3}}{b^{2/3}}+\frac{(-a)^{1/3}\,x}{b^{1/3}}+x^2}{x^2}}$$

$$\left.\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{\mathbb{i}\,(-a)^{1/3}}{b^{1/3}\,x}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\right)\Big/\left(6\,b\,\sqrt{e\,x}\,\sqrt{a+b\,x^3}\right)$$

Problem 548: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{A+B\,x^3}{(e\,x)^{3/2}\,\sqrt{a+b\,x^3}}\,dx$$

Optimal (type 4, 542 leaves, 5 steps):

$$-\frac{2\,A\,\sqrt{a+b\,x^3}}{a\,e\,\sqrt{e\,x}}+\frac{\left(1+\sqrt{3}\right)\,\left(2\,A\,b+a\,B\right)\,\sqrt{e\,x}\,\sqrt{a+b\,x^3}}{a\,b^{2/3}\,e^2\,\left(a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x\right)}-$$

$$\left(3^{1/4}\,\left(2\,A\,b+a\,B\right)\,\sqrt{e\,x}\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right)\,b^{1/3}\,x}{a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x}\right],\,\frac{1}{4}\,\left(2+\sqrt{3}\right)\right]\right)\Big/$$

$$\left(a^{2/3}\,b^{2/3}\,e^2\,\sqrt{\frac{b^{1/3}\,x\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)-$$

$$\left(\left(1-\sqrt{3}\right)\,\left(2\,A\,b+a\,B\right)\,\sqrt{e\,x}\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right)\,b^{1/3}\,x}{a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x}\right],\,\frac{1}{4}\,\left(2+\sqrt{3}\right)\right]\right)\Big/$$

$$\left(2\times 3^{1/4}\,a^{2/3}\,b^{2/3}\,e^2\,\sqrt{\frac{b^{1/3}\,x\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 355 leaves):

$$\left(x\left(-2A\left(a+bx^3\right)+\left((2Ab+aB)\left(-\left(-1+(-1)^{2/3}\right)a^{1/3}b^{1/3}x\left((-1)^{1/3}a^{1/3}-b^{1/3}x\right)\left((-1)^{2/3}a^{1/3}+b^{1/3}x\right)-(-1)^{2/3}a^{2/3}\left(a^{1/3}+b^{1/3}x\right)^2\sqrt{\frac{\left(1+(-1)^{1/3}\right)b^{1/3}x\left(a^{1/3}-(-1)^{1/3}b^{1/3}x\right)}{\left(a^{1/3}+b^{1/3}x\right)^2}}\sqrt{\frac{a^{1/3}+(-1)^{2/3}b^{1/3}x}{a^{1/3}+b^{1/3}x}}\left(\left(1+(-1)^{1/3}\right)\text{EllipticE}\left[\text{ArcSin}\left[\sqrt{\frac{\left(1+(-1)^{1/3}\right)b^{1/3}x}{a^{1/3}+b^{1/3}x}}\right],\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]-\left(1+(-1)^{2/3}\right)\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{\left(1+(-1)^{1/3}\right)b^{1/3}x}{a^{1/3}+b^{1/3}x}}\right],\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\right)\right)\right)\Big/\left(\left(-1+(-1)^{2/3}\right)a^{1/3}b\right)\right)\right)\Big/\left(a\,(e\,x)^{3/2}\sqrt{a+bx^3}\right)$$

Problem 550: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{A+Bx^3}{(e\,x)^{7/2}\sqrt{a+bx^3}}\,dx$$

Optimal (type 4, 246 leaves, 3 steps):

$$-\frac{2A\sqrt{a+bx^3}}{5ae\,(e\,x)^{5/2}}-\left((2Ab-5aB)\sqrt{e\,x}\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}x\right)^2}}\,\text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right)b^{1/3}x}{a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}x}\right],\frac{1}{4}\left(2+\sqrt{3}\right)\right]\right)\Big/\left(5\times3^{1/4}a^{4/3}e^4\sqrt{\frac{b^{1/3}x\left(a^{1/3}+b^{1/3}x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}x\right)^2}}\sqrt{a+bx^3}\right)$$

Result (type 4, 187 leaves):

$$\left(2x\left(-3A\left(a+bx^3\right)+\frac{1}{(-a)^{1/3}}\right.\right.$$

$$i\,3^{3/4}\,b^{1/3}\left(2Ab-5aB\right)x^4\sqrt{\frac{(-1)^{5/6}\left((-a)^{1/3}-b^{1/3}x\right)}{b^{1/3}x}}\sqrt{\frac{\frac{(-a)^{2/3}}{b^{2/3}}+\frac{(-a)^{1/3}x}{b^{1/3}}+x^2}{x^2}}$$

$$\left.\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-a)^{1/3}}{b^{1/3}x}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right)\Bigg/\left(15a\,(ex)^{7/2}\sqrt{a+bx^3}\right)$$

Problem 552: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{(ex)^{5/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx$$

Optimal (type 4, 286 leaves, 4 steps):

$$-\frac{(4Ab-7aB)\,e^2\sqrt{ex}}{6b^2\sqrt{a+bx^3}}+\frac{B\,(ex)^{7/2}}{2be\sqrt{a+bx^3}}+$$

$$\left((4Ab-7aB)\,e^2\sqrt{ex}\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}x\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right)b^{1/3}x}{a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}x}\right],\frac{1}{4}\left(2+\sqrt{3}\right)\right]\right)\Bigg/$$

$$\left(12\times 3^{1/4}\,a^{1/3}\,b^2\sqrt{\frac{b^{1/3}x\left(a^{1/3}+b^{1/3}x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}x\right)^2}}\sqrt{a+bx^3}\right)$$

Result (type 4, 202 leaves):

$$\left(e^2 \sqrt{e x} \left(3 (-a)^{1/3} \left(-4 A b + 7 a B + 3 b B x^3\right) - i\, 3^{3/4} b^{1/3} (4 A b - 7 a B)\, x \sqrt{\frac{(-1)^{5/6} \left((-a)^{1/3} - b^{1/3} x\right)}{b^{1/3} x}} \sqrt{\frac{\frac{(-a)^{2/3}}{b^{2/3}} + \frac{(-a)^{1/3} x}{b^{1/3}} + x^2}{x^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6} - \frac{i (-a)^{1/3}}{b^{1/3} x}}}{3^{1/4}}\right], (-1)^{1/3}\right]\right)\right) \Big/ \left(18 (-a)^{1/3} b^2 \sqrt{a + b x^3}\right)$$

Problem 553: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(e x)^{3/2} \left(A + B x^3\right)}{\left(a + b x^3\right)^{3/2}}\, dx$$

Optimal (type 4, 553 leaves, 5 steps):

$$\frac{2 (A b - a B) (e x)^{5/2}}{3 a b e \sqrt{a + b x^3}} - \frac{\left(1 + \sqrt{3}\right) (2 A b - 5 a B)\, e \sqrt{e x} \sqrt{a + b x^3}}{3 a b^{5/3} \left(a^{1/3} + \left(1 + \sqrt{3}\right) b^{1/3} x\right)} + \left((2 A b - 5 a B)\, e \sqrt{e x} \left(a^{1/3} + b^{1/3} x\right) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left(a^{1/3} + \left(1 + \sqrt{3}\right) b^{1/3} x\right)^2}}\ \text{EllipticE}\left[\text{ArcCos}\left[\frac{a^{1/3} + \left(1 - \sqrt{3}\right) b^{1/3} x}{a^{1/3} + \left(1 + \sqrt{3}\right) b^{1/3} x}\right], \frac{1}{4}\left(2 + \sqrt{3}\right)\right]\right) \Big/ \left(3^{3/4} a^{2/3} b^{5/3} \sqrt{\frac{b^{1/3} x \left(a^{1/3} + b^{1/3} x\right)}{\left(a^{1/3} + \left(1 + \sqrt{3}\right) b^{1/3} x\right)^2}} \sqrt{a + b x^3}\right) + \left(\left(1 - \sqrt{3}\right) (2 A b - 5 a B)\, e \sqrt{e x} \left(a^{1/3} + b^{1/3} x\right) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left(a^{1/3} + \left(1 + \sqrt{3}\right) b^{1/3} x\right)^2}}\ \text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3} + \left(1 - \sqrt{3}\right) b^{1/3} x}{a^{1/3} + \left(1 + \sqrt{3}\right) b^{1/3} x}\right], \frac{1}{4}\left(2 + \sqrt{3}\right)\right]\right) \Big/ \left(6 \times 3^{1/4} a^{2/3} b^{5/3} \sqrt{\frac{b^{1/3} x \left(a^{1/3} + b^{1/3} x\right)}{\left(a^{1/3} + \left(1 + \sqrt{3}\right) b^{1/3} x\right)^2}} \sqrt{a + b x^3}\right)$$

Result (type 4, 266 leaves):

$$\frac{1}{9\,a\,b^2\,\sqrt{a+b\,x^3}}\,x\,(e\,x)^{3/2}\left(6\,b\,(A\,b-a\,B)-\left(-2\,A\,b+5\,a\,B\right)\left(-3\left(b+\frac{a}{x^3}\right)+\frac{1}{(-a)^{2/3}\,x}(-1)^{1/6}\,3^{3/4}\,a\,b^{2/3}\,\sqrt{\frac{(-1)^{5/6}\left((-a)^{1/3}-b^{1/3}\,x\right)}{b^{1/3}\,x}}\,\sqrt{\frac{\frac{(-a)^{2/3}}{b^{2/3}}+\frac{(-a)^{1/3}\,x}{b^{1/3}}+x^2}{x^2}}\left(-i\,\sqrt{3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-a)^{1/3}}{b^{1/3}\,x}}}{3^{1/4}}\right],(-1)^{1/3}\right]+(-1)^{1/3}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-a)^{1/3}}{b^{1/3}\,x}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right)\right)$$

Problem 555: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{A+B\,x^3}{\sqrt{e\,x}\,\left(a+b\,x^3\right)^{3/2}}\,dx$$

Optimal (type 4, 258 leaves, 3 steps):

$$\frac{2\,(A\,b-a\,B)\,\sqrt{e\,x}}{3\,a\,b\,e\,\sqrt{a+b\,x^3}}+\left((2\,A\,b+a\,B)\,\sqrt{e\,x}\,\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}\,x\right)^2}}\,\text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right)b^{1/3}\,x}{a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}\,x}\right],\frac{1}{4}\left(2+\sqrt{3}\right)\right]\right)\Big/\left(3\times3^{1/4}\,a^{4/3}\,b\,e\,\sqrt{\frac{b^{1/3}\,x\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 193 leaves):

$$-\left(\left(6\,(-a)^{1/3}\,(A\,b-a\,B)\,x-\right.\right.$$

$$2\,i\,3^{3/4}\,b^{1/3}\,(2\,A\,b+a\,B)\,x^2\,\sqrt{\frac{(-1)^{5/6}\left((-a)^{1/3}-b^{1/3}\,x\right)}{b^{1/3}\,x}}\,\sqrt{\frac{\frac{(-a)^{2/3}}{b^{2/3}}+\frac{(-a)^{1/3}\,x}{b^{1/3}}+x^2}{x^2}}$$

$$\left.\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-a)^{1/3}}{b^{1/3}\,x}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\Big/\left(9\,(-a)^{4/3}\,b\,\sqrt{e\,x}\,\sqrt{a+b\,x^3}\right)\right)$$

Problem 556: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{A+B\,x^3}{(e\,x)^{3/2}\,\left(a+b\,x^3\right)^{3/2}}\,dx$$

Optimal (type 4, 585 leaves, 6 steps):

$$-\frac{2\,A}{a\,e\,\sqrt{e\,x}\,\sqrt{a+b\,x^3}}-\frac{2\,(4\,A\,b-a\,B)\,(e\,x)^{5/2}}{3\,a^2\,e^4\,\sqrt{a+b\,x^3}}+$$

$$\frac{2\left(1+\sqrt{3}\right)(4\,A\,b-a\,B)\,\sqrt{e\,x}\,\sqrt{a+b\,x^3}}{3\,a^2\,b^{2/3}\,e^2\left(a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}\,x\right)}-\left(2\,(4\,A\,b-a\,B)\,\sqrt{e\,x}\,\left(a^{1/3}+b^{1/3}\,x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}\,x\right)^2}}\;\text{EllipticE}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right)b^{1/3}\,x}{a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}\,x}\right],\,\frac{1}{4}\left(2+\sqrt{3}\right)\right]\right)\Big/$$

$$\left(3^{3/4}\,a^{5/3}\,b^{2/3}\,e^2\,\sqrt{\frac{b^{1/3}\,x\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)-$$

$$\left(\left(1-\sqrt{3}\right)(4\,A\,b-a\,B)\,\sqrt{e\,x}\,\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}\,x\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right)b^{1/3}\,x}{a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}\,x}\right],\,\frac{1}{4}\left(2+\sqrt{3}\right)\right]\right)\Big/$$

$$\left(3\times 3^{1/4}\,a^{5/3}\,b^{2/3}\,e^2\,\sqrt{\frac{b^{1/3}\,x\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 372 leaves):

$$\left(2\,x\left(-(A\,b-a\,B)\,x^3-3\,A\,(a+b\,x^3)+\left((4\,A\,b-a\,B)\left(-\left(-1+(-1)^{2/3}\right)a^{1/3}\,b^{1/3}\,x\left((-1)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)\left((-1)^{2/3}\,a^{1/3}+b^{1/3}\,x\right)-(-1)^{2/3}\,a^{2/3}\,(a^{1/3}+b^{1/3}\,x)^2\,\sqrt{\frac{\left(1+(-1)^{1/3}\right)b^{1/3}\,x\left(a^{1/3}-(-1)^{1/3}\,b^{1/3}\,x\right)}{(a^{1/3}+b^{1/3}\,x)^2}}\,\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{a^{1/3}+b^{1/3}\,x}}\left(\left(1+(-1)^{1/3}\right)\text{EllipticE}\left[\text{ArcSin}\left[\sqrt{\frac{\left(1+(-1)^{1/3}\right)b^{1/3}\,x}{a^{1/3}+b^{1/3}\,x}}\right],\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]-\left(1+(-1)^{2/3}\right)\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{\left(1+(-1)^{1/3}\right)b^{1/3}\,x}{a^{1/3}+b^{1/3}\,x}}\right],\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\right)\right)\right)\Big/\left(\left(-1+(-1)^{2/3}\right)a^{1/3}\,b\right)\right)\right)\Big/\left(3\,a^2\,(e\,x)^{3/2}\,\sqrt{a+b\,x^3}\right)$$

Problem 558: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{A+B\,x^3}{(e\,x)^{7/2}\,(a+b\,x^3)^{3/2}}\,dx$$

Optimal (type 4, 283 leaves, 4 steps):

$$-\frac{2\,A}{5\,a\,e\,(e\,x)^{5/2}\,\sqrt{a+b\,x^3}}-\frac{2\,(8\,A\,b-5\,a\,B)\,\sqrt{e\,x}}{15\,a^2\,e^4\,\sqrt{a+b\,x^3}}-\left(2\,(8\,A\,b-5\,a\,B)\,\sqrt{e\,x}\,(a^{1/3}+b^{1/3}\,x)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}\,x\right)^2}}\,\text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right)b^{1/3}\,x}{a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}\,x}\right],\frac{1}{4}\left(2+\sqrt{3}\right)\right]\right)\Big/\left(15\times 3^{1/4}\,a^{7/3}\,e^4\,\sqrt{\frac{b^{1/3}\,x\,(a^{1/3}+b^{1/3}\,x)}{\left(a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 202 leaves):

$$\left(x\left(-6\,(-a)^{1/3}\left(3\,a\,A+8\,A\,b\,x^3-5\,a\,B\,x^3\right)+4\,\mathrm{i}\,3^{3/4}\,b^{1/3}\,(8\,A\,b-5\,a\,B)\,x^4\sqrt{\frac{(-1)^{5/6}\left((-a)^{1/3}-b^{1/3}\,x\right)}{b^{1/3}\,x}}\sqrt{\frac{\frac{(-a)^{2/3}}{b^{2/3}}+\frac{(-a)^{1/3}\,x}{b^{1/3}}+x^2}{x^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{\mathrm{i}\,(-a)^{1/3}}{b^{1/3}\,x}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\right)\Big/\left(45\,(-a)^{7/3}\,(e\,x)^{7/2}\sqrt{a+b\,x^3}\right)$$

Problem 560: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{(e\,x)^{5/2}\left(A+B\,x^3\right)}{\left(a+b\,x^3\right)^{5/2}}\,dx$$

Optimal (type 4, 299 leaves, 4 steps):

$$\frac{2\,(A\,b-a\,B)\,(e\,x)^{7/2}}{9\,a\,b\,e\left(a+b\,x^3\right)^{3/2}}-\frac{2\,(2\,A\,b+7\,a\,B)\,e^2\sqrt{e\,x}}{27\,a\,b^2\sqrt{a+b\,x^3}}+\left((2\,A\,b+7\,a\,B)\,e^2\sqrt{e\,x}\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}\,x\right)^2}}\;\text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right)b^{1/3}\,x}{a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}\,x}\right],\,\frac{1}{4}\left(2+\sqrt{3}\right)\right]\right)\Big/\left(27\times 3^{1/4}\,a^{4/3}\,b^2\sqrt{\frac{b^{1/3}\,x\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}\,x\right)^2}}\sqrt{a+b\,x^3}\right)$$

Result (type 4, 216 leaves):

$$\left(2\,i\,e^2\sqrt{e\,x}\left(-3\,i\,(-a)^{1/3}\left(7\,a^2\,B-A\,b^2\,x^3+2\,a\,b\left(A+5\,B\,x^3\right)\right)+\right.\right.$$

$$3^{3/4}\,b^{1/3}\,(2\,A\,b+7\,a\,B)\sqrt{(-1)^{5/6}\left(-1+\frac{(-a)^{1/3}}{b^{1/3}\,x}\right)}\;x\sqrt{\frac{\frac{(-a)^{2/3}}{b^{2/3}}+\frac{(-a)^{1/3}\,x}{b^{1/3}}+x^2}{x^2}}\;\left(a+b\,x^3\right)$$

$$\left.\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-a)^{1/3}}{b^{1/3}\,x}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\right)\Big/\left(81\,(-a)^{4/3}\,b^2\left(a+b\,x^3\right)^{3/2}\right)$$

Problem 561: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{(e\,x)^{3/2}\left(A+B\,x^3\right)}{\left(a+b\,x^3\right)^{5/2}}\,dx$$

Optimal (type 4, 596 leaves, 6 steps):

$$\frac{2\,(A\,b-a\,B)\,(e\,x)^{5/2}}{9\,a\,b\,e\left(a+b\,x^3\right)^{3/2}}+\frac{2\,(4\,A\,b+5\,a\,B)\,(e\,x)^{5/2}}{27\,a^2\,b\,e\sqrt{a+b\,x^3}}-$$

$$\frac{2\left(1+\sqrt{3}\right)(4\,A\,b+5\,a\,B)\,e\sqrt{e\,x}\,\sqrt{a+b\,x^3}}{27\,a^2\,b^{5/3}\left(a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}\,x\right)}+\left(2\,(4\,A\,b+5\,a\,B)\,e\sqrt{e\,x}\,\left(a^{1/3}+b^{1/3}\,x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}\,x\right)^2}}\;\text{EllipticE}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right)b^{1/3}\,x}{a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}\,x}\right],\,\frac{1}{4}\left(2+\sqrt{3}\right)\right]\right)\Big/$$

$$\left(9\times3^{3/4}\,a^{5/3}\,b^{5/3}\sqrt{\frac{b^{1/3}\,x\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}\,x\right)^2}}\;\sqrt{a+b\,x^3}\right)+$$

$$\left(\left(1-\sqrt{3}\right)(4\,A\,b+5\,a\,B)\,e\sqrt{e\,x}\,\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}\,x\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right)b^{1/3}\,x}{a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}\,x}\right],\,\frac{1}{4}\left(2+\sqrt{3}\right)\right]\right)\Big/$$

$$\left(27\times3^{1/4}\,a^{5/3}\,b^{5/3}\sqrt{\frac{b^{1/3}\,x\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right)b^{1/3}\,x\right)^2}}\;\sqrt{a+b\,x^3}\right)$$

Result (type 4, 307 leaves):

$$\frac{1}{81\,(-a)^{8/3}\,b^2\,\sqrt{e\,x}\,\left(a+b\,x^3\right)^{3/2}}\,2\,e^2\left(3\,(-a)^{2/3}\,b\,x^3\,\left(2\,a^2\,B+4\,A\,b^2\,x^3+a\,b\,\left(7\,A+5\,B\,x^3\right)\right)-\left(4\,A\,b+5\,a\,B\right)\,\left(a+b\,x^3\right)\right.$$
$$\left(3\,(-a)^{2/3}\,\left(a+b\,x^3\right)+(-1)^{2/3}\,3^{3/4}\,a\,b^{2/3}\,x^2\,\sqrt{\frac{(-1)^{5/6}\,\left((-a)^{1/3}-b^{1/3}\,x\right)}{b^{1/3}\,x}}\right.$$
$$\sqrt{\frac{\frac{(-a)^{2/3}}{b^{2/3}}+\frac{(-a)^{1/3}\,x}{b^{1/3}}+x^2}{x^2}}\,\left(\sqrt{3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-a)^{1/3}}{b^{1/3}\,x}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+\right.$$
$$\left.\left.\left.(-1)^{5/6}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-a)^{1/3}}{b^{1/3}\,x}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\right)\right)$$

Problem 563: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{A+B\,x^3}{\sqrt{e\,x}\,\left(a+b\,x^3\right)^{5/2}}\,dx$$

Optimal (type 4, 297 leaves, 4 steps):

$$\frac{2\,\left(A\,b-a\,B\right)\,\sqrt{e\,x}}{9\,a\,b\,e\,\left(a+b\,x^3\right)^{3/2}}+\frac{2\,\left(8\,A\,b+a\,B\right)\,\sqrt{e\,x}}{27\,a^2\,b\,e\,\sqrt{a+b\,x^3}}+$$
$$\left(2\,\left(8\,A\,b+a\,B\right)\,\sqrt{e\,x}\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x\right)^2}}\right.$$
$$\left.\text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right)\,b^{1/3}\,x}{a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x}\right],\,\frac{1}{4}\,\left(2+\sqrt{3}\right)\right]\right)\Bigg/$$
$$\left(27\times3^{1/4}\,a^{7/3}\,b\,e\,\sqrt{\frac{b^{1/3}\,x\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right)\,b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 214 leaves):

$$\left(2\left(3(-a)^{1/3}x\left(3a(Ab-aB)+(8Ab+aB)\left(a+bx^3\right)\right)-2\,i\,3^{3/4}b^{1/3}(8Ab+aB)\sqrt{(-1)^{5/6}\left(-1+\frac{(-a)^{1/3}}{b^{1/3}x}\right)}\,x^2\sqrt{\frac{\frac{(-a)^{2/3}}{b^{2/3}}+\frac{(-a)^{1/3}x}{b^{1/3}}+x^2}{x^2}}\left(a+bx^3\right)\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-a)^{1/3}}{b^{1/3}x}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right)\Big/\left(81(-a)^{7/3}b\sqrt{ex}\left(a+bx^3\right)^{3/2}\right)$$

Problem 564: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{A+Bx^3}{(ex)^{3/2}\left(a+bx^3\right)^{5/2}}\,dx$$

Optimal (type 4, 624 leaves, 7 steps):

$$-\frac{2 A}{a e \sqrt{e x} \left(a+b x^3\right)^{3/2}}-\frac{2 (10 A b-a B) (e x)^{5/2}}{9 a^2 e^4 \left(a+b x^3\right)^{3/2}}-\frac{8 (10 A b-a B) (e x)^{5/2}}{27 a^3 e^4 \sqrt{a+b x^3}}+\frac{8 \left(1+\sqrt{3}\right) (10 A b-a B) \sqrt{e x} \sqrt{a+b x^3}}{27 a^3 b^{2/3} e^2 \left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)}-$$

$$\left(8 (10 A b-a B) \sqrt{e x} \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right) b^{1/3} x}{a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x}\right],\frac{1}{4}\left(2+\sqrt{3}\right)\right]\right)\Big/$$

$$\left(9\times 3^{3/4} a^{8/3} b^{2/3} e^2 \sqrt{\frac{b^{1/3} x \left(a^{1/3}+b^{1/3} x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)-$$

$$\left(4 \left(1-\sqrt{3}\right) (10 A b-a B) \sqrt{e x} \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right) b^{1/3} x}{a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x}\right],\frac{1}{4}\left(2+\sqrt{3}\right)\right]\right)\Big/$$

$$\left(27\times 3^{1/4} a^{8/3} b^{2/3} e^2 \sqrt{\frac{b^{1/3} x \left(a^{1/3}+b^{1/3} x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 401 leaves):

$$\frac{1}{27 a^3 (e x)^{3/2} \sqrt{a+b x^3}} 2 x \left(\frac{-40 A b^2 x^6+a^2 \left(-27 A+7 B x^3\right)+a \left(-70 A b x^3+4 b B x^6\right)}{a+b x^3}+\frac{1}{\left(-1+(-1)^{2/3}\right) a^{1/3} b} 4 (10 A b-a B) \left(-\left(-1+(-1)^{2/3}\right) a^{1/3} b^{1/3} x \left((-1)^{1/3} a^{1/3}-b^{1/3} x\right) \left((-1)^{2/3} a^{1/3}+b^{1/3} x\right)-(-1)^{2/3} a^{2/3} \left(a^{1/3}+b^{1/3} x\right)^2 \sqrt{\frac{\left(1+(-1)^{1/3}\right) b^{1/3} x \left(a^{1/3}-(-1)^{1/3} b^{1/3} x\right)}{\left(a^{1/3}+b^{1/3} x\right)^2}} \sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{a^{1/3}+b^{1/3} x}} \left(\left(1+(-1)^{1/3}\right) \text{EllipticE}\left[\text{ArcSin}\left[\sqrt{\frac{\left(1+(-1)^{1/3}\right) b^{1/3} x}{a^{1/3}+b^{1/3} x}}\right], \frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]-\left(1+(-1)^{2/3}\right) \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{\left(1+(-1)^{1/3}\right) b^{1/3} x}{a^{1/3}+b^{1/3} x}}\right], \frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\right)\right)\right)$$

Problem 566: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{A+B x^3}{(e x)^{7/2} \left(a+b x^3\right)^{5/2}} \, dx$$

Optimal (type 4, 320 leaves, 5 steps):

$$-\frac{2 A}{5 a e (e x)^{5/2} \left(a+b x^3\right)^{3/2}}-\frac{2 (14 A b-5 a B) \sqrt{e x}}{45 a^2 e^4 \left(a+b x^3\right)^{3/2}}-\frac{16 (14 A b-5 a B) \sqrt{e x}}{135 a^3 e^4 \sqrt{a+b x^3}}-\left(16 (14 A b-5 a B) \sqrt{e x} \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)^2}} \text{EllipticF}\left[\text{ArcCos}\left[\frac{a^{1/3}+\left(1-\sqrt{3}\right) b^{1/3} x}{a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x}\right], \frac{1}{4} \left(2+\sqrt{3}\right)\right]\right) \Big/ \left(135 \times 3^{1/4} a^{10/3} e^4 \sqrt{\frac{b^{1/3} x \left(a^{1/3}+b^{1/3} x\right)}{\left(a^{1/3}+\left(1+\sqrt{3}\right) b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 232 leaves):

$$-\left(\left(2\,\mathrm{i}\,\sqrt{e\,x}\,\left(3\,\mathrm{i}\,(-a)^{1/3}\,\left(112\,A\,b^2\,x^6+a^2\,\left(27\,A-55\,B\,x^3\right)+2\,a\,b\,x^3\,\left(77\,A-20\,B\,x^3\right)\right)+16\times 3^{3/4}\,b^{1/3}\right.\right.\right.$$

$$\left(14\,A\,b-5\,a\,B\right)\,\sqrt{(-1)^{5/6}\,\left(-1+\frac{(-a)^{1/3}}{b^{1/3}\,x}\right)}\,x^4\,\sqrt{\frac{\frac{(-a)^{2/3}}{b^{2/3}}+\frac{(-a)^{1/3}\,x}{b^{1/3}}+x^2}{x^2}}\,\left(a+b\,x^3\right)\,\text{EllipticF}\Big[$$

$$\left.\left.\left.\text{ArcSin}\Big[\frac{\sqrt{-(-1)^{5/6}-\frac{\mathrm{i}\,(-a)^{1/3}}{b^{1/3}\,x}}}{3^{1/4}}\Big],\,(-1)^{1/3}\Big]\right)\right)\Big/\left(405\,(-a)^{10/3}\,e^4\,x^3\,\left(a+b\,x^3\right)^{3/2}\right)\right)$$

Problem 567: Result unnecessarily involves higher level functions.

$$\int \frac{x^{14}}{\left(1-x^3\right)^{1/3}\,\left(1+x^3\right)}\,dx$$

Optimal (type 3, 127 leaves, 7 steps):

$$\frac{2}{5}\,\left(1-x^3\right)^{5/3}-\frac{1}{4}\,\left(1-x^3\right)^{8/3}+\frac{1}{11}\,\left(1-x^3\right)^{11/3}+$$

$$\frac{\text{ArcTan}\Big[\frac{1+2^{2/3}\,\left(1-x^3\right)^{1/3}}{\sqrt{3}}\Big]}{2^{1/3}\,\sqrt{3}}-\frac{\text{Log}\left[1+x^3\right]}{6\times 2^{1/3}}+\frac{\text{Log}\left[2^{1/3}-\left(1-x^3\right)^{1/3}\right]}{2\times 2^{1/3}}$$

Result (type 5, 74 leaves):

$$\frac{1}{220\,\left(1-x^3\right)^{1/3}}$$

$$\left(\left(-1+x^3\right)^2\,\left(53+15\,x^3+20\,x^6\right)-220\,\left(\frac{-1+x^3}{1+x^3}\right)^{1/3}\,\text{Hypergeometric2F1}\Big[\frac{1}{3},\,\frac{1}{3},\,\frac{4}{3},\,\frac{2}{1+x^3}\Big]\right)$$

Problem 568: Result unnecessarily involves higher level functions.

$$\int \frac{x^{11}}{\left(1-x^3\right)^{1/3}\,\left(1+x^3\right)}\,dx$$

Optimal (type 3, 128 leaves, 7 steps):

$$-\frac{1}{2}\,\left(1-x^3\right)^{2/3}+\frac{1}{5}\,\left(1-x^3\right)^{5/3}-\frac{1}{8}\,\left(1-x^3\right)^{8/3}-$$

$$\frac{\text{ArcTan}\Big[\frac{1+2^{2/3}\,\left(1-x^3\right)^{1/3}}{\sqrt{3}}\Big]}{2^{1/3}\,\sqrt{3}}+\frac{\text{Log}\left[1+x^3\right]}{6\times 2^{1/3}}-\frac{\text{Log}\left[2^{1/3}-\left(1-x^3\right)^{1/3}\right]}{2\times 2^{1/3}}$$

Result (type 5, 70 leaves):

$$\frac{1}{40\left(1-x^3\right)^{1/3}}\left(-17+19x^3-7x^6+5x^9+40\left(\frac{-1+x^3}{1+x^3}\right)^{1/3}\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{3},\frac{4}{3},\frac{2}{1+x^3}\right]\right)$$

Problem 569: Result unnecessarily involves higher level functions.

$$\int\frac{x^8}{\left(1-x^3\right)^{1/3}\left(1+x^3\right)}\,dx$$

Optimal (type 3, 97 leaves, 7 steps):

$$\frac{1}{5}\left(1-x^3\right)^{5/3}+\frac{\text{ArcTan}\left[\frac{1+2^{2/3}\left(1-x^3\right)^{1/3}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}}-\frac{\text{Log}\left[1+x^3\right]}{6\times2^{1/3}}+\frac{\text{Log}\left[2^{1/3}-\left(1-x^3\right)^{1/3}\right]}{2\times2^{1/3}}$$

Result (type 5, 61 leaves):

$$\frac{\left(-1+x^3\right)^2-5\left(\frac{-1+x^3}{1+x^3}\right)^{1/3}\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{3},\frac{4}{3},\frac{2}{1+x^3}\right]}{5\left(1-x^3\right)^{1/3}}$$

Problem 570: Result unnecessarily involves higher level functions.

$$\int\frac{x^5}{\left(1-x^3\right)^{1/3}\left(1+x^3\right)}\,dx$$

Optimal (type 3, 98 leaves, 6 steps):

$$-\frac{1}{2}\left(1-x^3\right)^{2/3}-\frac{\text{ArcTan}\left[\frac{1+2^{2/3}\left(1-x^3\right)^{1/3}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}}+\frac{\text{Log}\left[1+x^3\right]}{6\times2^{1/3}}-\frac{\text{Log}\left[2^{1/3}-\left(1-x^3\right)^{1/3}\right]}{2\times2^{1/3}}$$

Result (type 5, 58 leaves):

$$\frac{-1+x^3+2\left(\frac{-1+x^3}{1+x^3}\right)^{1/3}\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{1}{3},\frac{4}{3},\frac{2}{1+x^3}\right]}{2\left(1-x^3\right)^{1/3}}$$

Problem 572: Result unnecessarily involves higher level functions.

$$\int\frac{1}{x\left(1-x^3\right)^{1/3}\left(1+x^3\right)}\,dx$$

Optimal (type 3, 137 leaves, 10 steps):

$$\frac{\text{ArcTan}\left[\frac{1+2\left(1-x^3\right)^{1/3}}{\sqrt{3}}\right]}{\sqrt{3}}-\frac{\text{ArcTan}\left[\frac{1+2^{2/3}\left(1-x^3\right)^{1/3}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}}-\frac{\text{Log}[x]}{2}+$$
$$\frac{\text{Log}\left[1+x^3\right]}{6\times2^{1/3}}+\frac{1}{2}\text{Log}\left[1-\left(1-x^3\right)^{1/3}\right]-\frac{\text{Log}\left[2^{1/3}-\left(1-x^3\right)^{1/3}\right]}{2\times2^{1/3}}$$

Result (type 6, 111 leaves):

$$-\left(\left(7 x^3 \text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},\frac{1}{x^3},-\frac{1}{x^3}\right]\right)\Big/\left(4\left(1-x^3\right)^{1/3}\left(1+x^3\right)\left(7 x^3 \text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},\frac{1}{x^3},-\frac{1}{x^3}\right]-3 \text{AppellF1}\left[\frac{7}{3},\frac{1}{3},2,\frac{10}{3},\frac{1}{x^3},-\frac{1}{x^3}\right]+\text{AppellF1}\left[\frac{7}{3},\frac{4}{3},1,\frac{10}{3},\frac{1}{x^3},-\frac{1}{x^3}\right]\right)\right)\right)$$

Problem 573: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^4\left(1-x^3\right)^{1/3}\left(1+x^3\right)}\,dx$$

Optimal (type 3, 157 leaves, 11 steps):

$$-\frac{\left(1-x^3\right)^{2/3}}{3 x^3}-\frac{2 \text{ArcTan}\left[\frac{1+2\left(1-x^3\right)^{1/3}}{\sqrt{3}}\right]}{3\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1+2^{2/3}\left(1-x^3\right)^{1/3}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}}+\frac{\text{Log}[x]}{3}-\frac{\text{Log}\left[1+x^3\right]}{6\times 2^{1/3}}-\frac{1}{3}\text{Log}\left[1-\left(1-x^3\right)^{1/3}\right]+\frac{\text{Log}\left[2^{1/3}-\left(1-x^3\right)^{1/3}\right]}{2\times 2^{1/3}}$$

Result (type 6, 209 leaves):

$$\frac{1}{6 x^3\left(1-x^3\right)^{1/3}}\left(-2+2 x^3-\left(4 x^6 \text{AppellF1}\left[1,\frac{1}{3},1,2,x^3,-x^3\right]\right)\Big/\left(\left(1+x^3\right)\left(-6 \text{AppellF1}\left[1,\frac{1}{3},1,2,x^3,-x^3\right]+x^3\left(3 \text{AppellF1}\left[2,\frac{1}{3},2,3,x^3,-x^3\right]-\text{AppellF1}\left[2,\frac{4}{3},1,3,x^3,-x^3\right]\right)\right)\right)+\left(7 x^6 \text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},\frac{1}{x^3},-\frac{1}{x^3}\right]\right)\Big/\left(\left(1+x^3\right)\left(7 x^3 \text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},\frac{1}{x^3},-\frac{1}{x^3}\right]-3 \text{AppellF1}\left[\frac{7}{3},\frac{1}{3},2,\frac{10}{3},\frac{1}{x^3},-\frac{1}{x^3}\right]+\text{AppellF1}\left[\frac{7}{3},\frac{4}{3},1,\frac{10}{3},\frac{1}{x^3},-\frac{1}{x^3}\right]\right)\right)\right)$$

Problem 574: Result unnecessarily involves higher level functions.

$$\int \frac{x^6}{\left(1-x^3\right)^{1/3}\left(1+x^3\right)}\,dx$$

Optimal (type 3, 226 leaves, 15 steps):

$$-\frac{1}{3} x\left(1-x^3\right)^{2/3}+\frac{2 \text{ArcTan}\left[\frac{1-\frac{2 x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{3\sqrt{3}}-\frac{\text{ArcTan}\left[\frac{1-\frac{2\, 2^{1/3} x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}}+\frac{1}{9}\text{Log}\left[1+\frac{x^2}{\left(1-x^3\right)^{2/3}}-\frac{x}{\left(1-x^3\right)^{1/3}}\right]-\frac{2}{9}\text{Log}\left[1+\frac{x}{\left(1-x^3\right)^{1/3}}\right]-\frac{\text{Log}\left[1+\frac{2^{2/3} x^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3} x}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{1/3} x}{\left(1-x^3\right)^{1/3}}\right]}{3\times 2^{1/3}}$$

Result (type 6, 233 leaves):

$$\frac{1}{36}\left(-12\,x\,\left(1-x^3\right)^{2/3}+\left(42\,x^4\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},x^3,-x^3\right]\right)\Big/\left(\left(1-x^3\right)^{1/3}\left(1+x^3\right)\left(-7\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},x^3,-x^3\right]+x^3\left(3\,\text{AppellF1}\left[\frac{7}{3},\frac{1}{3},2,\frac{10}{3},x^3,-x^3\right]-\text{AppellF1}\left[\frac{7}{3},\frac{4}{3},1,\frac{10}{3},x^3,-x^3\right]\right)\right)\right)+2^{2/3}\left(2\sqrt{3}\,\text{ArcTan}\left[\frac{-1+\frac{2\times 2^{1/3}x}{\left(-1+x^3\right)^{1/3}}}{\sqrt{3}}\right]-\text{Log}\left[1+\frac{2^{2/3}x^2}{\left(-1+x^3\right)^{2/3}}-\frac{2^{1/3}x}{\left(-1+x^3\right)^{1/3}}\right]+2\,\text{Log}\left[1+\frac{2^{1/3}x}{\left(-1+x^3\right)^{1/3}}\right]\right)\right)$$

Problem 575: Result unnecessarily involves higher level functions.

$$\int\frac{x^3}{\left(1-x^3\right)^{1/3}\left(1+x^3\right)}\,dx$$

Optimal (type 3, 207 leaves, 14 steps):

$$-\frac{\text{ArcTan}\left[\frac{1-\frac{2x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1-\frac{2\times 2^{1/3}x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}}-\frac{1}{6}\text{Log}\left[1+\frac{x^2}{\left(1-x^3\right)^{2/3}}-\frac{x}{\left(1-x^3\right)^{1/3}}\right]+\frac{1}{3}\text{Log}\left[1+\frac{x}{\left(1-x^3\right)^{1/3}}\right]+\frac{\text{Log}\left[1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3}x}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{1/3}x}{\left(1-x^3\right)^{1/3}}\right]}{3\times 2^{1/3}}$$

Result (type 6, 115 leaves):

$$-\left(\left(7\,x^4\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},x^3,-x^3\right]\right)\Big/\left(4\left(1-x^3\right)^{1/3}\left(1+x^3\right)\left(-7\,\text{AppellF1}\left[\frac{4}{3},\frac{1}{3},1,\frac{7}{3},x^3,-x^3\right]+x^3\left(3\,\text{AppellF1}\left[\frac{7}{3},\frac{1}{3},2,\frac{10}{3},x^3,-x^3\right]-\text{AppellF1}\left[\frac{7}{3},\frac{4}{3},1,\frac{10}{3},x^3,-x^3\right]\right)\right)\right)\right)$$

Problem 580: Result unnecessarily involves higher level functions.

$$\int\frac{x^7}{\left(1-x^3\right)^{1/3}\left(1+x^3\right)}\,dx$$

Optimal (type 5, 313 leaves, 31 steps):

$$-\frac{1}{4}x^2\left(1-x^3\right)^{2/3}+\frac{\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{1/3}\sqrt{3}}-$$
$$\frac{1}{4}x^2\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right]-\frac{\text{Log}\left[2^{2/3}-\frac{1-x}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{1/3}}+$$
$$\frac{\text{Log}\left[1+\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{3\times 2^{1/3}}+\frac{\text{Log}\left[2\times 2^{1/3}+\frac{(1-x)^2}{\left(1-x^3\right)^{2/3}}+\frac{2^{2/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{12\times 2^{1/3}}$$

Result (type 6, 119 leaves):

$$\frac{1}{4}x^2\left(1-x^3\right)^{2/3}$$
$$\left(-1-\left(5\,\text{AppellF1}\left[\frac{2}{3},-\frac{2}{3},1,\frac{5}{3},x^3,-x^3\right]\right)\Big/\left(\left(1+x^3\right)\left(-5\,\text{AppellF1}\left[\frac{2}{3},-\frac{2}{3},1,\frac{5}{3},x^3,-x^3\right]+\right.\right.\right.$$
$$\left.\left.\left.x^3\left(3\,\text{AppellF1}\left[\frac{5}{3},-\frac{2}{3},2,\frac{8}{3},x^3,-x^3\right]+2\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},1,\frac{8}{3},x^3,-x^3\right]\right)\right)\right)\right)$$

Problem 581: Result unnecessarily involves higher level functions.

$$\int\frac{x^4}{\left(1-x^3\right)^{1/3}\left(1+x^3\right)}\,dx$$

Optimal (type 5, 293 leaves, 15 steps):

$$-\frac{\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3}\sqrt{3}}-\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{1/3}\sqrt{3}}+$$
$$\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right]-\frac{\text{Log}\left[1+\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{1/3}}+$$
$$\frac{\text{Log}\left[1+\frac{2^{1/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{3\times 2^{1/3}}-\frac{\text{Log}\left[2\times 2^{1/3}+\frac{(1-x)^2}{\left(1-x^3\right)^{2/3}}+\frac{2^{2/3}(1-x)}{\left(1-x^3\right)^{1/3}}\right]}{12\times 2^{1/3}}+\frac{\text{Log}\left[2^{2/3}+\frac{-1+x}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{1/3}}$$

Result (type 6, 115 leaves):

$$-\left(\left(8\,x^5\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},1,\frac{8}{3},x^3,-x^3\right]\right)\Big/\right.$$
$$\left(5\left(1-x^3\right)^{1/3}\left(1+x^3\right)\left(-8\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{3},1,\frac{8}{3},x^3,-x^3\right]+\right.\right.$$
$$\left.\left.\left.x^3\left(3\,\text{AppellF1}\left[\frac{8}{3},\frac{1}{3},2,\frac{11}{3},x^3,-x^3\right]-\text{AppellF1}\left[\frac{8}{3},\frac{4}{3},1,\frac{11}{3},x^3,-x^3\right]\right)\right)\right)\right)$$

Problem 582: Result unnecessarily involves higher level functions.

$$\int \frac{x}{\left(1-x^3\right)^{1/3} \left(1+x^3\right)} \, dx$$

Optimal (type 3, 272 leaves, 13 steps):

$$\frac{\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3} (1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3} \sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3} (1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{1/3} \sqrt{3}}+\frac{\text{Log}\left[1+\frac{2^{2/3} (1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3} (1-x)}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{1/3}}-$$
$$\frac{\text{Log}\left[1+\frac{2^{1/3} (1-x)}{\left(1-x^3\right)^{1/3}}\right]}{3\times 2^{1/3}}+\frac{\text{Log}\left[2\times 2^{1/3}+\frac{(1-x)^2}{\left(1-x^3\right)^{2/3}}+\frac{2^{2/3} (1-x)}{\left(1-x^3\right)^{1/3}}\right]}{12\times 2^{1/3}}-\frac{\text{Log}\left[2^{2/3}+\frac{-1+x}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{1/3}}$$

Result (type 6, 115 leaves):

$$-\left(\left(5\, x^2\, \text{AppellF1}\left[\frac{2}{3},\frac{1}{3},1,\frac{5}{3},x^3,-x^3\right]\right)\Big/\right.$$
$$\left(2\left(1-x^3\right)^{1/3} \left(1+x^3\right) \left(-5\, \text{AppellF1}\left[\frac{2}{3},\frac{1}{3},1,\frac{5}{3},x^3,-x^3\right]+\right.\right.$$
$$\left.\left.\left. x^3 \left(3\, \text{AppellF1}\left[\frac{5}{3},\frac{1}{3},2,\frac{8}{3},x^3,-x^3\right]-\text{AppellF1}\left[\frac{5}{3},\frac{4}{3},1,\frac{8}{3},x^3,-x^3\right]\right)\right)\right)\right)$$

Problem 583: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^2 \left(1-x^3\right)^{1/3} \left(1+x^3\right)} \, dx$$

Optimal (type 5, 312 leaves, 17 steps):

$$-\frac{\left(1-x^3\right)^{2/3}}{x}-\frac{\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3} (1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3} \sqrt{3}}-\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3} (1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{1/3} \sqrt{3}}-$$
$$\frac{1}{2} x^2\, \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right]+\frac{\text{Log}\left[2^{2/3}-\frac{1-x}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{1/3}}-$$
$$\frac{\text{Log}\left[1+\frac{2^{2/3} (1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3} (1-x)}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{1/3} (1-x)}{\left(1-x^3\right)^{1/3}}\right]}{3\times 2^{1/3}}-\frac{\text{Log}\left[2\times 2^{1/3}+\frac{(1-x)^2}{\left(1-x^3\right)^{2/3}}+\frac{2^{2/3} (1-x)}{\left(1-x^3\right)^{1/3}}\right]}{12\times 2^{1/3}}$$

Result (type 6, 229 leaves):

$$\frac{1}{5 x \left(1-x^3\right)^{1/3}}\left(-5+5 x^3+\left(25 x^3 \text{AppellF1}\left[\frac{2}{3},\frac{1}{3},1,\frac{5}{3},x^3,-x^3\right]\right)\Big/\left(\left(1+x^3\right)\left(-5 \text{AppellF1}\left[\frac{2}{3},\frac{1}{3},1,\frac{5}{3},x^3,-x^3\right]+x^3\left(3 \text{AppellF1}\left[\frac{5}{3},\frac{1}{3},2,\frac{8}{3},x^3,-x^3\right]-\text{AppellF1}\left[\frac{5}{3},\frac{4}{3},1,\frac{8}{3},x^3,-x^3\right]\right)\right)\right)+\left(8 x^6 \text{AppellF1}\left[\frac{5}{3},\frac{1}{3},1,\frac{8}{3},x^3,-x^3\right]\right)\Big/\left(\left(1+x^3\right)\left(-8 \text{AppellF1}\left[\frac{5}{3},\frac{1}{3},1,\frac{8}{3},x^3,-x^3\right]+x^3\left(3 \text{AppellF1}\left[\frac{8}{3},\frac{1}{3},2,\frac{11}{3},x^3,-x^3\right]-\text{AppellF1}\left[\frac{8}{3},\frac{4}{3},1,\frac{11}{3},x^3,-x^3\right]\right)\right)\right)\right)$$

Problem 584: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^5 \left(1-x^3\right)^{1/3} \left(1+x^3\right)} \, dx$$

Optimal (type 5, 331 leaves, 19 steps):

$$-\frac{\left(1-x^3\right)^{2/3}}{4 x^4}+\frac{\left(1-x^3\right)^{2/3}}{2 x}+\frac{\text{ArcTan}\left[\frac{1-\frac{2\ 2^{1/3} (1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{1/3} \sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1+\frac{2^{1/3} (1-x)}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2\times 2^{1/3} \sqrt{3}}+\frac{1}{4} x^2 \text{Hypergeometric2F1}\left[\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right]-\frac{\text{Log}\left[2^{2/3}-\frac{1-x}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{1/3}}+\frac{\text{Log}\left[1+\frac{2^{2/3} (1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3} (1-x)}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{1/3}}-\frac{\text{Log}\left[1+\frac{2^{1/3} (1-x)}{\left(1-x^3\right)^{1/3}}\right]}{3\times 2^{1/3}}+\frac{\text{Log}\left[2\times 2^{1/3}+\frac{(1-x)^2}{\left(1-x^3\right)^{2/3}}+\frac{2^{2/3} (1-x)}{\left(1-x^3\right)^{1/3}}\right]}{12\times 2^{1/3}}$$

Result (type 6, 234 leaves):

$$-\frac{1}{20 x^4 \left(1-x^3\right)^{1/3}}\left(5-15 x^3+10 x^6+\left(75 x^6 \text{AppellF1}\left[\frac{2}{3},\frac{1}{3},1,\frac{5}{3},x^3,-x^3\right]\right)\Big/\left(\left(1+x^3\right)\left(-5 \text{AppellF1}\left[\frac{2}{3},\frac{1}{3},1,\frac{5}{3},x^3,-x^3\right]+x^3\left(3 \text{AppellF1}\left[\frac{5}{3},\frac{1}{3},2,\frac{8}{3},x^3,-x^3\right]-\text{AppellF1}\left[\frac{5}{3},\frac{4}{3},1,\frac{8}{3},x^3,-x^3\right]\right)\right)\right)+\left(16 x^9 \text{AppellF1}\left[\frac{5}{3},\frac{1}{3},1,\frac{8}{3},x^3,-x^3\right]\right)\Big/\left(\left(1+x^3\right)\left(-8 \text{AppellF1}\left[\frac{5}{3},\frac{1}{3},1,\frac{8}{3},x^3,-x^3\right]+x^3\left(3 \text{AppellF1}\left[\frac{8}{3},\frac{1}{3},2,\frac{11}{3},x^3,-x^3\right]-\text{AppellF1}\left[\frac{8}{3},\frac{4}{3},1,\frac{11}{3},x^3,-x^3\right]\right)\right)\right)\right)$$

Problem 585: Result unnecessarily involves higher level functions.

$$\int \frac{x^{11}}{\left(1-x^3\right)^{2/3} \left(1+x^3\right)} \, dx$$

Optimal (type 3, 125 leaves, 7 steps):

$$-\left(1-x^3\right)^{1/3}+\frac{1}{4}\left(1-x^3\right)^{4/3}-\frac{1}{7}\left(1-x^3\right)^{7/3}+\frac{\text{ArcTan}\left[\frac{1+2^{2/3}\left(1-x^3\right)^{1/3}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}}+\frac{\text{Log}\left[1+x^3\right]}{6\times 2^{2/3}}-\frac{\text{Log}\left[2^{1/3}-\left(1-x^3\right)^{1/3}\right]}{2\times 2^{2/3}}$$

Result (type 5, 70 leaves):

$$\frac{1}{28\left(1-x^3\right)^{2/3}}\left(-25+26x^3-5x^6+4x^9+14\left(\frac{-1+x^3}{1+x^3}\right)^{2/3}\text{Hypergeometric2F1}\left[\frac{2}{3},\frac{2}{3},\frac{5}{3},\frac{2}{1+x^3}\right]\right)$$

Problem 586: Result unnecessarily involves higher level functions.

$$\int\frac{x^8}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$$

Optimal (type 3, 98 leaves, 7 steps):

$$\frac{1}{4}\left(1-x^3\right)^{4/3}-\frac{\text{ArcTan}\left[\frac{1+2^{2/3}\left(1-x^3\right)^{1/3}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}}-\frac{\text{Log}\left[1+x^3\right]}{6\times 2^{2/3}}+\frac{\text{Log}\left[2^{1/3}-\left(1-x^3\right)^{1/3}\right]}{2\times 2^{2/3}}$$

Result (type 5, 61 leaves):

$$\frac{\left(-1+x^3\right)^2-2\left(\frac{-1+x^3}{1+x^3}\right)^{2/3}\text{Hypergeometric2F1}\left[\frac{2}{3},\frac{2}{3},\frac{5}{3},\frac{2}{1+x^3}\right]}{4\left(1-x^3\right)^{2/3}}$$

Problem 587: Result unnecessarily involves higher level functions.

$$\int\frac{x^5}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$$

Optimal (type 3, 95 leaves, 6 steps):

$$-\left(1-x^3\right)^{1/3}+\frac{\text{ArcTan}\left[\frac{1+2^{2/3}\left(1-x^3\right)^{1/3}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}}+\frac{\text{Log}\left[1+x^3\right]}{6\times 2^{2/3}}-\frac{\text{Log}\left[2^{1/3}-\left(1-x^3\right)^{1/3}\right]}{2\times 2^{2/3}}$$

Result (type 5, 59 leaves):

$$\frac{-2+2x^3+\left(\frac{-1+x^3}{1+x^3}\right)^{2/3}\text{Hypergeometric2F1}\left[\frac{2}{3},\frac{2}{3},\frac{5}{3},\frac{2}{1+x^3}\right]}{2\left(1-x^3\right)^{2/3}}$$

Problem 589: Result unnecessarily involves higher level functions.

$$\int\frac{1}{x\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$$

Optimal (type 3, 137 leaves, 10 steps):

$$-\frac{\text{ArcTan}\left[\frac{1+2\left(1-x^3\right)^{1/3}}{\sqrt{3}}\right]}{\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1+2^{2/3}\left(1-x^3\right)^{1/3}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}}-\frac{\text{Log}[x]}{2}+\frac{\text{Log}\left[1+x^3\right]}{6\times 2^{2/3}}+\frac{1}{2}\text{Log}\left[1-\left(1-x^3\right)^{1/3}\right]-\frac{\text{Log}\left[2^{1/3}-\left(1-x^3\right)^{1/3}\right]}{2\times 2^{2/3}}$$

Result (type 6, 113 leaves):

$$-\left(\left(8\,x^3\,\text{AppellF1}\left[\frac{5}{3},\frac{2}{3},1,\frac{8}{3},\frac{1}{x^3},-\frac{1}{x^3}\right]\right)\Big/\left(5\left(1-x^3\right)^{2/3}\left(1+x^3\right)\left(8\,x^3\,\text{AppellF1}\left[\frac{5}{3},\frac{2}{3},1,\frac{8}{3},\frac{1}{x^3},-\frac{1}{x^3}\right]-3\,\text{AppellF1}\left[\frac{8}{3},\frac{2}{3},2,\frac{11}{3},\frac{1}{x^3},-\frac{1}{x^3}\right]+2\,\text{AppellF1}\left[\frac{8}{3},\frac{5}{3},1,\frac{11}{3},\frac{1}{x^3},-\frac{1}{x^3}\right]\right)\right)\right)$$

Problem 590: Result unnecessarily involves higher level functions.

$$\int\frac{1}{x^4\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$$

Optimal (type 3, 158 leaves, 11 steps):

$$-\frac{\left(1-x^3\right)^{1/3}}{3\,x^3}+\frac{\text{ArcTan}\left[\frac{1+2\left(1-x^3\right)^{1/3}}{\sqrt{3}}\right]}{3\sqrt{3}}-\frac{\text{ArcTan}\left[\frac{1+2^{2/3}\left(1-x^3\right)^{1/3}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}}+\frac{\text{Log}[x]}{6}-\frac{\text{Log}\left[1+x^3\right]}{6\times 2^{2/3}}-\frac{1}{6}\text{Log}\left[1-\left(1-x^3\right)^{1/3}\right]+\frac{\text{Log}\left[2^{1/3}-\left(1-x^3\right)^{1/3}\right]}{2\times 2^{2/3}}$$

Result (type 6, 110 leaves):

$$-\left(\left(11\,\text{AppellF1}\left[\frac{8}{3},\frac{2}{3},1,\frac{11}{3},\frac{1}{x^3},-\frac{1}{x^3}\right]\right)\Big/\left(8\left(1-x^3\right)^{2/3}\left(1+x^3\right)\left(11\,x^3\,\text{AppellF1}\left[\frac{8}{3},\frac{2}{3},1,\frac{11}{3},\frac{1}{x^3},-\frac{1}{x^3}\right]-3\,\text{AppellF1}\left[\frac{11}{3},\frac{2}{3},2,\frac{14}{3},\frac{1}{x^3},-\frac{1}{x^3}\right]+2\,\text{AppellF1}\left[\frac{11}{3},\frac{5}{3},1,\frac{14}{3},\frac{1}{x^3},-\frac{1}{x^3}\right]\right)\right)\right)$$

Problem 591: Result unnecessarily involves higher level functions.

$$\int\frac{x^4}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$$

Optimal (type 3, 207 leaves, 14 steps):

$$-\frac{\text{ArcTan}\left[\frac{1-\frac{2x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}}+\frac{1}{6}\text{Log}\left[1+\frac{x^2}{(1-x^3)^{2/3}}-\frac{x}{(1-x^3)^{1/3}}\right]-\frac{1}{3}\text{Log}\left[1+\frac{x}{(1-x^3)^{1/3}}\right]-\frac{\text{Log}\left[1+\frac{2^{2/3}x^2}{(1-x^3)^{2/3}}-\frac{2^{1/3}x}{(1-x^3)^{1/3}}\right]}{6\times 2^{2/3}}+\frac{\text{Log}\left[1+\frac{2^{1/3}x}{(1-x^3)^{1/3}}\right]}{3\times 2^{2/3}}$$

Result (type 6, 115 leaves):

$$-\left(\left(8x^5\,\text{AppellF1}\left[\frac{5}{3},\frac{2}{3},1,\frac{8}{3},x^3,-x^3\right]\right)\Big/\left(5\left(1-x^3\right)^{2/3}\left(1+x^3\right)\left(-8\,\text{AppellF1}\left[\frac{5}{3},\frac{2}{3},1,\frac{8}{3},x^3,-x^3\right]+x^3\left(3\,\text{AppellF1}\left[\frac{8}{3},\frac{2}{3},2,\frac{11}{3},x^3,-x^3\right]-2\,\text{AppellF1}\left[\frac{8}{3},\frac{5}{3},1,\frac{11}{3},x^3,-x^3\right]\right)\right)\right)\right)$$

Problem 592: Result unnecessarily involves higher level functions.

$$\int\frac{x}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$$

Optimal (type 3, 122 leaves, 7 steps):

$$-\frac{\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}}+\frac{\text{Log}\left[1+\frac{2^{2/3}x^2}{(1-x^3)^{2/3}}-\frac{2^{1/3}x}{(1-x^3)^{1/3}}\right]}{6\times 2^{2/3}}-\frac{\text{Log}\left[1+\frac{2^{1/3}x}{(1-x^3)^{1/3}}\right]}{3\times 2^{2/3}}$$

Result (type 5, 59 leaves):

$$\frac{x^2\left(\frac{1-x^3}{1+x^3}\right)^{2/3}\text{Hypergeometric2F1}\left[\frac{2}{3},\frac{2}{3},\frac{5}{3},\frac{2x^3}{1+x^3}\right]}{2\left(1-x^3\right)^{2/3}}$$

Problem 593: Result unnecessarily involves higher level functions.

$$\int\frac{1}{x^2\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$$

Optimal (type 3, 137 leaves, 8 steps):

$$-\frac{\left(1-x^3\right)^{1/3}}{x}+\frac{\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}x}{(1-x^3)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}}-\frac{\text{Log}\left[1+\frac{2^{2/3}x^2}{(1-x^3)^{2/3}}-\frac{2^{1/3}x}{(1-x^3)^{1/3}}\right]}{6\times 2^{2/3}}+\frac{\text{Log}\left[1+\frac{2^{1/3}x}{(1-x^3)^{1/3}}\right]}{3\times 2^{2/3}}$$

Result (type 5, 154 leaves):

$$\left(5\left(2+x^3-3x^6\right)\text{Hypergeometric2F1}\left[\frac{2}{3},1,\frac{5}{3},\frac{2x^3}{-1+x^3}\right]-12x^3\left(1+x^3\right)\text{Hypergeometric2F1}\left[\frac{5}{3},2,\frac{8}{3},\frac{2x^3}{-1+x^3}\right]\right)\Big/\left(2x\left(1-x^3\right)^{2/3}\left(5\left(2-5x^3+3x^6\right)+15\left(-1+x^6\right)\text{Hypergeometric2F1}\left[\frac{2}{3},1,\frac{5}{3},\frac{2x^3}{-1+x^3}\right]+18\left(x^3+x^6\right)\text{Hypergeometric2F1}\left[\frac{5}{3},2,\frac{8}{3},\frac{2x^3}{-1+x^3}\right]\right)\right)$$

Problem 594: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\frac{1}{x^5\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$$

Optimal (type 3, 140 leaves, 9 steps):

$$-\frac{\left(1-x^3\right)^{4/3}}{4x^4}-\frac{\text{ArcTan}\left[\frac{1-\frac{2\,2^{1/3}x}{\left(1-x^3\right)^{1/3}}}{\sqrt{3}}\right]}{2^{2/3}\sqrt{3}}+\frac{\text{Log}\left[1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{2^{1/3}x}{\left(1-x^3\right)^{1/3}}\right]}{6\times 2^{2/3}}-\frac{\text{Log}\left[1+\frac{2^{1/3}x}{\left(1-x^3\right)^{1/3}}\right]}{3\times 2^{2/3}}$$

Result (type 5, 680 leaves):

$$-\left(\left(\left(1-x^3\right)^{4/3}\left(5\left(-1-9x^3+x^6+9x^9+\left(4-13x^3-20x^6+9x^9\right)\text{Hypergeometric2F1}\left[\frac{2}{3},1,\frac{5}{3},\frac{2x^3}{-1+x^3}\right]\right)+216\left(x^6+x^9\right)\text{HypergeometricPFQ}\left[\left\{\frac{2}{3},2,2\right\},\left\{1,\frac{8}{3}\right\},\frac{2x^3}{-1+x^3}\right]+81x^3\left(1+x^3\right)^2\text{HypergeometricPFQ}\left[\left\{\frac{2}{3},2,2,2\right\},\left\{1,1,\frac{8}{3}\right\},\frac{2x^3}{-1+x^3}\right]\right)\right)\Big/\left(3x^4\left(-20+70x^3+60x^6-200x^9+40x^{12}+50x^{15}+40\,\text{Hypergeometric2F1}\left[\frac{2}{3},1,\frac{5}{3},\frac{2x^3}{-1+x^3}\right]-125x^3\,\text{Hypergeometric2F1}\left[\frac{2}{3},1,\frac{5}{3},\frac{2x^3}{-1+x^3}\right]+90x^6\,\text{Hypergeometric2F1}\left[\frac{2}{3},1,\frac{5}{3},\frac{2x^3}{-1+x^3}\right]+180x^9\,\text{Hypergeometric2F1}\left[\frac{2}{3},1,\frac{5}{3},\frac{2x^3}{-1+x^3}\right]-130x^{12}\,\text{Hypergeometric2F1}\left[\frac{2}{3},1,\frac{5}{3},\frac{2x^3}{-1+x^3}\right]-55x^{15}\,\text{Hypergeometric2F1}\left[\frac{2}{3},1,\frac{5}{3},\frac{2x^3}{-1+x^3}\right]+144x^6\left(-1-4x^3+x^6+4x^9\right)\text{HypergeometricPFQ}\left[\left\{\frac{2}{3},2,2\right\},\left\{1,\frac{8}{3}\right\},\frac{2x^3}{-1+x^3}\right]+432x^9\left(1+x^3\right)^2\text{HypergeometricPFQ}\left[\left\{\frac{5}{3},3,3\right\},\left\{2,\frac{11}{3}\right\},\frac{2x^3}{-1+x^3}\right]+27x^3\,\text{HypergeometricPFQ}\left[\left\{\frac{2}{3},2,2,2\right\},\left\{1,1,\frac{8}{3}\right\},\frac{2x^3}{-1+x^3}\right]-270x^6\,\text{HypergeometricPFQ}\left[\left\{\frac{2}{3},2,2,2\right\},\left\{1,1,\frac{8}{3}\right\},\frac{2x^3}{-1+x^3}\right]-324x^9\,\text{HypergeometricPFQ}\left[\left\{\frac{2}{3},2,2,2\right\},\left\{1,1,\frac{8}{3}\right\},\frac{2x^3}{-1+x^3}\right]+270x^{12}\,\text{HypergeometricPFQ}\left[\left\{\frac{2}{3},2,2,2\right\},\left\{1,1,\frac{8}{3}\right\},\frac{2x^3}{-1+x^3}\right]+297x^{15}\,\text{HypergeometricPFQ}\left[\left\{\frac{2}{3},2,2,2\right\},\left\{1,1,\frac{8}{3}\right\},\frac{2x^3}{-1+x^3}\right]+324x^6\,\text{HypergeometricPFQ}\left[\left\{\frac{5}{3},3,3,3\right\},\left\{2,2,\frac{11}{3}\right\},\frac{2x^3}{-1+x^3}\right]+972x^9\,\text{HypergeometricPFQ}\left[\left\{\frac{5}{3},3,3,3\right\},\left\{2,2,\frac{11}{3}\right\},\frac{2x^3}{-1+x^3}\right]+972x^{12}\,\text{HypergeometricPFQ}\left[\left\{\frac{5}{3},3,3,3\right\},\left\{2,2,\frac{11}{3}\right\},\frac{2x^3}{-1+x^3}\right]+324x^{15}\,\text{HypergeometricPFQ}\left[\left\{\frac{5}{3},3,3,3\right\},\left\{2,2,\frac{11}{3}\right\},\frac{2x^3}{-1+x^3}\right]\right)\right)\right)$$

Problem 595: Result more than twice size of optimal antiderivative.

$$\int \frac{x^6}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$$

Optimal (type 6, 26 leaves, 1 step):

$$\frac{1}{7} x^7 \text{AppellF1}\left[\frac{7}{3}, \frac{2}{3}, 1, \frac{10}{3}, x^3, -x^3\right]$$

Result (type 6, 115 leaves):

$$\frac{1}{2} x \left(1-x^3\right)^{1/3} \left(-1-\left(4 \text{AppellF1}\left[\frac{1}{3}, -\frac{1}{3}, 1, \frac{4}{3}, x^3, -x^3\right]\right) \Big/ \left(\left(1+x^3\right) \left(-4 \text{AppellF1}\left[\frac{1}{3}, -\frac{1}{3}, 1, \frac{4}{3}, x^3, -x^3\right] + x^3 \left(3 \text{AppellF1}\left[\frac{4}{3}, -\frac{1}{3}, 2, \frac{7}{3}, x^3, -x^3\right] + \text{AppellF1}\left[\frac{4}{3}, \frac{2}{3}, 1, \frac{7}{3}, x^3, -x^3\right]\right)\right)\right)\right)$$

Problem 596: Result more than twice size of optimal antiderivative.

$$\int \frac{x^3}{\left(1-x^3\right)^{2/3} \left(1+x^3\right)} \, dx$$

Optimal (type 6, 26 leaves, 1 step):

$$\frac{1}{4} x^4 \text{AppellF1}\left[\frac{4}{3}, \frac{2}{3}, 1, \frac{7}{3}, x^3, -x^3\right]$$

Result (type 6, 115 leaves):

$$-\left(\left(7 x^4 \text{AppellF1}\left[\frac{4}{3}, \frac{2}{3}, 1, \frac{7}{3}, x^3, -x^3\right]\right) \Big/ \left(4 \left(1-x^3\right)^{2/3} \left(1+x^3\right) \left(-7 \text{AppellF1}\left[\frac{4}{3}, \frac{2}{3}, 1, \frac{7}{3}, x^3, -x^3\right] + x^3 \left(3 \text{AppellF1}\left[\frac{7}{3}, \frac{2}{3}, 2, \frac{10}{3}, x^3, -x^3\right] - 2 \text{AppellF1}\left[\frac{7}{3}, \frac{5}{3}, 1, \frac{10}{3}, x^3, -x^3\right]\right)\right)\right)\right)$$

Problem 597: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(1-x^3\right)^{2/3} \left(1+x^3\right)} \, dx$$

Optimal (type 6, 21 leaves, 1 step):

$$x \text{AppellF1}\left[\frac{1}{3}, \frac{2}{3}, 1, \frac{4}{3}, x^3, -x^3\right]$$

Result (type 6, 111 leaves):

$$-\left(\left(4 x \text{AppellF1}\left[\frac{1}{3}, \frac{2}{3}, 1, \frac{4}{3}, x^3, -x^3\right]\right) \Big/ \left(\left(1-x^3\right)^{2/3} \left(1+x^3\right) \left(-4 \text{AppellF1}\left[\frac{1}{3}, \frac{2}{3}, 1, \frac{4}{3}, x^3, -x^3\right] + x^3 \left(3 \text{AppellF1}\left[\frac{4}{3}, \frac{2}{3}, 2, \frac{7}{3}, x^3, -x^3\right] - 2 \text{AppellF1}\left[\frac{4}{3}, \frac{5}{3}, 1, \frac{7}{3}, x^3, -x^3\right]\right)\right)\right)\right)$$

Problem 598: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^3 \left(1-x^3\right)^{2/3} \left(1+x^3\right)} \, dx$$

Optimal (type 6, 26 leaves, 1 step):

$$-\frac{\text{AppellF1}\left[-\frac{2}{3}, \frac{2}{3}, 1, \frac{1}{3}, x^3, -x^3\right]}{2 x^2}$$

Result (type 6, 120 leaves):

$$\frac{1}{2 x^2} \left(1-x^3\right)^{1/3} \left(-1+\left(4 x^3 \text{AppellF1}\left[\frac{1}{3}, -\frac{1}{3}, 1, \frac{4}{3}, x^3, -x^3\right]\right) \Big/ \left(\left(1+x^3\right) \left(-4 \text{AppellF1}\left[\frac{1}{3}, -\frac{1}{3}, 1, \frac{4}{3}, x^3, -x^3\right] + x^3 \left(3 \text{AppellF1}\left[\frac{4}{3}, -\frac{1}{3}, 2, \frac{7}{3}, x^3, -x^3\right] + \text{AppellF1}\left[\frac{4}{3}, \frac{2}{3}, 1, \frac{7}{3}, x^3, -x^3\right]\right)\right)\right)\right)$$

Problem 623: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{c+d x^4}}{x \left(a+b x^4\right)} \, dx$$

Optimal (type 3, 85 leaves, 6 steps):

$$-\frac{\sqrt{c} \, \text{ArcTanh}\left[\frac{\sqrt{c+d x^4}}{\sqrt{c}}\right]}{2 a} + \frac{\sqrt{b c-a d} \, \text{ArcTanh}\left[\frac{\sqrt{b} \sqrt{c+d x^4}}{\sqrt{b c-a d}}\right]}{2 a \sqrt{b}}$$

Result (type 6, 162 leaves):

$$-\left(\left(3 b d x^4 \sqrt{c+d x^4} \, \text{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, 1, \frac{3}{2}, -\frac{c}{d x^4}, -\frac{a}{b x^4}\right]\right) \Big/ \left(2 \left(a+b x^4\right) \left(3 b d x^4 \text{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, 1, \frac{3}{2}, -\frac{c}{d x^4}, -\frac{a}{b x^4}\right] - 2 a d \, \text{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}, 2, \frac{5}{2}, -\frac{c}{d x^4}, -\frac{a}{b x^4}\right] + b c \, \text{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, 1, \frac{5}{2}, -\frac{c}{d x^4}, -\frac{a}{b x^4}\right]\right)\right)\right)$$

Problem 625: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{c+d x^4}}{x^5 \left(a+b x^4\right)} \, dx$$

Optimal (type 3, 115 leaves, 7 steps):

$$-\frac{\sqrt{c+d x^4}}{4 a x^4}+\frac{(2 b c-a d) \text{ArcTanh}\left[\frac{\sqrt{c+d x^4}}{\sqrt{c}}\right]}{4 a^2 \sqrt{c}}-\frac{\sqrt{b} \sqrt{b c-a d} \text{ArcTanh}\left[\frac{\sqrt{b} \sqrt{c+d x^4}}{\sqrt{b c-a d}}\right]}{2 a^2}$$

Result (type 6, 407 leaves):

$$\left(\left(6 b c d x^8 \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^4}{c},-\frac{b x^4}{a}\right]\right)\Big/\left(-4 a c \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^4}{c},-\frac{b x^4}{a}\right]+x^4\left(2 b c \text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d x^4}{c},-\frac{b x^4}{a}\right]+a d \text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d x^4}{c},-\frac{b x^4}{a}\right]\right)\right)+\left(5 b d x^4\left(3 a c+b c x^4+4 a d x^4+3 b d x^8\right) \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^4},-\frac{a}{b x^4}\right]-3\left(a+b x^4\right)\left(c+d x^4\right)\left(2 a d \text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^4},-\frac{a}{b x^4}\right]+b c \text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^4},-\frac{a}{b x^4}\right]\right)\right)\Big/\left(a\left(-5 b d x^4 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^4},-\frac{a}{b x^4}\right]+2 a d \text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^4},-\frac{a}{b x^4}\right]+b c \text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^4},-\frac{a}{b x^4}\right]\right)\right)\right)\Big/\left(12 x^4\left(a+b x^4\right)\sqrt{c+d x^4}\right)$$

Problem 627: Result unnecessarily involves higher level functions.

$$\int \frac{x^6 \sqrt{c+d x^4}}{a+b x^4}\, dx$$

Optimal (type 4, 857 leaves, 13 steps):

$$\frac{x^3\sqrt{c+d x^4}}{5 b}+\frac{(2 b c-5 a d)\, x\sqrt{c+d x^4}}{5 b^2\sqrt{d}\left(\sqrt{c}+\sqrt{d}\, x^2\right)}-$$

$$\frac{a\sqrt{-\frac{b c-a d}{\sqrt{-a}\sqrt{b}}}\,\text{ArcTan}\left[\frac{\sqrt{-\frac{b c-a d}{\sqrt{-a}\sqrt{b}}}\, x}{\sqrt{c+d x^4}}\right]}{4 b^2}-\frac{a\sqrt{\frac{b c-a d}{\sqrt{-a}\sqrt{b}}}\,\text{ArcTan}\left[\frac{\sqrt{\frac{b c-a d}{\sqrt{-a}\sqrt{b}}}\, x}{\sqrt{c+d x^4}}\right]}{4 b^2}-$$

$$\left(c^{1/4}(2 b c-5 a d)\left(\sqrt{c}+\sqrt{d}\, x^2\right)\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\, x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(5 b^2 d^{3/4}\sqrt{c+d x^4}\right)+\left(c^{1/4}\left(b^2 c^2+a b c d-5 a^2 d^2\right)\left(\sqrt{c}+\sqrt{d}\, x^2\right)\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\, x^2\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(5 b^2 d^{3/4}(b c+a d)\sqrt{c+d x^4}\right)+$$

$$\left(a\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)(b c-a d)\left(\sqrt{c}+\sqrt{d}\, x^2\right)\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\, x^2\right)^2}}\right.$$

$$\left.\text{EllipticPi}\left[\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},\, 2\,\text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(8 b^{5/2} c^{1/4}\left(\sqrt{-a}\sqrt{b}\sqrt{c}-a\sqrt{d}\right) d^{1/4}\sqrt{c+d x^4}\right)-$$

$$\left(a\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)(b c-a d)\left(\sqrt{c}+\sqrt{d}\, x^2\right)\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\, x^2\right)^2}}\right.$$

$$\left.\text{EllipticPi}\left[-\frac{\sqrt{c}\left(\sqrt{b}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{c}}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{d}},\, 2\,\text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(8 b^{5/2} c^{1/4}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right) d^{1/4}\sqrt{c+d x^4}\right)$$

Result (type 6, 428 leaves) :

$$\left(x^3 \left(\left(49\, a^2\, c^2\, \text{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\frac{d\, x^4}{c}, -\frac{b\, x^4}{a}\right]\right) \Big/\right.\right.$$

$$\left(-7\, a\, c\, \text{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\frac{d\, x^4}{c}, -\frac{b\, x^4}{a}\right] + 2\, x^4 \left(2\, b\, c\, \text{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2,\right.\right.\right.$$

$$\left.\left.\left.\frac{11}{4}, -\frac{d\, x^4}{c}, -\frac{b\, x^4}{a}\right] + a\, d\, \text{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, -\frac{d\, x^4}{c}, -\frac{b\, x^4}{a}\right]\right)\right) +$$

$$\left(-11\, a\, c\, \left(7\, a\, c + 9\, b\, c\, x^4 + 2\, a\, d\, x^4 + 7\, b\, d\, x^8\right) \text{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\frac{d\, x^4}{c}, -\frac{b\, x^4}{a}\right] +\right.$$

$$14\, x^4\, \left(a + b\, x^4\right)\, \left(c + d\, x^4\right) \left(2\, b\, c\, \text{AppellF1}\left[\frac{11}{4}, \frac{1}{2}, 2, \frac{15}{4}, -\frac{d\, x^4}{c}, -\frac{b\, x^4}{a}\right] +\right.$$

$$\left.\left.a\, d\, \text{AppellF1}\left[\frac{11}{4}, \frac{3}{2}, 1, \frac{15}{4}, -\frac{d\, x^4}{c}, -\frac{b\, x^4}{a}\right]\right)\right) \Big/$$

$$\left(-11\, a\, c\, \text{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, -\frac{d\, x^4}{c}, -\frac{b\, x^4}{a}\right] +\right.$$

$$2\, x^4 \left(2\, b\, c\, \text{AppellF1}\left[\frac{11}{4}, \frac{1}{2}, 2, \frac{15}{4}, -\frac{d\, x^4}{c}, -\frac{b\, x^4}{a}\right] +\right.$$

$$\left.\left.\left.\left.a\, d\, \text{AppellF1}\left[\frac{11}{4}, \frac{3}{2}, 1, \frac{15}{4}, -\frac{d\, x^4}{c}, -\frac{b\, x^4}{a}\right]\right)\right)\right)\right) \Big/ \left(35\, b\, \left(a + b\, x^4\right)\, \sqrt{c + d\, x^4}\right)$$

Problem 628: Result unnecessarily involves higher level functions.

$$\int \frac{x^4\, \sqrt{c + d\, x^4}}{a + b\, x^4}\, dx$$

Optimal (type 4, 700 leaves, 10 steps):

$$\frac{x\sqrt{c+d x^4}}{3 b}-\frac{(b c-a d)\,\text{ArcTan}\left[\frac{\sqrt{\frac{\sqrt{-a}\left(\frac{b c}{a}-d\right)}{\sqrt{b}}}\,x}{\sqrt{c+d x^4}}\right]}{4 b^2\sqrt{-\frac{b c-a d}{\sqrt{-a}\sqrt{b}}}}-\frac{(b c-a d)\,\text{ArcTan}\left[\frac{\sqrt{\frac{b c-a d}{\sqrt{-a}\sqrt{b}}}\,x}{\sqrt{c+d x^4}}\right]}{4 b^2\sqrt{\frac{b c-a d}{\sqrt{-a}\sqrt{b}}}}+$$

$$\left(c^{3/4}(b c-2 a d)\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(3 b\, d^{1/4}(b c+a d)\sqrt{c+d x^4}\right)-\left(\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)(b c-a d)\left(\sqrt{c}+\sqrt{d}\,x^2\right)\right.$$

$$\left.\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,\text{EllipticPi}\left[-\frac{\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2}{4\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}},2\,\text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(8 b^2 c^{1/4}\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right) d^{1/4}\sqrt{c+d x^4}\right)-$$

$$\left(\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)(b c-a d)\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\right.$$

$$\left.\text{EllipticPi}\left[\frac{\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2}{4\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}},2\,\text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(8 b^2 c^{1/4}\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right) d^{1/4}\sqrt{c+d x^4}\right)$$

Result (type 6, 426 leaves) :

$$\left(x\left(\left(25 a^2 c^2 \text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]\right)\Big/\right.\right.$$
$$\left(-5 a c \text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]+2 x^4\left(2 b c\right.\right.$$
$$\left.\left.\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},2,\frac{9}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]+a d \text{AppellF1}\left[\frac{5}{4},\frac{3}{2},1,\frac{9}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]\right)\right)+$$
$$\left(-9 a c\left(5 a c+7 b c x^4+2 a d x^4+5 b d x^8\right) \text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]+\right.$$
$$10 x^4\left(a+b x^4\right)\left(c+d x^4\right)\left(2 b c \text{AppellF1}\left[\frac{9}{4},\frac{1}{2},2,\frac{13}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]+\right.$$
$$\left.\left.a d \text{AppellF1}\left[\frac{9}{4},\frac{3}{2},1,\frac{13}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]\right)\right)\Big/\left(-9 a c \text{AppellF1}\left[\frac{5}{4},\frac{1}{2},\right.\right.$$
$$1,\frac{9}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]+2 x^4\left(2 b c \text{AppellF1}\left[\frac{9}{4},\frac{1}{2},2,\frac{13}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]+\right.$$
$$\left.\left.\left.\left.a d \text{AppellF1}\left[\frac{9}{4},\frac{3}{2},1,\frac{13}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]\right)\right)\right)\right)\Big/\left(15 b\left(a+b x^4\right)\sqrt{c+d x^4}\right)$$

Problem 629: Result unnecessarily involves higher level functions.

$$\int \frac{x^2\sqrt{c+d x^4}}{a+b x^4}\,dx$$

Optimal (type 4, 786 leaves, 11 steps):

$$\frac{\sqrt{d}\, x \sqrt{c+d x^4}}{b\left(\sqrt{c}+\sqrt{d}\, x^2\right)}+\frac{\sqrt{-\frac{b c-a d}{\sqrt{-a}\sqrt{b}}}\,\text{ArcTan}\left[\frac{\sqrt{-\frac{b c-a d}{\sqrt{-a}\sqrt{b}}}\, x}{\sqrt{c+d x^4}}\right]}{4 b}+\frac{\sqrt{\frac{b c-a d}{\sqrt{-a}\sqrt{b}}}\,\text{ArcTan}\left[\frac{\sqrt{\frac{b c-a d}{\sqrt{-a}\sqrt{b}}}\, x}{\sqrt{c+d x^4}}\right]}{4 b}-$$

$$\frac{1}{b\sqrt{c+d x^4}} c^{1/4} d^{1/4}\left(\sqrt{c}+\sqrt{d}\, x^2\right)\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\, x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right],\frac{1}{2}\right]+$$

$$\left(a\, c^{1/4} d^{5/4}\left(\sqrt{c}+\sqrt{d}\, x^2\right)\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\, x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(b\left(b c+a d\right)\sqrt{c+d x^4}\right)-\left(\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\left(b c-a d\right)\left(\sqrt{c}+\sqrt{d}\, x^2\right)\right.$$

$$\left.\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\, x^2\right)^2}}\,\text{EllipticPi}\left[\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\,\text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(8 b^{3/2} c^{1/4}\left(\sqrt{-a}\sqrt{b}\sqrt{c}-a\sqrt{d}\right) d^{1/4}\sqrt{c+d x^4}\right)+$$

$$\left(\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\left(b c-a d\right)\left(\sqrt{c}+\sqrt{d}\, x^2\right)\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\, x^2\right)^2}}\right.$$

$$\left.\text{EllipticPi}\left[-\frac{\sqrt{c}\left(\sqrt{b}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{c}}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{d}},2\,\text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(8 b^{3/2} c^{1/4}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right) d^{1/4}\sqrt{c+d x^4}\right)$$

Result (type 6, 165 leaves):

$$\left(7 a c x^3\sqrt{c+d x^4}\,\text{AppellF1}\left[\frac{3}{4},-\frac{1}{2},1,\frac{7}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]\right)\Big/$$

$$\left(3\left(a+b x^4\right)\left(7 a c\,\text{AppellF1}\left[\frac{3}{4},-\frac{1}{2},1,\frac{7}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]+2 x^4\left(-2 b c\right.\right.\right.$$

$$\left.\left.\left.\text{AppellF1}\left[\frac{7}{4},-\frac{1}{2},2,\frac{11}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]+a d\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},1,\frac{11}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]\right)\right)\right)$$

Problem 630: Result unnecessarily involves higher level functions.

$$\int\frac{\sqrt{c+d x^4}}{a+b x^4}\,dx$$

Optimal (type 4, 679 leaves, 9 steps):

$$\frac{(b c-a d) \operatorname{ArcTan}\left[\frac{\sqrt{\frac{\sqrt{-a}\left(\frac{b c}{a}-d\right)}{\sqrt{b}}}\, x}{\sqrt{c+d x^4}}\right]}{4 a b \sqrt{-\frac{b c-a d}{\sqrt{-a} \sqrt{b}}}}+\frac{(b c-a d) \operatorname{ArcTan}\left[\frac{\sqrt{\frac{b c-a d}{\sqrt{-a} \sqrt{b}}}\, x}{\sqrt{c+d x^4}}\right]}{4 a b \sqrt{\frac{b c-a d}{\sqrt{-a} \sqrt{b}}}}+$$

$$\left(c^{3/4} d^{3/4} \left(\sqrt{c}+\sqrt{d}\, x^2\right) \sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\, x^2\right)^2}}\ \text{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right], \frac{1}{2}\right]\right) \Big/$$

$$\left((b c+a d) \sqrt{c+d x^4}\right)+\left(\left(\sqrt{b} \sqrt{c}+\sqrt{-a} \sqrt{d}\right) (b c-a d) \left(\sqrt{c}+\sqrt{d}\, x^2\right)\right.$$

$$\left.\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\, x^2\right)^2}}\ \text{EllipticPi}\left[-\frac{\left(\sqrt{b} \sqrt{c}-\sqrt{-a} \sqrt{d}\right)^2}{4 \sqrt{-a} \sqrt{b} \sqrt{c} \sqrt{d}}, 2 \operatorname{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right], \frac{1}{2}\right]\right) \Big/$$

$$\left(8 a b c^{1/4} \left(\sqrt{b} \sqrt{c}-\sqrt{-a} \sqrt{d}\right) d^{1/4} \sqrt{c+d x^4}\right)+$$

$$\left(\left(\sqrt{b} \sqrt{c}-\sqrt{-a} \sqrt{d}\right) (b c-a d) \left(\sqrt{c}+\sqrt{d}\, x^2\right) \sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\, x^2\right)^2}}\right.$$

$$\left.\text{EllipticPi}\left[\frac{\left(\sqrt{b} \sqrt{c}+\sqrt{-a} \sqrt{d}\right)^2}{4 \sqrt{-a} \sqrt{b} \sqrt{c} \sqrt{d}}, 2 \operatorname{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right], \frac{1}{2}\right]\right) \Big/$$

$$\left(8 a b c^{1/4} \left(\sqrt{b} \sqrt{c}+\sqrt{-a} \sqrt{d}\right) d^{1/4} \sqrt{c+d x^4}\right)$$

Result (type 6, 161 leaves):

$$\left(5 a c x \sqrt{c+d x^4}\ \text{AppellF1}\left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, -\frac{d x^4}{c}, -\frac{b x^4}{a}\right]\right) \Big/$$

$$\left(\left(a+b x^4\right) \left(5 a c\ \text{AppellF1}\left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, -\frac{d x^4}{c}, -\frac{b x^4}{a}\right]+2 x^4 \left(-2 b c\right.\right.\right.$$

$$\left.\left.\left.\text{AppellF1}\left[\frac{5}{4}, -\frac{1}{2}, 2, \frac{9}{4}, -\frac{d x^4}{c}, -\frac{b x^4}{a}\right]+a d\ \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, -\frac{d x^4}{c}, -\frac{b x^4}{a}\right]\right)\right)\right)$$

Problem 631: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{c+d x^4}}{x^2 \left(a+b x^4\right)}\, dx$$

Optimal (type 4, 809 leaves, 13 steps):

$$-\frac{\sqrt{c+d\,x^4}}{a\,x}+\frac{\sqrt{d}\;x\,\sqrt{c+d\,x^4}}{a\,\left(\sqrt{c}+\sqrt{d}\;x^2\right)}-\frac{\sqrt{-\frac{b\,c-a\,d}{\sqrt{-a}\,\sqrt{b}}}\;\text{ArcTan}\left[\frac{\sqrt{-\frac{b\,c-a\,d}{\sqrt{-a}\,\sqrt{b}}}\;x}{\sqrt{c+d\,x^4}}\right]}{4\,a}-$$

$$\frac{\sqrt{\frac{b\,c-a\,d}{\sqrt{-a}\,\sqrt{b}}}\;\text{ArcTan}\left[\frac{\sqrt{\frac{b\,c-a\,d}{\sqrt{-a}\,\sqrt{b}}}\;x}{\sqrt{c+d\,x^4}}\right]}{4\,a}-\frac{1}{a\,\sqrt{c+d\,x^4}}$$

$$c^{1/4}\,d^{1/4}\,\left(\sqrt{c}+\sqrt{d}\;x^2\right)\,\sqrt{\frac{c+d\,x^4}{\left(\sqrt{c}+\sqrt{d}\;x^2\right)^2}}\;\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}\,x}{c^{1/4}}\right],\frac{1}{2}\right]+$$

$$\left(b\,c^{5/4}\,d^{1/4}\,\left(\sqrt{c}+\sqrt{d}\;x^2\right)\,\sqrt{\frac{c+d\,x^4}{\left(\sqrt{c}+\sqrt{d}\;x^2\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}\,x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(a\,\left(b\,c+a\,d\right)\,\sqrt{c+d\,x^4}\right)-\left(\left(\sqrt{b}\;\sqrt{c}-\sqrt{-a}\;\sqrt{d}\right)\,\left(b\,c-a\,d\right)\,\left(\sqrt{c}+\sqrt{d}\;x^2\right)\right.$$

$$\left.\sqrt{\frac{c+d\,x^4}{\left(\sqrt{c}+\sqrt{d}\;x^2\right)^2}}\;\text{EllipticPi}\left[\frac{\left(\sqrt{b}\;\sqrt{c}+\sqrt{-a}\;\sqrt{d}\right)^2}{4\,\sqrt{-a}\;\sqrt{b}\;\sqrt{c}\;\sqrt{d}},\,2\,\text{ArcTan}\left[\frac{d^{1/4}\,x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(8\,\sqrt{b}\;c^{1/4}\,\left((-a)^{3/2}\,\sqrt{b}\;\sqrt{c}+a^2\,\sqrt{d}\right)\,d^{1/4}\,\sqrt{c+d\,x^4}\right)-$$

$$\left(\left(\sqrt{b}\;\sqrt{c}+\sqrt{-a}\;\sqrt{d}\right)\,\left(b\,c-a\,d\right)\,\left(\sqrt{c}+\sqrt{d}\;x^2\right)\,\sqrt{\frac{c+d\,x^4}{\left(\sqrt{c}+\sqrt{d}\;x^2\right)^2}}\right.$$

$$\left.\text{EllipticPi}\left[-\frac{\sqrt{c}\,\left(\sqrt{b}-\frac{\sqrt{-a}\,\sqrt{d}}{\sqrt{c}}\right)^2}{4\,\sqrt{-a}\;\sqrt{b}\;\sqrt{d}},\,2\,\text{ArcTan}\left[\frac{d^{1/4}\,x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(8\,a\,\sqrt{b}\;c^{1/4}\,\left(\sqrt{-a}\;\sqrt{b}\;\sqrt{c}+a\,\sqrt{d}\right)\,d^{1/4}\,\sqrt{c+d\,x^4}\right)$$

Result (type 6, 343 leaves) :

$$\frac{1}{21\,x\,\sqrt{c+d\,x^4}}\left(-\frac{21\,\left(c+d\,x^4\right)}{a}+\left(49\,c\,\left(b\,c-2\,a\,d\right)\,x^4\,\text{AppellF1}\left[\frac{3}{4},\frac{1}{2},1,\frac{7}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\Big/\left(\left(a+b\,x^4\right)\left(-7\,a\,c\,\text{AppellF1}\left[\frac{3}{4},\frac{1}{2},1,\frac{7}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+2\,x^4\,\left(2\,b\,c\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},2,\frac{11}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{7}{4},\frac{3}{2},1,\frac{11}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\right)\right)-\left(33\,b\,c\,d\,x^8\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},1,\frac{11}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\Big/\left(\left(a+b\,x^4\right)\left(-11\,a\,c\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},1,\frac{11}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+2\,x^4\,\left(2\,b\,c\,\text{AppellF1}\left[\frac{11}{4},\frac{1}{2},2,\frac{15}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{11}{4},\frac{3}{2},1,\frac{15}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\right)\right)\right)$$

Problem 632: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{c+d\,x^4}}{x^4\,\left(a+b\,x^4\right)}\,dx$$

Optimal (type 4, 703 leaves, 10 steps):

$$-\frac{\sqrt{c+d x^4}}{3 a x^3}-\frac{(b c-a d)\,\text{ArcTan}\left[\frac{\sqrt{\frac{\sqrt{-a}\left(\frac{b c}{a}-d\right)}{\sqrt{b}}}\,x}{\sqrt{c+d x^4}}\right]}{4 a^2 \sqrt{-\frac{b c-a d}{\sqrt{-a}\,\sqrt{b}}}}-\frac{(b c-a d)\,\text{ArcTan}\left[\frac{\sqrt{\frac{b c-a d}{\sqrt{-a}\,\sqrt{b}}}\,x}{\sqrt{c+d x^4}}\right]}{4 a^2 \sqrt{\frac{b c-a d}{\sqrt{-a}\,\sqrt{b}}}}-$$

$$\left(d^{3/4}\,(2 b c-a d)\,\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(3 a c^{1/4}\,(b c+a d)\,\sqrt{c+d x^4}\right)-\left(\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)(b c-a d)\left(\sqrt{c}+\sqrt{d}\,x^2\right)\right.$$

$$\left.\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\;\text{EllipticPi}\left[-\frac{\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2}{4\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}},2\,\text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(8 a^2 c^{1/4}\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right) d^{1/4}\sqrt{c+d x^4}\right)-$$

$$\left(\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)(b c-a d)\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\right.$$

$$\left.\text{EllipticPi}\left[\frac{\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2}{4\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}},2\,\text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(8 a^2 c^{1/4}\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right) d^{1/4}\sqrt{c+d x^4}\right)$$

Result (type 6, 344 leaves) :

$$\frac{1}{15 x^3 \sqrt{c+d x^4}}$$

$$\left(-\frac{5\left(c+d x^4\right)}{a}+\left(25 c\,(3 b c-2 a d)\,x^4\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]\right)\Big/\left(\left(a+b x^4\right)\right.\right.$$

$$\left(-5 a c\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]+2 x^4\left(2 b c\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},2,\frac{9}{4},-\frac{d x^4}{c},\right.\right.\right.$$

$$\left.\left.\left.\left.-\frac{b x^4}{a}\right]+a d\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{2},1,\frac{9}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]\right)\right)\right)+$$

$$\left(9 b c d x^8\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]\right)\Big/\left(\left(a+b x^4\right)\right.$$

$$\left(-9 a c\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]+2 x^4\left(2 b c\,\text{AppellF1}\left[\frac{9}{4},\frac{1}{2},2,\right.\right.\right.$$

$$\left.\left.\left.\left.\left.\frac{13}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]+a d\,\text{AppellF1}\left[\frac{9}{4},\frac{3}{2},1,\frac{13}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]\right)\right)\right)\right)$$

Problem 633: Result more than twice size of optimal antiderivative.

$$\int \frac{(e\,x)^{3/2}\,\sqrt{c+d\,x^4}}{a+b\,x^4}\,dx$$

Optimal (type 6, 71 leaves, 3 steps):

$$\frac{2\,(e\,x)^{5/2}\,\sqrt{c+d\,x^4}\,\text{AppellF1}\left[\frac{5}{8},1,-\frac{1}{2},\frac{13}{8},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]}{5\,a\,e\,\sqrt{1+\frac{d\,x^4}{c}}}$$

Result (type 6, 170 leaves):

$$\left(26\,a\,c\,x\,(e\,x)^{3/2}\,\sqrt{c+d\,x^4}\,\text{AppellF1}\left[\frac{5}{8},-\frac{1}{2},1,\frac{13}{8},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\Big/ \left(5\,\left(a+b\,x^4\right)\left(13\,a\,c\,\text{AppellF1}\left[\frac{5}{8},-\frac{1}{2},1,\frac{13}{8},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+4\,x^4\left(-2\,b\,c\,\text{AppellF1}\left[\frac{13}{8},-\frac{1}{2},2,\frac{21}{8},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{13}{8},\frac{1}{2},1,\frac{21}{8},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\right)\right)$$

Problem 634: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{e\,x}\,\sqrt{c+d\,x^4}}{a+b\,x^4}\,dx$$

Optimal (type 6, 71 leaves, 3 steps):

$$\frac{2\,(e\,x)^{3/2}\,\sqrt{c+d\,x^4}\,\text{AppellF1}\left[\frac{3}{8},1,-\frac{1}{2},\frac{11}{8},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]}{3\,a\,e\,\sqrt{1+\frac{d\,x^4}{c}}}$$

Result (type 6, 170 leaves):

$$\left(22\,a\,c\,x\,\sqrt{e\,x}\,\sqrt{c+d\,x^4}\,\text{AppellF1}\left[\frac{3}{8},-\frac{1}{2},1,\frac{11}{8},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\Big/ \left(3\,\left(a+b\,x^4\right)\left(11\,a\,c\,\text{AppellF1}\left[\frac{3}{8},-\frac{1}{2},1,\frac{11}{8},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+4\,x^4\left(-2\,b\,c\,\text{AppellF1}\left[\frac{11}{8},-\frac{1}{2},2,\frac{19}{8},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{11}{8},\frac{1}{2},1,\frac{19}{8},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\right)\right)$$

Problem 635: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{c+d x^4}}{\sqrt{e x}\,\left(a+b x^4\right)}\,dx$$

Optimal (type 6, 69 leaves, 3 steps):

$$\frac{2\sqrt{e x}\,\sqrt{c+d x^4}\,\text{AppellF1}\left[\frac{1}{8},1,-\frac{1}{2},\frac{9}{8},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]}{a e \sqrt{1+\frac{d x^4}{c}}}$$

Result (type 6, 168 leaves):

$$\left(18 a c x \sqrt{c+d x^4}\,\text{AppellF1}\left[\frac{1}{8},-\frac{1}{2},1,\frac{9}{8},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]\right)\Big/$$
$$\left(\sqrt{e x}\,\left(a+b x^4\right)\left(9 a c\,\text{AppellF1}\left[\frac{1}{8},-\frac{1}{2},1,\frac{9}{8},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]+4 x^4\left(-2 b c\right.\right.\right.$$
$$\left.\left.\left.\text{AppellF1}\left[\frac{9}{8},-\frac{1}{2},2,\frac{17}{8},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]+a d\,\text{AppellF1}\left[\frac{9}{8},\frac{1}{2},1,\frac{17}{8},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]\right)\right)\right)$$

Problem 636: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{c+d x^4}}{(e x)^{3/2}\,\left(a+b x^4\right)}\,dx$$

Optimal (type 6, 69 leaves, 3 steps):

$$-\frac{2\sqrt{c+d x^4}\,\text{AppellF1}\left[-\frac{1}{8},1,-\frac{1}{2},\frac{7}{8},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]}{a e \sqrt{e x}\,\sqrt{1+\frac{d x^4}{c}}}$$

Result (type 6, 348 leaves):

$$\left(2 x \left(-\frac{35 \left(c+d x^4\right)}{a}+\left(75 c \left(b c-4 a d\right) x^4 \text{AppellF1}\left[\frac{7}{8},\frac{1}{2},1,\frac{15}{8},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]\right)\Big/\left(\left(a+b x^4\right) \left(-15 a c \text{AppellF1}\left[\frac{7}{8},\frac{1}{2},1,\frac{15}{8},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]+4 x^4 \left(2 b c \text{AppellF1}\left[\frac{15}{8},\frac{1}{2},2,\frac{23}{8},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]+a d \text{AppellF1}\left[\frac{15}{8},\frac{3}{2},1,\frac{23}{8},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]\right)\right)\right)-\left(161 b c d x^8 \text{AppellF1}\left[\frac{15}{8},\frac{1}{2},1,\frac{23}{8},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]\right)\Big/\left(\left(a+b x^4\right) \left(-23 a c \text{AppellF1}\left[\frac{15}{8},\frac{1}{2},1,\frac{23}{8},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]+4 x^4 \left(2 b c \text{AppellF1}\left[\frac{23}{8},\frac{1}{2},2,\frac{31}{8},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]+a d \text{AppellF1}\left[\frac{23}{8},\frac{3}{2},1,\frac{31}{8},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]\right)\right)\right)\right)\right)\Big/\left(35 (e x)^{3/2} \sqrt{c+d x^4}\right)$$

Problem 640: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x \left(a+b x^4\right) \sqrt{c+d x^4}} \, dx$$

Optimal (type 3, 85 leaves, 6 steps):

$$-\frac{\text{ArcTanh}\left[\frac{\sqrt{c+d x^4}}{\sqrt{c}}\right]}{2 a \sqrt{c}}+\frac{\sqrt{b} \text{ArcTanh}\left[\frac{\sqrt{b} \sqrt{c+d x^4}}{\sqrt{b c-a d}}\right]}{2 a \sqrt{b c-a d}}$$

Result (type 6, 162 leaves):

$$\left(5 b d x^4 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^4},-\frac{a}{b x^4}\right]\right)\Big/\left(6 \left(a+b x^4\right) \sqrt{c+d x^4} \left(-5 b d x^4 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^4},-\frac{a}{b x^4}\right]+2 a d \text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^4},-\frac{a}{b x^4}\right]+b c \text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^4},-\frac{a}{b x^4}\right]\right)\right)$$

Problem 641: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{x^5 \left(a+b x^4\right) \sqrt{c+d x^4}} \, dx$$

Optimal (type 3, 117 leaves, 7 steps):

$$-\frac{\sqrt{c+d x^4}}{4 a c x^4}+\frac{\left(2 b c+a d\right) \text{ArcTanh}\left[\frac{\sqrt{c+d x^4}}{\sqrt{c}}\right]}{4 a^2 c^{3/2}}-\frac{b^{3/2} \text{ArcTanh}\left[\frac{\sqrt{b} \sqrt{c+d x^4}}{\sqrt{b c-a d}}\right]}{2 a^2 \sqrt{b c-a d}}$$

Result (type 6, 409 leaves) :

$$\left(\left(6\, b\, d\, x^8\, \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\Big/\left(-4\,a\,c\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+x^4\left(2\,b\,c\,\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+a\,d\,\text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\right)+\left(5\,b\,d\,x^4\left(3\,a\,c+b\,c\,x^4+2\,a\,d\,x^4+3\,b\,d\,x^8\right)\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^4},-\frac{a}{b\,x^4}\right]-3\left(a+b\,x^4\right)\left(c+d\,x^4\right)\left(2\,a\,d\,\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d\,x^4},-\frac{a}{b\,x^4}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d\,x^4},-\frac{a}{b\,x^4}\right]\right)\right)\Big/\left(a\,c\left(-5\,b\,d\,x^4\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^4},-\frac{a}{b\,x^4}\right]+2\,a\,d\,\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d\,x^4},-\frac{a}{b\,x^4}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d\,x^4},-\frac{a}{b\,x^4}\right]\right)\right)\right)\Big/\left(12\,x^4\left(a+b\,x^4\right)\sqrt{c+d\,x^4}\right)$$

Problem 647: Result unnecessarily involves higher level functions.

$$\int \frac{x^8}{\left(a+b\,x^4\right)\sqrt{c+d\,x^4}}\,dx$$

Optimal (type 4, 872 leaves, 10 steps) :

$$\frac{x\sqrt{c+d x^4}}{3 b d}-\frac{(-a)^{5/4}\,\text{ArcTan}\left[\frac{\sqrt{b c-a d}\, x}{(-a)^{1/4} b^{1/4}\sqrt{c+d x^4}}\right]}{4 b^{7/4}\sqrt{b c-a d}}-\frac{(-a)^{5/4}\,\text{ArcTan}\left[\frac{\sqrt{-b c+a d}\, x}{(-a)^{1/4} b^{1/4}\sqrt{c+d x^4}}\right]}{4 b^{7/4}\sqrt{-b c+a d}}+$$

$$\left(a^2\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right) d^{1/4}\left(\sqrt{c}+\sqrt{d}\, x^2\right)\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\, x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(4 b^2 c^{1/4}(b c+a d)\sqrt{c+d x^4}\right)+\left(a\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right) d^{1/4}\left(\sqrt{c}+\sqrt{d}\, x^2\right)\right.$$

$$\left.\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\, x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(4 b^2 c^{1/4}(b c+a d)\sqrt{c+d x^4}\right)-$$

$$\left((b c+3 a d)\left(\sqrt{c}+\sqrt{d}\, x^2\right)\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\, x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(6 b^2 c^{1/4} d^{5/4}\sqrt{c+d x^4}\right)+$$

$$\left(a\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}\, x^2\right)\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\, x^2\right)^2}}\,\text{EllipticPi}\left[\right.\right.$$

$$\left.\left.-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},\,2\,\text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(8 b^2 c^{1/4} d^{1/4}(b c+a d)\sqrt{c+d x^4}\right)+$$

$$\left(a\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}\, x^2\right)\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\, x^2\right)^2}}\,\text{EllipticPi}\left[\right.\right.$$

$$\left.\left.\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},\,2\,\text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(8 b^2 c^{1/4} d^{1/4}(b c+a d)\sqrt{c+d x^4}\right)$$

Result (type 6, 429 leaves) :

$$\left(x\left(\left(25\,a^2\,c^2\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\Big/\right.\right.$$
$$\left(-5\,a\,c\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+2\,x^4\left(2\,b\,c\right.\right.$$
$$\left.\left.\left.\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},2,\frac{9}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{2},1,\frac{9}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\right)\right)+$$
$$\left(-9\,a\,c\left(5\,a\,c+4\,b\,c\,x^4+2\,a\,d\,x^4+5\,b\,d\,x^8\right)\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+\right.$$
$$10\,x^4\left(a+b\,x^4\right)\left(c+d\,x^4\right)\left(2\,b\,c\,\text{AppellF1}\left[\frac{9}{4},\frac{1}{2},2,\frac{13}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+\right.$$
$$\left.\left.a\,d\,\text{AppellF1}\left[\frac{9}{4},\frac{3}{2},1,\frac{13}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\right)\Big/\left(-9\,a\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},\right.\right.$$
$$1,\frac{9}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+2\,x^4\left(2\,b\,c\,\text{AppellF1}\left[\frac{9}{4},\frac{1}{2},2,\frac{13}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+\right.$$
$$\left.\left.\left.\left.a\,d\,\text{AppellF1}\left[\frac{9}{4},\frac{3}{2},1,\frac{13}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\right)\right)\right)\Big/\left(15\,b\,d\left(a+b\,x^4\right)\sqrt{c+d\,x^4}\right)$$

Problem 648: Result unnecessarily involves higher level functions.

$$\int\frac{x^4}{\left(a+b\,x^4\right)\sqrt{c+d\,x^4}}\,dx$$

Optimal (type 4, 638 leaves, 9 steps):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{\frac{\sqrt{-a}\left(\frac{b c}{a}-d\right)}{\sqrt{b}}}\ x}{\sqrt{c+d x^4}}\right]}{4 b \sqrt{-\frac{b c-a d}{\sqrt{-a}\ \sqrt{b}}}}-\frac{\text{ArcTan}\left[\frac{\sqrt{\frac{b c-a d}{\sqrt{-a}\ \sqrt{b}}}\ x}{\sqrt{c+d x^4}}\right]}{4 b \sqrt{\frac{b c-a d}{\sqrt{-a}\ \sqrt{b}}}}+$$

$$\frac{c^{3/4}\left(\sqrt{c}+\sqrt{d}\ x^2\right)\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\ x^2\right)^2}}\ \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right],\frac{1}{2}\right]}{2 d^{1/4}\left(b c+a d\right)\sqrt{c+d x^4}}-$$

$$\left(\left(\sqrt{b}\ \sqrt{c}+\sqrt{-a}\ \sqrt{d}\right)\left(\sqrt{c}+\sqrt{d}\ x^2\right)\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\ x^2\right)^2}}\right.$$

$$\left.\text{EllipticPi}\left[-\frac{\left(\sqrt{b}\ \sqrt{c}-\sqrt{-a}\ \sqrt{d}\right)^2}{4\sqrt{-a}\ \sqrt{b}\ \sqrt{c}\ \sqrt{d}},2 \text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(8 b c^{1/4}\left(\sqrt{b}\ \sqrt{c}-\sqrt{-a}\ \sqrt{d}\right) d^{1/4}\sqrt{c+d x^4}\right)-\left(\left(\sqrt{b}\ \sqrt{c}-\sqrt{-a}\ \sqrt{d}\right)\left(\sqrt{c}+\sqrt{d}\ x^2\right)\right.$$

$$\left.\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\ x^2\right)^2}}\ \text{EllipticPi}\left[\frac{\left(\sqrt{b}\ \sqrt{c}+\sqrt{-a}\ \sqrt{d}\right)^2}{4\sqrt{-a}\ \sqrt{b}\ \sqrt{c}\ \sqrt{d}},2 \text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(8 b c^{1/4}\left(\sqrt{b}\ \sqrt{c}+\sqrt{-a}\ \sqrt{d}\right) d^{1/4}\sqrt{c+d x^4}\right)$$

Result (type 6, 165 leaves):

$$-\left(\left(9 a c x^5 \text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]\right)\Big/\right.$$

$$\left(5\left(a+b x^4\right)\sqrt{c+d x^4}\left(-9 a c \text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]+2 x^4\left(2 b c \text{AppellF1}\left[\frac{9}{4},\right.\right.\right.\right.$$

$$\left.\left.\left.\left.\left.\frac{1}{2},2,\frac{13}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]+a d \text{AppellF1}\left[\frac{9}{4},\frac{3}{2},1,\frac{13}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]\right)\right)\right)\right)$$

Problem 649: Result unnecessarily involves higher level functions.

$$\int\frac{1}{\left(a+b x^4\right)\sqrt{c+d x^4}}\,dx$$

Optimal (type 4, 638 leaves, 7 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{\frac{\sqrt{-a}\left(\frac{b c}{a}-d\right)}{\sqrt{b}}}\, x}{\sqrt{c+d x^4}}\right]}{4 a \sqrt{-\frac{b c-a d}{\sqrt{-a}\sqrt{b}}}}+\frac{\text{ArcTan}\left[\frac{\sqrt{\frac{b c-a d}{\sqrt{-a}\sqrt{b}}}\, x}{\sqrt{c+d x^4}}\right]}{4 a \sqrt{\frac{b c-a d}{\sqrt{-a}\sqrt{b}}}}+$$

$$\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}\, x^2\right)\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\, x^2\right)^2}}\ \text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right],\frac{1}{2}\right]}{2 c^{1/4}\left(b c+a d\right)\sqrt{c+d x^4}}+$$

$$\left(\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\left(\sqrt{c}+\sqrt{d}\, x^2\right)\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\, x^2\right)^2}}\right.$$

$$\left.\text{EllipticPi}\left[-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\,\text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(8 a c^{1/4}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)d^{1/4}\sqrt{c+d x^4}\right)+\left(\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\left(\sqrt{c}+\sqrt{d}\, x^2\right)\right.$$

$$\left.\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\, x^2\right)^2}}\ \text{EllipticPi}\left[\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\,\text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(8 a c^{1/4}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)d^{1/4}\sqrt{c+d x^4}\right)$$

Result (type 6, 161 leaves):

$$-\left(\left(5 a c x\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]\right)\Big/\right.$$

$$\left(\left(a+b x^4\right)\sqrt{c+d x^4}\left(-5 a c\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]+2 x^4\left(2 b c\right.\right.\right.$$

$$\left.\left.\left.\left.\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},2,\frac{9}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]+a d\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{2},1,\frac{9}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]\right)\right)\right)\right)$$

Problem 650: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^4\left(a+b x^4\right)\sqrt{c+d x^4}}\, dx$$

Optimal (type 4, 677 leaves, 10 steps):

$$-\frac{\sqrt{c+d x^4}}{3 a c x^3}-\frac{b \operatorname{ArcTan}\left[\frac{\sqrt{\frac{\sqrt{-a}\left(\frac{b c}{a}-d\right)}{\sqrt{b}}}\, x}{\sqrt{c+d x^4}}\right]}{4 a^2 \sqrt{-\frac{b c-a d}{\sqrt{-a}\, \sqrt{b}}}}-\frac{b \operatorname{ArcTan}\left[\frac{\sqrt{\frac{b c-a d}{\sqrt{-a}\, \sqrt{b}}}\, x}{\sqrt{c+d x^4}}\right]}{4 a^2 \sqrt{\frac{b c-a d}{\sqrt{-a}\, \sqrt{b}}}}-$$

$$\left(d^{3/4} (4 b c+a d) \left(\sqrt{c}+\sqrt{d}\, x^2\right) \sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\, x^2\right)^2}}\ \text{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right], \frac{1}{2}\right]\right) \Big/$$

$$\left(6 a c^{5/4} (b c+a d) \sqrt{c+d x^4}\right)-\Bigg(b \left(\sqrt{b}\, \sqrt{c}+\sqrt{-a}\, \sqrt{d}\right) \left(\sqrt{c}+\sqrt{d}\, x^2\right)$$

$$\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\, x^2\right)^2}}\ \text{EllipticPi}\left[-\frac{\left(\sqrt{b}\, \sqrt{c}-\sqrt{-a}\, \sqrt{d}\right)^2}{4 \sqrt{-a}\, \sqrt{b}\, \sqrt{c}\, \sqrt{d}}, 2 \operatorname{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right], \frac{1}{2}\right]\Bigg) \Big/$$

$$\left(8 a^2 c^{1/4} \left(\sqrt{b}\, \sqrt{c}-\sqrt{-a}\, \sqrt{d}\right) d^{1/4} \sqrt{c+d x^4}\right)-\Bigg(b \left(\sqrt{b}\, \sqrt{c}-\sqrt{-a}\, \sqrt{d}\right) \left(\sqrt{c}+\sqrt{d}\, x^2\right)$$

$$\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\, x^2\right)^2}}\ \text{EllipticPi}\left[\frac{\left(\sqrt{b}\, \sqrt{c}+\sqrt{-a}\, \sqrt{d}\right)^2}{4 \sqrt{-a}\, \sqrt{b}\, \sqrt{c}\, \sqrt{d}}, 2 \operatorname{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right], \frac{1}{2}\right]\Bigg) \Big/$$

$$\left(8 a^2 c^{1/4} \left(\sqrt{b}\, \sqrt{c}+\sqrt{-a}\, \sqrt{d}\right) d^{1/4} \sqrt{c+d x^4}\right)$$

Result (type 6, 344 leaves):

$$\frac{1}{15 x^3 \sqrt{c+d x^4}}$$

$$\Bigg(-\frac{5 \left(c+d x^4\right)}{a c}+\left(25 (3 b c+a d) x^4 \text{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, -\frac{d x^4}{c}, -\frac{b x^4}{a}\right]\right) \Big/ \Bigg(\left(a+b x^4\right)$$

$$\Bigg(-5 a c\, \text{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, -\frac{d x^4}{c}, -\frac{b x^4}{a}\right]+2 x^4 \Bigg(2 b c\, \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, -\frac{d x^4}{c},\right.$$

$$\left.-\frac{b x^4}{a}\right]+a d\, \text{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, -\frac{d x^4}{c}, -\frac{b x^4}{a}\right]\Bigg)\Bigg)\Bigg)+$$

$$\left(9 b d x^8 \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, -\frac{d x^4}{c}, -\frac{b x^4}{a}\right]\right) \Big/ \Bigg(\left(a+b x^4\right)$$

$$\Bigg(-9 a c\, \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, -\frac{d x^4}{c}, -\frac{b x^4}{a}\right]+2 x^4 \Bigg(2 b c\, \text{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, 2,\right.$$

$$\left.\frac{13}{4}, -\frac{d x^4}{c}, -\frac{b x^4}{a}\right]+a d\, \text{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, -\frac{d x^4}{c}, -\frac{b x^4}{a}\right]\Bigg)\Bigg)\Bigg)\Bigg)$$

Problem 651: Result unnecessarily involves higher level functions.

$$\int \frac{x^6}{\left(a+b x^4\right)\sqrt{c+d x^4}}\,dx$$

Optimal (type 4, 804 leaves, 11 steps):

$$\frac{x\sqrt{c+d x^4}}{b\sqrt{d}\left(\sqrt{c}+\sqrt{d}\,x^2\right)}-\frac{a\sqrt{-\frac{b c-a d}{\sqrt{-a}\,\sqrt{b}}}\,\text{ArcTan}\left[\frac{\sqrt{-\frac{b c-a d}{\sqrt{-a}\,\sqrt{b}}}\,x}{\sqrt{c+d x^4}}\right]}{4 b\,(b c-a d)}-\frac{a\sqrt{\frac{b c-a d}{\sqrt{-a}\,\sqrt{b}}}\,\text{ArcTan}\left[\frac{\sqrt{\frac{b c-a d}{\sqrt{-a}\,\sqrt{b}}}\,x}{\sqrt{c+d x^4}}\right]}{4 b\,(b c-a d)}-$$

$$\frac{c^{1/4}\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right],\frac{1}{2}\right]}{b\,d^{3/4}\sqrt{c+d x^4}}+$$

$$\left(c^{1/4}(b c+2 a d)\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(2 b\,d^{3/4}(b c+a d)\sqrt{c+d x^4}\right)+\left(a\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)\left(\sqrt{c}+\sqrt{d}\,x^2\right)\right.$$

$$\left.\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,\text{EllipticPi}\left[\frac{\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2}{4\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}},2\,\text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(8 b^{3/2} c^{1/4}\left(\sqrt{-a}\,\sqrt{b}\,\sqrt{c}-a\sqrt{d}\right)d^{1/4}\sqrt{c+d x^4}\right)-\left(a\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)\left(\sqrt{c}+\sqrt{d}\,x^2\right)\right.$$

$$\left.\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,\text{EllipticPi}\left[-\frac{\sqrt{c}\left(\sqrt{b}-\frac{\sqrt{-a}\,\sqrt{d}}{\sqrt{c}}\right)^2}{4\sqrt{-a}\,\sqrt{b}\,\sqrt{d}},2\,\text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(8 b^{3/2} c^{1/4}\left(\sqrt{-a}\,\sqrt{b}\,\sqrt{c}+a\sqrt{d}\right)d^{1/4}\sqrt{c+d x^4}\right)$$

Result (type 6, 165 leaves):

$$-\left(\left(11 a c x^7\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},1,\frac{11}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]\right)\Big/\right.$$

$$\left(7\left(a+b x^4\right)\sqrt{c+d x^4}\left(-11 a c\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},1,\frac{11}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]+2 x^4\left(2 b c\,\text{AppellF1}\left[\right.\right.\right.\right.$$

$$\left.\left.\left.\left.\left.\frac{11}{4},\frac{1}{2},2,\frac{15}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]+a d\,\text{AppellF1}\left[\frac{11}{4},\frac{3}{2},1,\frac{15}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]\right)\right)\right)\right)$$

Problem 652: Result unnecessarily involves higher level functions.

$$\int \frac{x^2}{\left(a+b x^4\right) \sqrt{c+d x^4}} \, dx$$

Optimal (type 4, 656 leaves, 7 steps):

$$\frac{\sqrt{-\frac{b c-a d}{\sqrt{-a} \sqrt{b}}} \operatorname{ArcTan}\left[\frac{\sqrt{-\frac{b c-a d}{\sqrt{-a} \sqrt{b}}} x}{\sqrt{c+d x^4}}\right]}{4 (b c-a d)}+\frac{\sqrt{\frac{b c-a d}{\sqrt{-a} \sqrt{b}}} \operatorname{ArcTan}\left[\frac{\sqrt{\frac{b c-a d}{\sqrt{-a} \sqrt{b}}} x}{\sqrt{c+d x^4}}\right]}{4 (b c-a d)}-$$

$$\left(c^{1/4} d^{1/4} \left(\sqrt{c}+\sqrt{d} x^2\right) \sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d} x^2\right)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right], \frac{1}{2}\right]\right) \Big/$$

$$\left(2 (b c+a d) \sqrt{c+d x^4}\right)-\left(\left(\sqrt{b} \sqrt{c}-\sqrt{-a} \sqrt{d}\right) \left(\sqrt{c}+\sqrt{d} x^2\right)\right.$$

$$\left.\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d} x^2\right)^2}} \operatorname{EllipticPi}\left[\frac{\left(\sqrt{b} \sqrt{c}+\sqrt{-a} \sqrt{d}\right)^2}{4 \sqrt{-a} \sqrt{b} \sqrt{c} \sqrt{d}}, 2 \operatorname{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right], \frac{1}{2}\right]\right) \Big/$$

$$\left(8 \sqrt{b} c^{1/4} \left(\sqrt{-a} \sqrt{b} \sqrt{c}-a \sqrt{d}\right) d^{1/4} \sqrt{c+d x^4}\right)+\left(\left(\sqrt{b} \sqrt{c}+\sqrt{-a} \sqrt{d}\right) \left(\sqrt{c}+\sqrt{d} x^2\right)\right.$$

$$\left.\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d} x^2\right)^2}} \operatorname{EllipticPi}\left[-\frac{\sqrt{c} \left(\sqrt{b}-\frac{\sqrt{-a} \sqrt{d}}{\sqrt{c}}\right)^2}{4 \sqrt{-a} \sqrt{b} \sqrt{d}}, 2 \operatorname{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right], \frac{1}{2}\right]\right) \Big/$$

$$\left(8 \sqrt{b} c^{1/4} \left(\sqrt{-a} \sqrt{b} \sqrt{c}+a \sqrt{d}\right) d^{1/4} \sqrt{c+d x^4}\right)$$

Result (type 6, 165 leaves):

$$-\left(\left(7 a c x^3 \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\frac{d x^4}{c}, -\frac{b x^4}{a}\right]\right) \Big/\right.$$

$$\left(3 \left(a+b x^4\right) \sqrt{c+d x^4} \left(-7 a c \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\frac{d x^4}{c}, -\frac{b x^4}{a}\right]+2 x^4 \left(2 b c \operatorname{AppellF1}\left[\frac{7}{4},\right.\right.\right.\right.$$

$$\left.\left.\left.\left.\left.\frac{1}{2}, 2, \frac{11}{4}, -\frac{d x^4}{c}, -\frac{b x^4}{a}\right]+a d \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, -\frac{d x^4}{c}, -\frac{b x^4}{a}\right]\right)\right)\right)\right)$$

Problem 653: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^2 \left(a+b x^4\right) \sqrt{c+d x^4}} \, dx$$

Optimal (type 4, 833 leaves, 13 steps):

$$-\frac{\sqrt{c+d x^4}}{a c x}+\frac{\sqrt{d}\, x \sqrt{c+d x^4}}{a c \left(\sqrt{c}+\sqrt{d}\, x^2\right)}-\frac{b \sqrt{-\frac{b c-a d}{\sqrt{-a} \sqrt{b}}}\, \text{ArcTan}\left[\frac{\sqrt{-\frac{b c-a d}{\sqrt{-a} \sqrt{b}}}\, x}{\sqrt{c+d x^4}}\right]}{4 a (b c-a d)}-\frac{b \sqrt{\frac{b c-a d}{\sqrt{-a} \sqrt{b}}}\, \text{ArcTan}\left[\frac{\sqrt{\frac{b c-a d}{\sqrt{-a} \sqrt{b}}}\, x}{\sqrt{c+d x^4}}\right]}{4 a (b c-a d)}-$$

$$\frac{d^{1/4} \left(\sqrt{c}+\sqrt{d}\, x^2\right) \sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\, x^2\right)^2}}\, \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right], \frac{1}{2}\right]}{a c^{3/4} \sqrt{c+d x^4}}+$$

$$\left(d^{1/4} (2 b c+a d) \left(\sqrt{c}+\sqrt{d}\, x^2\right) \sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\, x^2\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right], \frac{1}{2}\right]\right) \Big/ \left(2 a c^{3/4} (b c+a d) \sqrt{c+d x^4}\right)+$$

$$\left(\sqrt{b} \left(\frac{\sqrt{b}\, c^{1/4}}{d^{1/4}}-\frac{\sqrt{-a}\, d^{1/4}}{c^{1/4}}\right) \left(\sqrt{c}+\sqrt{d}\, x^2\right) \sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\, x^2\right)^2}}\, \text{EllipticPi}\left[\frac{\left(\sqrt{b} \sqrt{c}+\sqrt{-a} \sqrt{d}\right)^2}{4 \sqrt{-a} \sqrt{b} \sqrt{c} \sqrt{d}}, 2 \text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right], \frac{1}{2}\right]\right) \Big/ \left(8 a \left(\sqrt{-a} \sqrt{b} \sqrt{c}-a \sqrt{d}\right) \sqrt{c+d x^4}\right)-$$

$$\left(\sqrt{b} \left(\frac{\sqrt{b}\, c^{1/4}}{d^{1/4}}+\frac{\sqrt{-a}\, d^{1/4}}{c^{1/4}}\right) \left(\sqrt{c}+\sqrt{d}\, x^2\right) \sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\, x^2\right)^2}}\, \text{EllipticPi}\left[-\frac{\sqrt{c} \left(\sqrt{b}-\frac{\sqrt{-a} \sqrt{d}}{\sqrt{c}}\right)^2}{4 \sqrt{-a} \sqrt{b} \sqrt{d}}, 2 \text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right], \frac{1}{2}\right]\right) \Big/ \left(8 a \left(\sqrt{-a} \sqrt{b} \sqrt{c}+a \sqrt{d}\right) \sqrt{c+d x^4}\right)$$

Result (type 6, 344 leaves):

$$\frac{1}{21\,x\,\sqrt{c+d\,x^4}}\left(-\frac{21\,(c+d\,x^4)}{a\,c}+\left(49\,(b\,c-a\,d)\,x^4\,\text{AppellF1}\left[\frac{3}{4},\frac{1}{2},1,\frac{7}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\Big/\left((a+b\,x^4)\left(-7\,a\,c\,\text{AppellF1}\left[\frac{3}{4},\frac{1}{2},1,\frac{7}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+2\,x^4\left(2\,b\,c\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},2,\frac{11}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{7}{4},\frac{3}{2},1,\frac{11}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\right)\right)-\left(33\,b\,d\,x^8\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},1,\frac{11}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\Big/\left((a+b\,x^4)\left(-11\,a\,c\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},1,\frac{11}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+2\,x^4\left(2\,b\,c\,\text{AppellF1}\left[\frac{11}{4},\frac{1}{2},2,\frac{15}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{11}{4},\frac{3}{2},1,\frac{15}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\right)\right)\right)$$

Problem 658: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\frac{1}{x\,(a+b\,x^4)^2\,\sqrt{c+d\,x^4}}\,dx$$

Optimal (type 3, 132 leaves, 7 steps):

$$\frac{b\,\sqrt{c+d\,x^4}}{4\,a\,(b\,c-a\,d)\,(a+b\,x^4)}-\frac{\text{ArcTanh}\left[\frac{\sqrt{c+d\,x^4}}{\sqrt{c}}\right]}{2\,a^2\,\sqrt{c}}+\frac{\sqrt{b}\,(2\,b\,c-3\,a\,d)\,\text{ArcTanh}\left[\frac{\sqrt{b}\,\sqrt{c+d\,x^4}}{\sqrt{b\,c-a\,d}}\right]}{4\,a^2\,(b\,c-a\,d)^{3/2}}$$

Result (type 6, 396 leaves):

$$\left(b\left(\left(6\,c\,d\,x^4\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\Big/\left(-4\,a\,c\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+x^4\left(2\,b\,c\,\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+a\,d\,\text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\right)+\left(5\,d\,x^4\,(2\,a\,d+b\,(c+3\,d\,x^4))\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^4},-\frac{a}{b\,x^4}\right]-3\,(c+d\,x^4)\left(2\,a\,d\,\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d\,x^4},-\frac{a}{b\,x^4}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d\,x^4},-\frac{a}{b\,x^4}\right]\right)\right)\Big/\left(a\left(-5\,b\,d\,x^4\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^4},-\frac{a}{b\,x^4}\right]+2\,a\,d\,\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d\,x^4},-\frac{a}{b\,x^4}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d\,x^4},-\frac{a}{b\,x^4}\right]\right)\right)\right)\right)\Big/\left(12\,(-b\,c+a\,d)\,(a+b\,x^4)\,\sqrt{c+d\,x^4}\right)$$

Problem 659: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{x^5 \left(a+b x^4\right)^2 \sqrt{c+d x^4}} \, dx$$

Optimal (type 3, 185 leaves, 8 steps):

$$-\frac{b (2 b c-a d) \sqrt{c+d x^4}}{4 a^2 c (b c-a d) \left(a+b x^4\right)}-\frac{\sqrt{c+d x^4}}{4 a c x^4 \left(a+b x^4\right)}+\frac{(4 b c+a d) \operatorname{ArcTanh}\left[\frac{\sqrt{c+d x^4}}{\sqrt{c}}\right]}{4 a^3 c^{3/2}}-\frac{b^{3/2} (4 b c-5 a d) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{c+d x^4}}{\sqrt{b c-a d}}\right]}{4 a^3 (b c-a d)^{3/2}}$$

Result (type 6, 489 leaves):

$$\left(\left(6 a b d (-2 b c+a d) x^8 \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^4}{c},-\frac{b x^4}{a}\right]\right)\Big/\left((-b c+a d) \left(-4 a c \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^4}{c},-\frac{b x^4}{a}\right]+x^4 \left(2 b c \text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d x^4}{c},-\frac{b x^4}{a}\right]+a d \text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d x^4}{c},-\frac{b x^4}{a}\right]\right)\right)\right)+\left(5 b d x^4 \left(-a^2 d \left(3 c+2 d x^4\right)+2 b^2 c x^4 \left(c+3 d x^4\right)+3 a b \left(c^2+c d x^4-d^2 x^8\right)\right) \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^4},-\frac{a}{b x^4}\right]+3 \left(c+d x^4\right) \left(a^2 d-2 b^2 c x^4+a b \left(-c+d x^4\right)\right) \left(2 a d \text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^4},-\frac{a}{b x^4}\right]+b c \text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^4},-\frac{a}{b x^4}\right]\right)\right)\Big/\left(c (b c-a d) \left(-5 b d x^4 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^4},-\frac{a}{b x^4}\right]+2 a d \text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^4},-\frac{a}{b x^4}\right]+b c \text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^4},-\frac{a}{b x^4}\right]\right)\right)\right)\Big/\left(12 a^2 x^4 \left(a+b x^4\right) \sqrt{c+d x^4}\right)$$

Problem 666: Result unnecessarily involves higher level functions.

$$\int \frac{x^8}{\left(a+b x^4\right)^2 \sqrt{c+d x^4}} \, dx$$

Optimal (type 4, 996 leaves, 10 steps):

$$\frac{a\,x\,\sqrt{c+d\,x^4}}{4\,b\,(b\,c-a\,d)\,(a+b\,x^4)}-\frac{(-a)^{1/4}\,(5\,b\,c-3\,a\,d)\,\text{ArcTan}\left[\frac{\sqrt{b\,c-a\,d}\;x}{(-a)^{1/4}\,b^{1/4}\,\sqrt{c+d\,x^4}}\right]}{16\,b^{7/4}\,(b\,c-a\,d)^{3/2}}+$$

$$\frac{(-a)^{1/4}\,(5\,b\,c-3\,a\,d)\,\text{ArcTan}\left[\frac{\sqrt{-b\,c+a\,d}\;x}{(-a)^{1/4}\,b^{1/4}\,\sqrt{c+d\,x^4}}\right]}{16\,b^{7/4}\,(-b\,c+a\,d)^{3/2}}+$$

$$\left((4\,b\,c-3\,a\,d)\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+d\,x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}\,x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(8\,b^2\,c^{1/4}\,d^{1/4}\,(b\,c-a\,d)\,\sqrt{c+d\,x^4}\right)-$$

$$\left(a\left(\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)d^{1/4}\,(5\,b\,c-3\,a\,d)\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+d\,x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}\,x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(16\,b^2\,c^{1/4}\,(b\,c-a\,d)\,(b\,c+a\,d)\,\sqrt{c+d\,x^4}\right)-$$

$$\left(\sqrt{-a}\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)d^{1/4}\,(5\,b\,c-3\,a\,d)\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+d\,x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}\,x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(16\,b^2\,c^{1/4}\,(b\,c-a\,d)\,(b\,c+a\,d)\,\sqrt{c+d\,x^4}\right)-$$

$$\left(\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2(5\,b\,c-3\,a\,d)\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+d\,x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\right.$$

$$\left.\text{EllipticPi}\left[-\frac{\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2}{4\,\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}},2\,\text{ArcTan}\left[\frac{d^{1/4}\,x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(32\,b^2\,c^{1/4}\,d^{1/4}\,(b\,c-a\,d)\,(b\,c+a\,d)\,\sqrt{c+d\,x^4}\right)-$$

$$\left(\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2(5\,b\,c-3\,a\,d)\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+d\,x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\right.$$

$$\left.\text{EllipticPi}\left[\frac{\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2}{4\,\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}},2\,\text{ArcTan}\left[\frac{d^{1/4}\,x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(32\, b^2\, c^{1/4}\, d^{1/4}\, (b\, c - a\, d)\, (b\, c + a\, d)\, \sqrt{c + d\, x^4}\right)$$

Result (type 6, 420 leaves):

$$\left(a\, x\, \left(\left(25\, a\, c^2\, \text{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, -\frac{d\, x^4}{c}, -\frac{b\, x^4}{a}\right]\right) \Big/ \left(-5\, a\, c\, \text{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, -\frac{d\, x^4}{c}, -\frac{b\, x^4}{a}\right] + 2\, x^4\, \left(2\, b\, c\, \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, -\frac{d\, x^4}{c}, -\frac{b\, x^4}{a}\right] + a\, d\, \text{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, -\frac{d\, x^4}{c}, -\frac{b\, x^4}{a}\right]\right)\right) + \left(-9\, c\, \left(5\, a\, c + 4\, b\, c\, x^4 + 2\, a\, d\, x^4\right)\, \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, -\frac{d\, x^4}{c}, -\frac{b\, x^4}{a}\right] + 10\, x^4\, \left(c + d\, x^4\right)\, \left(2\, b\, c\, \text{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, -\frac{d\, x^4}{c}, -\frac{b\, x^4}{a}\right] + a\, d\, \text{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, -\frac{d\, x^4}{c}, -\frac{b\, x^4}{a}\right]\right)\right) \Big/ \left(-9\, a\, c\, \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, -\frac{d\, x^4}{c}, -\frac{b\, x^4}{a}\right] + 2\, x^4\, \left(2\, b\, c\, \text{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, -\frac{d\, x^4}{c}, -\frac{b\, x^4}{a}\right] + a\, d\, \text{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, -\frac{d\, x^4}{c}, -\frac{b\, x^4}{a}\right]\right)\right)\right)\right) \Big/ \left(20\, b\, (b\, c - a\, d)\, \left(a + b\, x^4\right)\, \sqrt{c + d\, x^4}\right)$$

Problem 667: Result unnecessarily involves higher level functions.

$$\int \frac{x^4}{\left(a + b\, x^4\right)^2\, \sqrt{c + d\, x^4}}\, dx$$

Optimal (type 4, 908 leaves, 10 steps):

$$-\frac{x\sqrt{c+d x^4}}{4\left(b c-a d\right)\left(a+b x^4\right)}-\frac{\left(b c+a d\right)\text{ArcTan}\left[\frac{\sqrt{b c-a d}\,x}{(-a)^{1/4} b^{1/4}\sqrt{c+d x^4}}\right]}{16\,(-a)^{3/4} b^{3/4}\left(b c-a d\right)^{3/2}}+\frac{\left(b c+a d\right)\text{ArcTan}\left[\frac{\sqrt{-b c+a d}\,x}{(-a)^{1/4} b^{1/4}\sqrt{c+d x^4}}\right]}{16\,(-a)^{3/4} b^{3/4}\left(-b c+a d\right)^{3/2}}+$$

$$\left(\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)d^{1/4}\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(16\,b\,c^{1/4}\left(b c-a d\right)\sqrt{c+d x^4}\right)+\left(\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)d^{1/4}\left(\sqrt{c}+\sqrt{d}\,x^2\right)\right.$$

$$\left.\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(16\,a\,b\,c^{1/4}\left(b c-a d\right)\sqrt{c+d x^4}\right)-$$

$$\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right],\frac{1}{2}\right]}{8\,b\,c^{1/4}\left(b c-a d\right)\sqrt{c+d x^4}}+$$

$$\left(\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,\text{EllipticPi}\left[\right.\right.$$

$$\left.\left.-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\,\text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(32\,a\,b\,c^{1/4} d^{1/4}\left(b c-a d\right)\sqrt{c+d x^4}\right)+$$

$$\left(\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,\text{EllipticPi}\left[\right.\right.$$

$$\left.\left.\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\,\text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(32\,a\,b\,c^{1/4} d^{1/4}\left(b c-a d\right)\sqrt{c+d x^4}\right)$$

Result (type 6, 331 leaves):

$$\left(x\left(5\left(c+d x^4\right)+\left(25 a c^2 \text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]\right)\Big/\left(-5 a c \text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]+2 x^4\left(2 b c \text{AppellF1}\left[\frac{5}{4},\frac{1}{2},2,\frac{9}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]+a d \text{AppellF1}\left[\frac{5}{4},\frac{3}{2},1,\frac{9}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]\right)\right)-\left(9 a c d x^4 \text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]\right)\Big/\left(-9 a c \text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]+2 x^4\left(2 b c \text{AppellF1}\left[\frac{9}{4},\frac{1}{2},2,\frac{13}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]+a d \text{AppellF1}\left[\frac{9}{4},\frac{3}{2},1,\frac{13}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]\right)\right)\right)\right)\Big/\left(20\left(-b c+a d\right)\left(a+b x^4\right)\sqrt{c+d x^4}\right)$$

Problem 668: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(a+b x^4\right)^2 \sqrt{c+d x^4}}\,dx$$

Optimal (type 4, 983 leaves, 10 steps):

$$\frac{b\,x\,\sqrt{c+d\,x^4}}{4\,a\,\left(b\,c-a\,d\right)\,\left(a+b\,x^4\right)}+\frac{b^{1/4}\,\left(3\,b\,c-5\,a\,d\right)\,\text{ArcTan}\left[\frac{\sqrt{b\,c-a\,d}\,\,x}{(-a)^{1/4}\,b^{1/4}\,\sqrt{c+d\,x^4}}\right]}{16\,(-a)^{7/4}\,\left(b\,c-a\,d\right)^{3/2}}-$$

$$\frac{b^{1/4}\,\left(3\,b\,c-5\,a\,d\right)\,\text{ArcTan}\left[\frac{\sqrt{-b\,c+a\,d}\,\,x}{(-a)^{1/4}\,b^{1/4}\,\sqrt{c+d\,x^4}}\right]}{16\,(-a)^{7/4}\,\left(-b\,c+a\,d\right)^{3/2}}+$$

$$\frac{d^{3/4}\,\left(\sqrt{c}+\sqrt{d}\,\,x^2\right)\,\sqrt{\frac{c+d\,x^4}{\left(\sqrt{c}+\sqrt{d}\,\,x^2\right)^2}}\,\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}\,x}{c^{1/4}}\right],\,\frac{1}{2}\right]}{8\,a\,c^{1/4}\,\left(b\,c-a\,d\right)\,\sqrt{c+d\,x^4}}+$$

$$\left(\left(\frac{\sqrt{b}\,\,\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\,d^{1/4}\,\left(3\,b\,c-5\,a\,d\right)\,\left(\sqrt{c}+\sqrt{d}\,\,x^2\right)\,\sqrt{\frac{c+d\,x^4}{\left(\sqrt{c}+\sqrt{d}\,\,x^2\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}\,x}{c^{1/4}}\right],\,\frac{1}{2}\right]\right)\Big/\left(16\,a\,c^{1/4}\,\left(b\,c-a\,d\right)\,\left(b\,c+a\,d\right)\,\sqrt{c+d\,x^4}\right)+$$

$$\left(\left(\sqrt{b}\,\,\sqrt{c}-\sqrt{-a}\,\,\sqrt{d}\right)\,d^{1/4}\,\left(3\,b\,c-5\,a\,d\right)\,\left(\sqrt{c}+\sqrt{d}\,\,x^2\right)\,\sqrt{\frac{c+d\,x^4}{\left(\sqrt{c}+\sqrt{d}\,\,x^2\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}\,x}{c^{1/4}}\right],\,\frac{1}{2}\right]\right)\Big/\left(16\,(-a)^{3/2}\,c^{1/4}\,\left(b\,c-a\,d\right)\,\left(b\,c+a\,d\right)\,\sqrt{c+d\,x^4}\right)+$$

$$\left(\left(\sqrt{b}\,\,\sqrt{c}+\sqrt{-a}\,\,\sqrt{d}\right)^2\,\left(3\,b\,c-5\,a\,d\right)\,\left(\sqrt{c}+\sqrt{d}\,\,x^2\right)\,\sqrt{\frac{c+d\,x^4}{\left(\sqrt{c}+\sqrt{d}\,\,x^2\right)^2}}\right.$$

$$\left.\text{EllipticPi}\left[-\frac{\left(\sqrt{b}\,\,\sqrt{c}-\sqrt{-a}\,\,\sqrt{d}\right)^2}{4\,\sqrt{-a}\,\,\sqrt{b}\,\,\sqrt{c}\,\,\sqrt{d}},\,2\,\text{ArcTan}\left[\frac{d^{1/4}\,x}{c^{1/4}}\right],\,\frac{1}{2}\right]\right)\Big/$$

$$\left(32\,a^2\,c^{1/4}\,d^{1/4}\,\left(b\,c-a\,d\right)\,\left(b\,c+a\,d\right)\,\sqrt{c+d\,x^4}\right)+$$

$$\left(\left(\sqrt{b}\,\,\sqrt{c}-\sqrt{-a}\,\,\sqrt{d}\right)^2\,\left(3\,b\,c-5\,a\,d\right)\,\left(\sqrt{c}+\sqrt{d}\,\,x^2\right)\,\sqrt{\frac{c+d\,x^4}{\left(\sqrt{c}+\sqrt{d}\,\,x^2\right)^2}}\right.$$

$$\left.\text{EllipticPi}\left[\frac{\left(\sqrt{b}\,\,\sqrt{c}+\sqrt{-a}\,\,\sqrt{d}\right)^2}{4\,\sqrt{-a}\,\,\sqrt{b}\,\,\sqrt{c}\,\,\sqrt{d}},\,2\,\text{ArcTan}\left[\frac{d^{1/4}\,x}{c^{1/4}}\right],\,\frac{1}{2}\right]\right)\Big/$$

$$\left(32\,a^2\,c^{1/4}\,d^{1/4}\,\left(b\,c-a\,d\right)\,\left(b\,c+a\,d\right)\,\sqrt{c+d\,x^4}\right)$$

Result (type 6, 341 leaves) :

$$\left(x\left(-\frac{5\,b\,\left(c+d\,x^4\right)}{a}+\left(25\,c\,\left(3\,b\,c-4\,a\,d\right)\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\Big/\left(-5\,a\,c\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+2\,x^4\left(2\,b\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},2,\frac{9}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{2},1,\frac{9}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\right)+\left(9\,b\,c\,d\,x^4\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\Big/\left(-9\,a\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+2\,x^4\left(2\,b\,c\,\text{AppellF1}\left[\frac{9}{4},\frac{1}{2},2,\frac{13}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{9}{4},\frac{3}{2},1,\frac{13}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\right)\right)\right)\Big/\left(20\,\left(-b\,c+a\,d\right)\,\left(a+b\,x^4\right)\,\sqrt{c+d\,x^4}\right)$$

Problem 669: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^4\,\left(a+b\,x^4\right)^2\,\sqrt{c+d\,x^4}}\,dx$$

Optimal (type 4, 1046 leaves, 11 steps):

$$-\frac{(7 b c-4 a d) \sqrt{c+d x^4}}{12 a^2 c (b c-a d) x^3}+\frac{b \sqrt{c+d x^4}}{4 a (b c-a d) x^3 (a+b x^4)}+$$

$$\frac{b^{5/4} (7 b c-9 a d) \text{ArcTan}\left[\frac{\sqrt{b c-a d}\, x}{(-a)^{1/4} b^{1/4} \sqrt{c+d x^4}}\right]}{16 (-a)^{11/4} (b c-a d)^{3/2}}-\frac{b^{5/4} (7 b c-9 a d) \text{ArcTan}\left[\frac{\sqrt{-b c+a d}\, x}{(-a)^{1/4} b^{1/4} \sqrt{c+d x^4}}\right]}{16 (-a)^{11/4} (-b c+a d)^{3/2}}-$$

$$\left(d^{3/4} (7 b c-4 a d) \left(\sqrt{c}+\sqrt{d} x^2\right) \sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d} x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right], \frac{1}{2}\right]\right) \Big/$$

$$\left(24 a^2 c^{5/4} (b c-a d) \sqrt{c+d x^4}\right)+$$

$$\left(b \left(\sqrt{b} \sqrt{c}-\sqrt{-a} \sqrt{d}\right) d^{1/4} (7 b c-9 a d) \left(\sqrt{c}+\sqrt{d} x^2\right) \sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d} x^2\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[2 \text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right], \frac{1}{2}\right]\right) \Big/ \left(16 (-a)^{5/2} c^{1/4} (b c-a d) (b c+a d) \sqrt{c+d x^4}\right)-$$

$$\left(b \left(\sqrt{b} \sqrt{c}+\sqrt{-a} \sqrt{d}\right) d^{1/4} (7 b c-9 a d) \left(\sqrt{c}+\sqrt{d} x^2\right) \sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d} x^2\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[2 \text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right], \frac{1}{2}\right]\right) \Big/ \left(16 (-a)^{5/2} c^{1/4} (b c-a d) (b c+a d) \sqrt{c+d x^4}\right)-$$

$$\left(b \left(\sqrt{b} \sqrt{c}+\sqrt{-a} \sqrt{d}\right)^2 (7 b c-9 a d) \left(\sqrt{c}+\sqrt{d} x^2\right) \sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d} x^2\right)^2}}\right.$$

$$\left.\text{EllipticPi}\left[-\frac{\left(\sqrt{b} \sqrt{c}-\sqrt{-a} \sqrt{d}\right)^2}{4 \sqrt{-a} \sqrt{b} \sqrt{c} \sqrt{d}}, 2 \text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right], \frac{1}{2}\right]\right) \Big/$$

$$\left(32 a^3 c^{1/4} d^{1/4} (b c-a d) (b c+a d) \sqrt{c+d x^4}\right)-$$

$$\left(b \left(\sqrt{b} \sqrt{c}-\sqrt{-a} \sqrt{d}\right)^2 (7 b c-9 a d) \left(\sqrt{c}+\sqrt{d} x^2\right) \sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d} x^2\right)^2}}\right.$$

$$\left.\text{EllipticPi}\left[\frac{\left(\sqrt{b} \sqrt{c}+\sqrt{-a} \sqrt{d}\right)^2}{4 \sqrt{-a} \sqrt{b} \sqrt{c} \sqrt{d}}, 2 \text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right], \frac{1}{2}\right]\right) \Big/$$

$$\left(32 a^3 c^{1/4} d^{1/4} (b c-a d) (b c+a d) \sqrt{c+d x^4}\right)$$

Result (type 6, 399 leaves):

$$\left(\frac{5\left(c+d x^4\right)\left(-4 a^2 d+7 b^2 c x^4+4 a b\left(c-d x^4\right)\right)}{c}+\left(25 a\left(-21 b^2 c^2+20 a b c d+4 a^2 d^2\right) x^4 \text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]\right)\Big/\left(-5 a c\, \text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]+2 x^4\left(2 b c\, \text{AppellF1}\left[\frac{5}{4},\frac{1}{2},2,\frac{9}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]+a d\, \text{AppellF1}\left[\frac{5}{4},\frac{3}{2},1,\frac{9}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]\right)\right)+\left(9 a b d\left(-7 b c+4 a d\right) x^8 \text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]\right)\Big/\left(-9 a c\, \text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]+2 x^4\left(2 b c\, \text{AppellF1}\left[\frac{9}{4},\frac{1}{2},2,\frac{13}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]+a d\, \text{AppellF1}\left[\frac{9}{4},\frac{3}{2},1,\frac{13}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]\right)\right)\right)\Big/\left(60 a^2\left(-b c+a d\right) x^3\left(a+b x^4\right)\sqrt{c+d x^4}\right)$$

Problem 670: Result unnecessarily involves higher level functions.

$$\int \frac{x^6}{\left(a+b x^4\right)^2 \sqrt{c+d x^4}}\, dx$$

Optimal (type 4, 1146 leaves, 13 steps):

$$\frac{\sqrt{d}\, x \sqrt{c+d x^4}}{4 b\left(b c-a d\right)\left(\sqrt{c}+\sqrt{d}\, x^2\right)}-\frac{x^3\sqrt{c+d x^4}}{4\left(b c-a d\right)\left(a+b x^4\right)}+\frac{\left(3 b c-a d\right) \text{ArcTan}\left[\frac{\sqrt{b c-a d}\, x}{(-a)^{1/4} b^{1/4}\sqrt{c+d x^4}}\right]}{16\left(-a\right)^{1/4} b^{5/4}\left(b c-a d\right)^{3/2}}+\frac{\left(3 b c-a d\right) \text{ArcTan}\left[\frac{\sqrt{-b c+a d}\, x}{(-a)^{1/4} b^{1/4}\sqrt{c+d x^4}}\right]}{16\left(-a\right)^{1/4} b^{5/4}\left(-b c+a d\right)^{3/2}}-\left(c^{1/4} d^{1/4}\left(\sqrt{c}+\sqrt{d}\, x^2\right)\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\, x^2\right)^2}}\, \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(4 b\left(b c-a d\right)\sqrt{c+d x^4}\right)+\left(c^{1/4} d^{1/4}\left(\sqrt{c}+\sqrt{d}\, x^2\right)\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\, x^2\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(8 b\left(b c-a d\right)\sqrt{c+d x^4}\right)-$$

$$\left(\left(\sqrt{c}-\frac{\sqrt{-a}\,\sqrt{d}}{\sqrt{b}}\right) d^{1/4}\,(3\,b\,c-a\,d)\,\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+d\,x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}\,x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(16\,b\,c^{1/4}\,(b\,c-a\,d)\,(b\,c+a\,d)\,\sqrt{c+d\,x^4}\right)-$$

$$\left(\left(\sqrt{c}+\frac{\sqrt{-a}\,\sqrt{d}}{\sqrt{b}}\right) d^{1/4}\,(3\,b\,c-a\,d)\,\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+d\,x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}\,x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(16\,b\,c^{1/4}\,(b\,c-a\,d)\,(b\,c+a\,d)\,\sqrt{c+d\,x^4}\right)+$$

$$\left(\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2(3\,b\,c-a\,d)\,\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+d\,x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\right.$$

$$\left.\text{EllipticPi}\left[-\frac{\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2}{4\,\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}},2\,\text{ArcTan}\left[\frac{d^{1/4}\,x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(32\,\sqrt{-a}\,b^{3/2}\,c^{1/4}\,d^{1/4}\,(b\,c-a\,d)\,(b\,c+a\,d)\,\sqrt{c+d\,x^4}\right)-$$

$$\left(\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2(3\,b\,c-a\,d)\,\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+d\,x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\right.$$

$$\left.\text{EllipticPi}\left[\frac{\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2}{4\,\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}},2\,\text{ArcTan}\left[\frac{d^{1/4}\,x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(32\,\sqrt{-a}\,b^{3/2}\,c^{1/4}\,d^{1/4}\,(b\,c-a\,d)\,(b\,c+a\,d)\,\sqrt{c+d\,x^4}\right)$$

Result (type 6, 333 leaves) :

$$\left(x^3\left(7\left(c+d x^4\right)+\left(49 a c^2 \text{AppellF1}\left[\frac{3}{4},\frac{1}{2},1,\frac{7}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]\right)\Big/\left(-7 a c \text{AppellF1}\left[\frac{3}{4},\frac{1}{2},1,\frac{7}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]+2 x^4\left(2 b c \text{AppellF1}\left[\frac{7}{4},\frac{1}{2},2,\frac{11}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]+a d \text{AppellF1}\left[\frac{7}{4},\frac{3}{2},1,\frac{11}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]\right)\right)+\left(11 a c d x^4 \text{AppellF1}\left[\frac{7}{4},\frac{1}{2},1,\frac{11}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]\right)\Big/\left(-11 a c \text{AppellF1}\left[\frac{7}{4},\frac{1}{2},1,\frac{11}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]+2 x^4\left(2 b c \text{AppellF1}\left[\frac{11}{4},\frac{1}{2},2,\frac{15}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]+a d \text{AppellF1}\left[\frac{11}{4},\frac{3}{2},1,\frac{15}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]\right)\right)\right)\right)\Big/\left(28\left(-b c+a d\right)\left(a+b x^4\right)\sqrt{c+d x^4}\right)$$

Problem 671: Result unnecessarily involves higher level functions.

$$\int \frac{x^2}{\left(a+b x^4\right)^2 \sqrt{c+d x^4}}\, dx$$

Optimal (type 4, 1144 leaves, 13 steps):

$$-\frac{\sqrt{d}\; x \sqrt{c+d x^4}}{4 a (b c-a d)\left(\sqrt{c}+\sqrt{d}\, x^2\right)}+\frac{b x^3 \sqrt{c+d x^4}}{4 a (b c-a d)\left(a+b x^4\right)}-\frac{(b c-3 a d) \text{ArcTan}\left[\frac{\sqrt{b c-a d}\; x}{(-a)^{1/4} b^{1/4} \sqrt{c+d x^4}}\right]}{16 (-a)^{5/4} b^{1/4} (b c-a d)^{3/2}}-\frac{(b c-3 a d) \text{ArcTan}\left[\frac{\sqrt{-b c+a d}\; x}{(-a)^{1/4} b^{1/4} \sqrt{c+d x^4}}\right]}{16 (-a)^{5/4} b^{1/4} (-b c+a d)^{3/2}}+$$

$$\left(c^{1/4} d^{1/4}\left(\sqrt{c}+\sqrt{d}\, x^2\right)\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\, x^2\right)^2}}\; \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(4 a (b c-a d)\sqrt{c+d x^4}\right)-$$

$$\left(c^{1/4} d^{1/4}\left(\sqrt{c}+\sqrt{d}\, x^2\right)\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\, x^2\right)^2}}\; \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(8 a (b c-a d)\sqrt{c+d x^4}\right)-$$

$$\left(\left(\sqrt{c}-\frac{\sqrt{-a}\,\sqrt{d}}{\sqrt{b}}\right) d^{1/4} (b c-3 a d)\left(\sqrt{c}+\sqrt{d}\, x^2\right)\sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d}\, x^2\right)^2}}\; \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(16 a c^{1/4} (b c-a d)(b c+a d)\sqrt{c+d x^4}\right)-$$

$$\left(\left(\sqrt{c}+\frac{\sqrt{-a}\,\sqrt{d}}{\sqrt{b}}\right) d^{1/4}\,(b\,c-3\,a\,d)\,\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+d\,x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}\,x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(16\,a\,c^{1/4}\,(b\,c-a\,d)\,(b\,c+a\,d)\,\sqrt{c+d\,x^4}\right)-$$

$$\left(\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2(b\,c-3\,a\,d)\,\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+d\,x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\right.$$

$$\left.\text{EllipticPi}\left[-\frac{\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2}{4\,\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}},2\,\text{ArcTan}\left[\frac{d^{1/4}\,x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(32\,(-a)^{3/2}\sqrt{b}\,c^{1/4}\,d^{1/4}\,(b\,c-a\,d)\,(b\,c+a\,d)\,\sqrt{c+d\,x^4}\right)+$$

$$\left(\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2(b\,c-3\,a\,d)\,\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+d\,x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\right.$$

$$\left.\text{EllipticPi}\left[\frac{\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2}{4\,\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}},2\,\text{ArcTan}\left[\frac{d^{1/4}\,x}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(32\,(-a)^{3/2}\sqrt{b}\,c^{1/4}\,d^{1/4}\,(b\,c-a\,d)\,(b\,c+a\,d)\,\sqrt{c+d\,x^4}\right)$$

Result (type 6, 342 leaves):

$$\left(x^3\left(-\frac{21\,b\,(c+d\,x^4)}{a}+\left(49\,c\,(b\,c-4\,a\,d)\,\text{AppellF1}\left[\frac{3}{4},\frac{1}{2},1,\frac{7}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\Big/\right.\right.$$

$$\left(-7\,a\,c\,\text{AppellF1}\left[\frac{3}{4},\frac{1}{2},1,\frac{7}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+2\,x^4\left(2\,b\,c\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},2,\right.\right.\right.$$

$$\left.\left.\left.\frac{11}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{7}{4},\frac{3}{2},1,\frac{11}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\right)-$$

$$\left(33\,b\,c\,d\,x^4\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},1,\frac{11}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\Big/\left(-11\,a\,c\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},\right.\right.$$

$$\left.1,\frac{11}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+2\,x^4\left(2\,b\,c\,\text{AppellF1}\left[\frac{11}{4},\frac{1}{2},2,\frac{15}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+a\,d\right.$$

$$\left.\left.\left.\left.\text{AppellF1}\left[\frac{11}{4},\frac{3}{2},1,\frac{15}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\right)\right)\right)\Big/\left(84\,(-b\,c+a\,d)\,(a+b\,x^4)\,\sqrt{c+d\,x^4}\right)$$

Problem 672: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^2 \left(a+b x^4\right)^2 \sqrt{c+d x^4}} \, dx$$

Optimal (type 4, 1225 leaves, 14 steps):

$$-\frac{(5 b c-4 a d) \sqrt{c+d x^4}}{4 a^2 c (b c-a d) x}+\frac{\sqrt{d} (5 b c-4 a d) x \sqrt{c+d x^4}}{4 a^2 c (b c-a d) \left(\sqrt{c}+\sqrt{d} x^2\right)}+\frac{b \sqrt{c+d x^4}}{4 a (b c-a d) x \left(a+b x^4\right)}-$$

$$\frac{b^{3/4} (5 b c-7 a d) \text{ArcTan}\left[\frac{\sqrt{b c-a d}\, x}{(-a)^{1/4} b^{1/4} \sqrt{c+d x^4}}\right]}{16 (-a)^{9/4} (b c-a d)^{3/2}}-\frac{b^{3/4} (5 b c-7 a d) \text{ArcTan}\left[\frac{\sqrt{-b c+a d}\, x}{(-a)^{1/4} b^{1/4} \sqrt{c+d x^4}}\right]}{16 (-a)^{9/4} (-b c+a d)^{3/2}}-$$

$$\left(d^{1/4} (5 b c-4 a d) \left(\sqrt{c}+\sqrt{d} x^2\right) \sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d} x^2\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right], \frac{1}{2}\right]\right) \Big/$$

$$\left(4 a^2 c^{3/4} (b c-a d) \sqrt{c+d x^4}\right)+$$

$$\left(d^{1/4} (5 b c-4 a d) \left(\sqrt{c}+\sqrt{d} x^2\right) \sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d} x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right], \frac{1}{2}\right]\right) \Big/$$

$$\left(8 a^2 c^{3/4} (b c-a d) \sqrt{c+d x^4}\right)+$$

$$\left(b \left(\sqrt{c}-\frac{\sqrt{-a} \sqrt{d}}{\sqrt{b}}\right) d^{1/4} (5 b c-7 a d) \left(\sqrt{c}+\sqrt{d} x^2\right) \sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d} x^2\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[2 \text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right], \frac{1}{2}\right]\right) \Big/ \left(16 a^2 c^{1/4} (b c-a d) (b c+a d) \sqrt{c+d x^4}\right)+$$

$$\left(b \left(\sqrt{c}+\frac{\sqrt{-a} \sqrt{d}}{\sqrt{b}}\right) d^{1/4} (5 b c-7 a d) \left(\sqrt{c}+\sqrt{d} x^2\right) \sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d} x^2\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[2 \text{ArcTan}\left[\frac{d^{1/4} x}{c^{1/4}}\right], \frac{1}{2}\right]\right) \Big/ \left(16 a^2 c^{1/4} (b c-a d) (b c+a d) \sqrt{c+d x^4}\right)-$$

$$\left(\sqrt{b} \left(\sqrt{b} \sqrt{c}+\sqrt{-a} \sqrt{d}\right)^2 (5 b c-7 a d) \left(\sqrt{c}+\sqrt{d} x^2\right) \sqrt{\frac{c+d x^4}{\left(\sqrt{c}+\sqrt{d} x^2\right)^2}}\right.$$

$$\text{EllipticPi}\left[-\frac{\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2}{4\,\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}},\,2\,\text{ArcTan}\left[\frac{d^{1/4}\,x}{c^{1/4}}\right],\,\frac{1}{2}\right]\Bigg) \Big/ \left(32\,(-a)^{5/2}\,c^{1/4}\,d^{1/4}\,(b\,c-a\,d)\,(b\,c+a\,d)\,\sqrt{c+d\,x^4}\right)+$$

$$\left(\sqrt{b}\,\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2\,(5\,b\,c-7\,a\,d)\,\left(\sqrt{c}+\sqrt{d}\,x^2\right)\,\sqrt{\frac{c+d\,x^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\right.$$

$$\left.\text{EllipticPi}\left[\frac{\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2}{4\,\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}},\,2\,\text{ArcTan}\left[\frac{d^{1/4}\,x}{c^{1/4}}\right],\,\frac{1}{2}\right]\right) \Big/ \left(32\,(-a)^{5/2}\,c^{1/4}\,d^{1/4}\,(b\,c-a\,d)\,(b\,c+a\,d)\,\sqrt{c+d\,x^4}\right)$$

Result (type 6, 399 leaves):

$$\left(\frac{21\,(c+d\,x^4)\,\left(-4\,a^2\,d+5\,b^2\,c\,x^4+4\,a\,b\,(c-d\,x^4)\right)}{c}-\right.$$

$$\left(49\,a\,(5\,b^2\,c^2-12\,a\,b\,c\,d+4\,a^2\,d^2)\,x^4\,\text{AppellF1}\left[\frac{3}{4},\frac{1}{2},1,\frac{7}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\Big/$$

$$\left(-7\,a\,c\,\text{AppellF1}\left[\frac{3}{4},\frac{1}{2},1,\frac{7}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+2\,x^4\,\left(2\,b\,c\right.\right.$$

$$\left.\left.\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},2,\frac{11}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{7}{4},\frac{3}{2},1,\frac{11}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\right)+$$

$$\left(33\,a\,b\,d\,(5\,b\,c-4\,a\,d)\,x^8\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},1,\frac{11}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\Big/$$

$$\left(-11\,a\,c\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},1,\frac{11}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+\right.$$

$$2\,x^4\,\left(2\,b\,c\,\text{AppellF1}\left[\frac{11}{4},\frac{1}{2},2,\frac{15}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]+a\,d\right.$$

$$\left.\left.\left.\left.\text{AppellF1}\left[\frac{11}{4},\frac{3}{2},1,\frac{15}{4},-\frac{d\,x^4}{c},-\frac{b\,x^4}{a}\right]\right)\right)\right)\Big/\left(84\,a^2\,(-b\,c+a\,d)\,x\,(a+b\,x^4)\,\sqrt{c+d\,x^4}\right)$$

Problem 676: Result more than twice size of optimal antiderivative.

$$\int\frac{(e\,x)^m}{(a+b\,x^4)\,\sqrt{c+d\,x^4}}\,dx$$

Optimal (type 6, 81 leaves, 2 steps):

$$\frac{(e\,x)^{1+m}\,\sqrt{1+\frac{d\,x^4}{c}}\,\text{AppellF1}\left[\frac{1+m}{4},1,\frac{1}{2},\frac{5+m}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]}{a\,e\,(1+m)\,\sqrt{c+d\,x^4}}$$

Result (type 6, 282 leaves):

$$\frac{1}{(1+m)\sqrt{c+d x^4}} \left(x (e x)^m \left(-\left(\left(a b c (5+m) \left(c+d x^4\right) \text{AppellF1}\left[\frac{1+m}{4}, -\frac{1}{2}, 1, \frac{5+m}{4}, -\frac{d x^4}{c}, -\frac{b x^4}{a}\right] \right) \Big/ \left((-b c+a d) \left(a+b x^4\right) \left(a c (5+m) \text{AppellF1}\left[\frac{1+m}{4}, -\frac{1}{2}, 1, \frac{5+m}{4}, -\frac{d x^4}{c}, -\frac{b x^4}{a}\right] + 2 x^4 \left(-2 b c \text{AppellF1}\left[\frac{5+m}{4}, -\frac{1}{2}, 2, \frac{9+m}{4}, -\frac{d x^4}{c}, -\frac{b x^4}{a}\right] + a d \text{AppellF1}\left[\frac{5+m}{4}, \frac{1}{2}, 1, \frac{9+m}{4}, -\frac{d x^4}{c}, -\frac{b x^4}{a}\right] \right) \right) \right) \right) - \frac{d \sqrt{1+\frac{d x^4}{c}} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{4}, \frac{5+m}{4}, -\frac{d x^4}{c}\right]}{b c - a d} \right)$$

Problem 677: Result more than twice size of optimal antiderivative.

$$\int \frac{(e x)^m}{\left(a+b x^4\right)^2 \sqrt{c+d x^4}} \, dx$$

Optimal (type 6, 81 leaves, 2 steps):

$$\frac{(e x)^{1+m} \sqrt{1+\frac{d x^4}{c}} \text{AppellF1}\left[\frac{1+m}{4}, 2, \frac{1}{2}, \frac{5+m}{4}, -\frac{b x^4}{a}, -\frac{d x^4}{c}\right]}{a^2 e (1+m) \sqrt{c+d x^4}}$$

Result (type 6, 488 leaves):

$$\frac{1}{(1+m)\sqrt{c+d x^4}} x\,(e x)^m \Bigg(-\bigg(\Big(a b c d\,(5+m)\,(c+d x^4)\,\text{AppellF1}\Big[\frac{1+m}{4},-\frac{1}{2},1,\frac{5+m}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\Big]\Big)\Big/ \Big((b c-a d)^2\,(a+b x^4)\,\Big(a c\,(5+m)\,\text{AppellF1}\Big[\frac{1+m}{4},-\frac{1}{2},1,\frac{5+m}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\Big]+ 2 x^4\Big(-2 b c\,\text{AppellF1}\Big[\frac{5+m}{4},-\frac{1}{2},2,\frac{9+m}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\Big]+ a d\,\text{AppellF1}\Big[\frac{5+m}{4},\frac{1}{2},1,\frac{9+m}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\Big]\Big)\Big)\Big)\bigg)- \Big(a b c\,(5+m)\,(c+d x^4)\,\text{AppellF1}\Big[\frac{1+m}{4},2,-\frac{1}{2},\frac{5+m}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\Big]\Big)\Big/ \Big((-b c+a d)\,(a+b x^4)^2\,\Big(a c\,(5+m)\,\text{AppellF1}\Big[\frac{1+m}{4},2,-\frac{1}{2},\frac{5+m}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\Big]+ 2 x^4\Big(a d\,\text{AppellF1}\Big[\frac{5+m}{4},2,\frac{1}{2},\frac{9+m}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\Big]- 4 b c\,\text{AppellF1}\Big[\frac{5+m}{4},3,-\frac{1}{2},\frac{9+m}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\Big]\Big)\Big)\Big)+ \frac{d^2\sqrt{1+\frac{d x^4}{c}}\,\text{Hypergeometric2F1}\Big[\frac{1}{2},\frac{1+m}{4},\frac{5+m}{4},-\frac{d x^4}{c}\Big]}{(b c-a d)^2}\Bigg)$$

Problem 678: Result more than twice size of optimal antiderivative.

$$\int \frac{(e x)^m}{(a+b x^4)^3\sqrt{c+d x^4}}\,dx$$

Optimal (type 6, 81 leaves, 2 steps):

$$\frac{(e x)^{1+m}\sqrt{1+\frac{d x^4}{c}}\,\text{AppellF1}\Big[\frac{1+m}{4},3,\frac{1}{2},\frac{5+m}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\Big]}{a^3 e\,(1+m)\sqrt{c+d x^4}}$$

Result (type 6, 209 leaves):

$$-\left(\left(a c (5+m) x (e x)^m \text{AppellF1}\left[\frac{1+m}{4}, 3, \frac{1}{2}, \frac{5+m}{4}, -\frac{b x^4}{a}, -\frac{d x^4}{c}\right]\right)\Big/\left((1+m) \left(a+b x^4\right)^3 \sqrt{c+d x^4} \left(-a c (5+m) \text{AppellF1}\left[\frac{1+m}{4}, 3, \frac{1}{2}, \frac{5+m}{4}, -\frac{b x^4}{a}, -\frac{d x^4}{c}\right] + 2 x^4 \left(a d \text{AppellF1}\left[\frac{5+m}{4}, 3, \frac{3}{2}, \frac{9+m}{4}, -\frac{b x^4}{a}, -\frac{d x^4}{c}\right] + 6 b c \text{AppellF1}\left[\frac{5+m}{4}, 4, \frac{1}{2}, \frac{9+m}{4}, -\frac{b x^4}{a}, -\frac{d x^4}{c}\right]\right)\right)\right)\right)$$

Problem 682: Result more than twice size of optimal antiderivative.

$$\int \frac{(e x)^m}{\left(a+b x^4\right) \left(c+d x^4\right)^{3/2}} \, dx$$

Optimal (type 6, 84 leaves, 2 steps):

$$\frac{(e x)^{1+m} \sqrt{1+\frac{d x^4}{c}} \text{AppellF1}\left[\frac{1+m}{4}, 1, \frac{3}{2}, \frac{5+m}{4}, -\frac{b x^4}{a}, -\frac{d x^4}{c}\right]}{a c e (1+m) \sqrt{c+d x^4}}$$

Result (type 6, 329 leaves):

$$\left(x (e x)^m \left(\left(a b^2 c (5+m) \left(c+d x^4\right) \text{AppellF1}\left[\frac{1+m}{4}, -\frac{1}{2}, 1, \frac{5+m}{4}, -\frac{d x^4}{c}, -\frac{b x^4}{a}\right]\right)\Big/\left(\left(a+b x^4\right) \left(a c (5+m) \text{AppellF1}\left[\frac{1+m}{4}, -\frac{1}{2}, 1, \frac{5+m}{4}, -\frac{d x^4}{c}, -\frac{b x^4}{a}\right] + 2 x^4 \left(-2 b c \text{AppellF1}\left[\frac{5+m}{4}, -\frac{1}{2}, 2, \frac{9+m}{4}, -\frac{d x^4}{c}, -\frac{b x^4}{a}\right] + a d \text{AppellF1}\left[\frac{5+m}{4}, \frac{1}{2}, 1, \frac{9+m}{4}, -\frac{d x^4}{c}, -\frac{b x^4}{a}\right]\right)\right)\right) - b d \sqrt{1+\frac{d x^4}{c}} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{4}, \frac{5+m}{4}, -\frac{d x^4}{c}\right] - \frac{d (b c - a d) \sqrt{1+\frac{d x^4}{c}} \text{Hypergeometric2F1}\left[\frac{3}{2}, \frac{1+m}{4}, \frac{5+m}{4}, -\frac{d x^4}{c}\right]}{c}\right)\right)\Big/\left((b c - a d)^2 (1+m) \sqrt{c+d x^4}\right)$$

Problem 683: Result more than twice size of optimal antiderivative.

$$\int \frac{(e\,x)^m}{\left(a+b\,x^4\right)^2\,\left(c+d\,x^4\right)^{3/2}}\,dx$$

Optimal (type 6, 84 leaves, 2 steps):

$$\frac{(e\,x)^{1+m}\,\sqrt{1+\frac{d\,x^4}{c}}\,\text{AppellF1}\left[\frac{1+m}{4},2,\frac{3}{2},\frac{5+m}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]}{a^2\,c\,e\,(1+m)\,\sqrt{c+d\,x^4}}$$

Result (type 6, 210 leaves):

$$-\left(\left(a\,c\,(5+m)\,x\,(e\,x)^m\,\text{AppellF1}\left[\frac{1+m}{4},2,\frac{3}{2},\frac{5+m}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\Big/\left((1+m)\,\left(a+b\,x^4\right)^2\,\left(c+d\,x^4\right)^{3/2}\,\left(-a\,c\,(5+m)\,\text{AppellF1}\left[\frac{1+m}{4},2,\frac{3}{2},\frac{5+m}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+2\,x^4\,\left(3\,a\,d\,\text{AppellF1}\left[\frac{5+m}{4},2,\frac{5}{2},\frac{9+m}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+4\,b\,c\,\text{AppellF1}\left[\frac{5+m}{4},3,\frac{3}{2},\frac{9+m}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\right)\right)\right)$$

Problem 684: Result more than twice size of optimal antiderivative.

$$\int \frac{(e\,x)^m}{\left(a+b\,x^4\right)^3\,\left(c+d\,x^4\right)^{3/2}}\,dx$$

Optimal (type 6, 84 leaves, 2 steps):

$$\frac{(e\,x)^{1+m}\,\sqrt{1+\frac{d\,x^4}{c}}\,\text{AppellF1}\left[\frac{1+m}{4},3,\frac{3}{2},\frac{5+m}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]}{a^3\,c\,e\,(1+m)\,\sqrt{c+d\,x^4}}$$

Result (type 6, 209 leaves):

$$-\left(\left(a\,c\,(5+m)\,x\,(e\,x)^m\,\text{AppellF1}\left[\frac{1+m}{4},3,\frac{3}{2},\frac{5+m}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\Big/\left((1+m)\,\left(a+b\,x^4\right)^3\,\left(c+d\,x^4\right)^{3/2}\,\left(-a\,c\,(5+m)\,\text{AppellF1}\left[\frac{1+m}{4},3,\frac{3}{2},\frac{5+m}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+6\,x^4\,\left(a\,d\,\text{AppellF1}\left[\frac{5+m}{4},3,\frac{5}{2},\frac{9+m}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+2\,b\,c\,\text{AppellF1}\left[\frac{5+m}{4},4,\frac{3}{2},\frac{9+m}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\right)\right)\right)$$

Problem 688: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x\left(a+b x^6\right)\sqrt{c+d x^6}}\,dx$$

Optimal (type 3, 85 leaves, 6 steps):

$$-\frac{\text{ArcTanh}\left[\frac{\sqrt{c+d x^6}}{\sqrt{c}}\right]}{3 a \sqrt{c}}+\frac{\sqrt{b}\ \text{ArcTanh}\left[\frac{\sqrt{b}\ \sqrt{c+d x^6}}{\sqrt{b c-a d}}\right]}{3 a \sqrt{b c-a d}}$$

Result (type 6, 162 leaves):

$$\left(5 b d x^6 \,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^6},-\frac{a}{b x^6}\right]\right)\Big/ \left(9\left(a+b x^6\right)\sqrt{c+d x^6}\left(-5 b d x^6\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^6},-\frac{a}{b x^6}\right]+ 2 a d\,\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^6},-\frac{a}{b x^6}\right]+b c\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^6},-\frac{a}{b x^6}\right]\right)\right)$$

Problem 689: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{x^7\left(a+b x^6\right)\sqrt{c+d x^6}}\,dx$$

Optimal (type 3, 117 leaves, 7 steps):

$$-\frac{\sqrt{c+d x^6}}{6 a c x^6}+\frac{\left(2 b c+a d\right)\text{ArcTanh}\left[\frac{\sqrt{c+d x^6}}{\sqrt{c}}\right]}{6 a^2 c^{3/2}}-\frac{b^{3/2}\,\text{ArcTanh}\left[\frac{\sqrt{b}\ \sqrt{c+d x^6}}{\sqrt{b c-a d}}\right]}{3 a^2 \sqrt{b c-a d}}$$

Result (type 6, 410 leaves):

$$\left(\left(6 b d x^{12}\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^6}{c},-\frac{b x^6}{a}\right]\right)\Big/\left(-4 a c\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^6}{c},-\frac{b x^6}{a}\right]+ x^6\left(2 b c\,\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d x^6}{c},-\frac{b x^6}{a}\right]+a d\,\text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d x^6}{c},-\frac{b x^6}{a}\right]\right)\right)+ \left(5 b d x^6\left(a\left(3 c+2 d x^6\right)+b x^6\left(c+3 d x^6\right)\right)\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^6},-\frac{a}{b x^6}\right]- 3\left(a+b x^6\right)\left(c+d x^6\right)\left(2 a d\,\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^6},-\frac{a}{b x^6}\right]+ b c\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^6},-\frac{a}{b x^6}\right]\right)\right)\Big/ \left(a c\left(-5 b d x^6\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^6},-\frac{a}{b x^6}\right]+2 a d\,\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^6},-\frac{a}{b x^6}\right]+b c\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^6},-\frac{a}{b x^6}\right]\right)\right)\right)\Big/\left(18 x^6\left(a+b x^6\right)\sqrt{c+d x^6}\right)$$

Problem 695: Result more than twice size of optimal antiderivative.

$$\int \frac{x^4}{\left(a+b x^6\right) \sqrt{c+d x^6}} \, dx$$

Optimal (type 6, 64 leaves, 2 steps):

$$\frac{x^5 \sqrt{1+\frac{d x^6}{c}} \text{AppellF1}\left[\frac{5}{6}, 1, \frac{1}{2}, \frac{11}{6}, -\frac{b x^6}{a}, -\frac{d x^6}{c}\right]}{5 a \sqrt{c+d x^6}}$$

Result (type 6, 165 leaves):

$$-\left(\left(11 a c x^5 \text{AppellF1}\left[\frac{5}{6}, \frac{1}{2}, 1, \frac{11}{6}, -\frac{d x^6}{c}, -\frac{b x^6}{a}\right]\right) \Big/ \left(5 \left(a+b x^6\right) \sqrt{c+d x^6} \left(-11 a c \text{AppellF1}\left[\frac{5}{6}, \frac{1}{2}, 1, \frac{11}{6}, -\frac{d x^6}{c}, -\frac{b x^6}{a}\right] + 3 x^6 \left(2 b c \text{AppellF1}\left[\frac{11}{6}, \frac{1}{2}, 2, \frac{17}{6}, -\frac{d x^6}{c}, -\frac{b x^6}{a}\right] + a d \text{AppellF1}\left[\frac{11}{6}, \frac{3}{2}, 1, \frac{17}{6}, -\frac{d x^6}{c}, -\frac{b x^6}{a}\right]\right)\right)\right)\right)$$

Problem 696: Result more than twice size of optimal antiderivative.

$$\int \frac{x^3}{\left(a+b x^6\right) \sqrt{c+d x^6}} \, dx$$

Optimal (type 6, 64 leaves, 3 steps):

$$\frac{x^4 \sqrt{1+\frac{d x^6}{c}} \text{AppellF1}\left[\frac{2}{3}, 1, \frac{1}{2}, \frac{5}{3}, -\frac{b x^6}{a}, -\frac{d x^6}{c}\right]}{4 a \sqrt{c+d x^6}}$$

Result (type 6, 165 leaves):

$$-\left(\left(5 a c x^4 \text{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{d x^6}{c}, -\frac{b x^6}{a}\right]\right) \Big/ \left(2 \left(a+b x^6\right) \sqrt{c+d x^6} \left(-10 a c \text{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{d x^6}{c}, -\frac{b x^6}{a}\right] + 3 x^6 \left(2 b c \text{AppellF1}\left[\frac{5}{3}, \frac{1}{2}, 2, \frac{8}{3}, -\frac{d x^6}{c}, -\frac{b x^6}{a}\right] + a d \text{AppellF1}\left[\frac{5}{3}, \frac{3}{2}, 1, \frac{8}{3}, -\frac{d x^6}{c}, -\frac{b x^6}{a}\right]\right)\right)\right)\right)$$

Problem 697: Result more than twice size of optimal antiderivative.

$$\int \frac{x}{\left(a+b x^6\right) \sqrt{c+d x^6}} \, dx$$

Optimal (type 6, 64 leaves, 3 steps):

$$\frac{x^2 \sqrt{1+\frac{d x^6}{c}}\ \text{AppellF1}\left[\frac{1}{3}, 1, \frac{1}{2}, \frac{4}{3}, -\frac{b x^6}{a}, -\frac{d x^6}{c}\right]}{2 a \sqrt{c+d x^6}}$$

Result (type 6, 163 leaves):

$$-\left(\left(4 a c x^2 \text{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{d x^6}{c}, -\frac{b x^6}{a}\right]\right) \Big/ \left(\left(a+b x^6\right) \sqrt{c+d x^6} \left(-8 a c \text{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{d x^6}{c}, -\frac{b x^6}{a}\right] + 3 x^6 \left(2 b c \text{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 2, \frac{7}{3}, -\frac{d x^6}{c}, -\frac{b x^6}{a}\right] + a d \text{AppellF1}\left[\frac{4}{3}, \frac{3}{2}, 1, \frac{7}{3}, -\frac{d x^6}{c}, -\frac{b x^6}{a}\right]\right)\right)\right)\right)$$

Problem 698: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(a+b x^6\right) \sqrt{c+d x^6}} \, dx$$

Optimal (type 6, 59 leaves, 2 steps):

$$\frac{x \sqrt{1+\frac{d x^6}{c}}\ \text{AppellF1}\left[\frac{1}{6}, 1, \frac{1}{2}, \frac{7}{6}, -\frac{b x^6}{a}, -\frac{d x^6}{c}\right]}{a \sqrt{c+d x^6}}$$

Result (type 6, 161 leaves):

$$-\left(\left(7 a c x \text{AppellF1}\left[\frac{1}{6}, \frac{1}{2}, 1, \frac{7}{6}, -\frac{d x^6}{c}, -\frac{b x^6}{a}\right]\right) \Big/ \left(\left(a+b x^6\right) \sqrt{c+d x^6} \left(-7 a c \text{AppellF1}\left[\frac{1}{6}, \frac{1}{2}, 1, \frac{7}{6}, -\frac{d x^6}{c}, -\frac{b x^6}{a}\right] + 3 x^6 \left(2 b c \text{AppellF1}\left[\frac{7}{6}, \frac{1}{2}, 2, \frac{13}{6}, -\frac{d x^6}{c}, -\frac{b x^6}{a}\right] + a d \text{AppellF1}\left[\frac{7}{6}, \frac{3}{2}, 1, \frac{13}{6}, -\frac{d x^6}{c}, -\frac{b x^6}{a}\right]\right)\right)\right)\right)$$

Problem 699: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^2 \left(a+b x^6\right) \sqrt{c+d x^6}} \, dx$$

Optimal (type 6, 62 leaves, 2 steps):

$$-\frac{\sqrt{1+\frac{d x^6}{c}}\ \text{AppellF1}\left[-\frac{1}{6}, 1, \frac{1}{2}, \frac{5}{6}, -\frac{b x^6}{a}, -\frac{d x^6}{c}\right]}{a x \sqrt{c+d x^6}}$$

Result (type 6, 344 leaves):

$$\frac{1}{55\,x\,\sqrt{c+d\,x^6}}\left(-\frac{55\,(c+d\,x^6)}{a\,c}+\left(121\,(b\,c-2\,a\,d)\,x^6\,\text{AppellF1}\left[\frac{5}{6},\frac{1}{2},1,\frac{11}{6},-\frac{d\,x^6}{c},-\frac{b\,x^6}{a}\right]\right)\Big/\left((a+b\,x^6)\left(-11\,a\,c\,\text{AppellF1}\left[\frac{5}{6},\frac{1}{2},1,\frac{11}{6},-\frac{d\,x^6}{c},-\frac{b\,x^6}{a}\right]+3\,x^6\left(2\,b\,c\,\text{AppellF1}\left[\frac{11}{6},\frac{1}{2},2,\frac{17}{6},-\frac{d\,x^6}{c},-\frac{b\,x^6}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{11}{6},\frac{3}{2},1,\frac{17}{6},-\frac{d\,x^6}{c},-\frac{b\,x^6}{a}\right]\right)\right)\right)-\left(170\,b\,d\,x^{12}\,\text{AppellF1}\left[\frac{11}{6},\frac{1}{2},1,\frac{17}{6},-\frac{d\,x^6}{c},-\frac{b\,x^6}{a}\right]\right)\Big/\left((a+b\,x^6)\left(-17\,a\,c\,\text{AppellF1}\left[\frac{11}{6},\frac{1}{2},1,\frac{17}{6},-\frac{d\,x^6}{c},-\frac{b\,x^6}{a}\right]+3\,x^6\left(2\,b\,c\,\text{AppellF1}\left[\frac{17}{6},\frac{1}{2},2,\frac{23}{6},-\frac{d\,x^6}{c},-\frac{b\,x^6}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{17}{6},\frac{3}{2},1,\frac{23}{6},-\frac{d\,x^6}{c},-\frac{b\,x^6}{a}\right]\right)\right)\right)\right)$$

Problem 700: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^3\,(a+b\,x^6)\,\sqrt{c+d\,x^6}}\,dx$$

Optimal (type 6, 64 leaves, 3 steps):

$$-\frac{\sqrt{1+\frac{d\,x^6}{c}}\,\text{AppellF1}\left[-\frac{1}{3},1,\frac{1}{2},\frac{2}{3},-\frac{b\,x^6}{a},-\frac{d\,x^6}{c}\right]}{2\,a\,x^2\,\sqrt{c+d\,x^6}}$$

Result (type 6, 345 leaves):

$$\frac{1}{20\,x^2\,\sqrt{c+d\,x^6}}\left(-\frac{10\,(c+d\,x^6)}{a\,c}+\left(25\,(2\,b\,c-a\,d)\,x^6\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d\,x^6}{c},-\frac{b\,x^6}{a}\right]\right)\Big/\left((a+b\,x^6)\left(-10\,a\,c\,\text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d\,x^6}{c},-\frac{b\,x^6}{a}\right]+3\,x^6\left(2\,b\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d\,x^6}{c},-\frac{b\,x^6}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d\,x^6}{c},-\frac{b\,x^6}{a}\right]\right)\right)\right)-\left(16\,b\,d\,x^{12}\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d\,x^6}{c},-\frac{b\,x^6}{a}\right]\right)\Big/\left((a+b\,x^6)\left(-16\,a\,c\,\text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d\,x^6}{c},-\frac{b\,x^6}{a}\right]+3\,x^6\left(2\,b\,c\,\text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d\,x^6}{c},-\frac{b\,x^6}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d\,x^6}{c},-\frac{b\,x^6}{a}\right]\right)\right)\right)\right)$$

Problem 701: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^5 \left(a+b x^6\right) \sqrt{c+d x^6}} \, dx$$

Optimal (type 6, 64 leaves, 3 steps):

$$-\frac{\sqrt{1+\frac{d x^6}{c}} \text{ AppellF1}\left[-\frac{2}{3},1,\frac{1}{2},\frac{1}{3},-\frac{b x^6}{a},-\frac{d x^6}{c}\right]}{4 a x^4 \sqrt{c+d x^6}}$$

Result (type 6, 344 leaves):

$$\frac{1}{16 x^4 \sqrt{c+d x^6}} \left(-\frac{4 \left(c+d x^6\right)}{a c}+\left(16 \left(4 b c+a d\right) x^6 \text{ AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]\right) \Big/ \left(\left(a+b x^6\right) \left(-8 a c \text{ AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]+3 x^6 \left(2 b c \text{ AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]+a d \text{ AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]\right)\right)\right)+\left(7 b d x^{12} \text{ AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]\right) \Big/ \left(\left(a+b x^6\right) \left(-14 a c \text{ AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]+3 x^6 \left(2 b c \text{ AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]+a d \text{ AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]\right)\right)\right)\right)$$

Problem 705: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{x \left(a+b x^6\right)^2 \sqrt{c+d x^6}} \, dx$$

Optimal (type 3, 132 leaves, 7 steps):

$$\frac{b \sqrt{c+d x^6}}{6 a \left(b c-a d\right) \left(a+b x^6\right)}-\frac{\text{ArcTanh}\left[\frac{\sqrt{c+d x^6}}{\sqrt{c}}\right]}{3 a^2 \sqrt{c}}+\frac{\sqrt{b} \left(2 b c-3 a d\right) \text{ArcTanh}\left[\frac{\sqrt{b} \sqrt{c+d x^6}}{\sqrt{b c-a d}}\right]}{6 a^2 \left(b c-a d\right)^{3/2}}$$

Result (type 6, 396 leaves):

$$\left(b\left(\left(6\,c\,d\,x^6\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d\,x^6}{c},-\frac{b\,x^6}{a}\right]\right)\Big/\right.\right.$$

$$\left(-4\,a\,c\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d\,x^6}{c},-\frac{b\,x^6}{a}\right]+x^6\right.$$

$$\left.\left.\left(2\,b\,c\,\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d\,x^6}{c},-\frac{b\,x^6}{a}\right]+a\,d\,\text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d\,x^6}{c},-\frac{b\,x^6}{a}\right]\right)\right)+$$

$$\left(5\,d\,x^6\left(2\,a\,d+b\left(c+3\,d\,x^6\right)\right)\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^6},-\frac{a}{b\,x^6}\right]-\right.$$

$$3\left(c+d\,x^6\right)\left(2\,a\,d\,\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d\,x^6},-\frac{a}{b\,x^6}\right]+\right.$$

$$\left.\left.b\,c\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d\,x^6},-\frac{a}{b\,x^6}\right]\right)\right)\Big/\left(a\left(-5\,b\,d\,x^6\right.\right.$$

$$\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^6},-\frac{a}{b\,x^6}\right]+2\,a\,d\,\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d\,x^6},-\frac{a}{b\,x^6}\right]+$$

$$\left.\left.\left.\left.b\,c\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d\,x^6},-\frac{a}{b\,x^6}\right]\right)\right)\right)\right)\Big/\left(18\left(-b\,c+a\,d\right)\left(a+b\,x^6\right)\sqrt{c+d\,x^6}\right)$$

Problem 706: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\frac{1}{x^7\left(a+b\,x^6\right)^2\sqrt{c+d\,x^6}}\,dx$$

Optimal (type 3, 185 leaves, 8 steps):

$$-\frac{b\left(2\,b\,c-a\,d\right)\sqrt{c+d\,x^6}}{6\,a^2\,c\left(b\,c-a\,d\right)\left(a+b\,x^6\right)}-\frac{\sqrt{c+d\,x^6}}{6\,a\,c\,x^6\left(a+b\,x^6\right)}+$$

$$\frac{\left(4\,b\,c+a\,d\right)\text{ArcTanh}\left[\frac{\sqrt{c+d\,x^6}}{\sqrt{c}}\right]}{6\,a^3\,c^{3/2}}-\frac{b^{3/2}\left(4\,b\,c-5\,a\,d\right)\text{ArcTanh}\left[\frac{\sqrt{b}\,\sqrt{c+d\,x^6}}{\sqrt{b\,c-a\,d}}\right]}{6\,a^3\left(b\,c-a\,d\right)^{3/2}}$$

Result (type 6, 489 leaves):

$$\left(\left(6\,a\,b\,d\,(-2\,b\,c+a\,d)\,x^{12}\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d\,x^6}{c},-\frac{b\,x^6}{a}\right]\right)\Big/\left((-b\,c+a\,d)\left(-4\,a\,c\,\text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d\,x^6}{c},-\frac{b\,x^6}{a}\right]+x^6\left(2\,b\,c\,\text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d\,x^6}{c},-\frac{b\,x^6}{a}\right]+a\,d\,\text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d\,x^6}{c},-\frac{b\,x^6}{a}\right]\right)\right)\right)+\right.$$
$$\left(5\,b\,d\,x^6\left(-a^2\,d\,(3\,c+2\,d\,x^6)+2\,b^2\,c\,x^6\,(c+3\,d\,x^6)+3\,a\,b\,(c^2+c\,d\,x^6-d^2\,x^{12})\right)\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^6},-\frac{a}{b\,x^6}\right]+\right.$$
$$3\,(c+d\,x^6)\,(a^2\,d-2\,b^2\,c\,x^6+a\,b\,(-c+d\,x^6))\left(2\,a\,d\,\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d\,x^6},-\frac{a}{b\,x^6}\right]+\right.$$
$$\left.\left.b\,c\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d\,x^6},-\frac{a}{b\,x^6}\right]\right)\right)\Big/$$
$$\left(c\,(b\,c-a\,d)\left(-5\,b\,d\,x^6\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d\,x^6},-\frac{a}{b\,x^6}\right]+2\,a\,d\,\text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d\,x^6},-\frac{a}{b\,x^6}\right]+\right.\right.$$
$$\left.\left.\left.b\,c\,\text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d\,x^6},-\frac{a}{b\,x^6}\right]\right)\right)\right)\Big/\left(18\,a^2\,x^6\,(a+b\,x^6)\,\sqrt{c+d\,x^6}\right)$$

Problem 712: Result more than twice size of optimal antiderivative.

$$\int\frac{x^4}{(a+b\,x^6)^2\,\sqrt{c+d\,x^6}}\,dx$$

Optimal (type 6, 64 leaves, 2 steps):

$$\frac{x^5\,\sqrt{1+\frac{d\,x^6}{c}}\,\text{AppellF1}\left[\frac{5}{6},2,\frac{1}{2},\frac{11}{6},-\frac{b\,x^6}{a},-\frac{d\,x^6}{c}\right]}{5\,a^2\,\sqrt{c+d\,x^6}}$$

Result (type 6, 342 leaves):

$$\left(x^5 \left(-\frac{55 b \left(c+d x^6\right)}{a}+\left(121 c \left(b c-6 a d\right) \text{AppellF1}\left[\frac{5}{6},\frac{1}{2},1,\frac{11}{6},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]\right) \Big/ \left(-11 a c \text{AppellF1}\left[\frac{5}{6},\frac{1}{2},1,\frac{11}{6},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]+3 x^6 \left(2 b c \text{AppellF1}\left[\frac{11}{6},\frac{1}{2},2,\frac{17}{6},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]+a d \text{AppellF1}\left[\frac{11}{6},\frac{3}{2},1,\frac{17}{6},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]\right)\right)-\left(170 b c d x^6 \text{AppellF1}\left[\frac{11}{6},\frac{1}{2},1,\frac{17}{6},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]\right) \Big/ \left(-17 a c \text{AppellF1}\left[\frac{11}{6},\frac{1}{2},1,\frac{17}{6},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]+3 x^6 \left(2 b c \text{AppellF1}\left[\frac{17}{6},\frac{1}{2},2,\frac{23}{6},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]+a d \text{AppellF1}\left[\frac{17}{6},\frac{3}{2},1,\frac{23}{6},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]\right)\right)\right)\right) \Big/ \left(330 \left(-b c+a d\right) \left(a+b x^6\right) \sqrt{c+d x^6}\right)$$

Problem 713: Result more than twice size of optimal antiderivative.

$$\int \frac{x^3}{\left(a+b x^6\right)^2 \sqrt{c+d x^6}} \, dx$$

Optimal (type 6, 64 leaves, 3 steps):

$$\frac{x^4 \sqrt{1+\frac{d x^6}{c}} \, \text{AppellF1}\left[\frac{2}{3},2,\frac{1}{2},\frac{5}{3},-\frac{b x^6}{a},-\frac{d x^6}{c}\right]}{4 a^2 \sqrt{c+d x^6}}$$

Result (type 6, 342 leaves):

$$\left(x^4 \left(-\frac{5 b \left(c+d x^6\right)}{a}+\left(25 c \left(b c-3 a d\right) \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]\right) \Big/ \left(-10 a c \text{AppellF1}\left[\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]+3 x^6 \left(2 b c \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},2,\frac{8}{3},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]+a d \text{AppellF1}\left[\frac{5}{3},\frac{3}{2},1,\frac{8}{3},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]\right)\right)-\left(8 b c d x^6 \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]\right) \Big/ \left(-16 a c \text{AppellF1}\left[\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]+3 x^6 \left(2 b c \text{AppellF1}\left[\frac{8}{3},\frac{1}{2},2,\frac{11}{3},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]+a d \text{AppellF1}\left[\frac{8}{3},\frac{3}{2},1,\frac{11}{3},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]\right)\right)\right)\right) \Big/ \left(30 \left(-b c+a d\right) \left(a+b x^6\right) \sqrt{c+d x^6}\right)$$

Problem 714: Result more than twice size of optimal antiderivative.

$$\int \frac{x}{\left(a+b x^6\right)^2 \sqrt{c+d x^6}} \, dx$$

Optimal (type 6, 64 leaves, 3 steps):

$$\frac{x^2 \sqrt{1+\frac{d x^6}{c}} \,\text{AppellF1}\left[\frac{1}{3}, 2, \frac{1}{2}, \frac{4}{3}, -\frac{b x^6}{a}, -\frac{d x^6}{c}\right]}{2 a^2 \sqrt{c+d x^6}}$$

Result (type 6, 343 leaves):

$$\left(x^2 \left(-\frac{4 b \left(c+d x^6\right)}{a}+\left(32 c \left(2 b c-3 a d\right) \text{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{d x^6}{c}, -\frac{b x^6}{a}\right]\right) \Big/ \left(-8 a c \,\text{AppellF1}\left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{d x^6}{c}, -\frac{b x^6}{a}\right]+3 x^6 \left(2 b c \,\text{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 2, \frac{7}{3}, -\frac{d x^6}{c}, -\frac{b x^6}{a}\right]+a d \,\text{AppellF1}\left[\frac{4}{3}, \frac{3}{2}, 1, \frac{7}{3}, -\frac{d x^6}{c}, -\frac{b x^6}{a}\right]\right)\right)+\left(7 b c d x^6 \,\text{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, -\frac{d x^6}{c}, -\frac{b x^6}{a}\right]\right) \Big/ \left(-14 a c \,\text{AppellF1}\left[\frac{4}{3}, \frac{1}{2}, 1, \frac{7}{3}, -\frac{d x^6}{c}, -\frac{b x^6}{a}\right]+3 x^6 \left(2 b c \,\text{AppellF1}\left[\frac{7}{3}, \frac{1}{2}, 2, \frac{10}{3}, -\frac{d x^6}{c}, -\frac{b x^6}{a}\right]+a d \,\text{AppellF1}\left[\frac{7}{3}, \frac{3}{2}, 1, \frac{10}{3}, -\frac{d x^6}{c}, -\frac{b x^6}{a}\right]\right)\right)\right)\right) \Big/ \left(24 \left(-b c+a d\right) \left(a+b x^6\right) \sqrt{c+d x^6}\right)$$

Problem 715: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(a+b x^6\right)^2 \sqrt{c+d x^6}} \, dx$$

Optimal (type 6, 59 leaves, 2 steps):

$$\frac{x \sqrt{1+\frac{d x^6}{c}} \,\text{AppellF1}\left[\frac{1}{6}, 2, \frac{1}{2}, \frac{7}{6}, -\frac{b x^6}{a}, -\frac{d x^6}{c}\right]}{a^2 \sqrt{c+d x^6}}$$

Result (type 6, 341 leaves):

$$\left(x\left(-\frac{7b\left(c+dx^6\right)}{a}+\left(49c\left(5bc-6ad\right)\text{AppellF1}\left[\frac{1}{6},\frac{1}{2},1,\frac{7}{6},-\frac{dx^6}{c},-\frac{bx^6}{a}\right]\right)\Big/\left(-7ac\,\text{AppellF1}\left[\frac{1}{6},\frac{1}{2},1,\frac{7}{6},-\frac{dx^6}{c},-\frac{bx^6}{a}\right]+3x^6\left(2bc\,\text{AppellF1}\left[\frac{7}{6},\frac{1}{2},2,\frac{13}{6},-\frac{dx^6}{c},-\frac{bx^6}{a}\right]+ad\,\text{AppellF1}\left[\frac{7}{6},\frac{3}{2},1,\frac{13}{6},-\frac{dx^6}{c},-\frac{bx^6}{a}\right]\right)\right)+\left(26bcdx^6\,\text{AppellF1}\left[\frac{7}{6},\frac{1}{2},1,\frac{13}{6},-\frac{dx^6}{c},-\frac{bx^6}{a}\right]\right)\Big/\left(-13ac\,\text{AppellF1}\left[\frac{7}{6},\frac{1}{2},1,\frac{13}{6},-\frac{dx^6}{c},-\frac{bx^6}{a}\right]+3x^6\left(2bc\,\text{AppellF1}\left[\frac{13}{6},\frac{1}{2},2,\frac{19}{6},-\frac{dx^6}{c},-\frac{bx^6}{a}\right]+ad\,\text{AppellF1}\left[\frac{13}{6},\frac{3}{2},1,\frac{19}{6},-\frac{dx^6}{c},-\frac{bx^6}{a}\right]\right)\right)\right)\right)\Big/\left(42\left(-bc+ad\right)\left(a+bx^6\right)\sqrt{c+dx^6}\right)$$

Problem 716: Result more than twice size of optimal antiderivative.

$$\int\frac{1}{x^2\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$$

Optimal (type 6, 62 leaves, 2 steps):

$$-\frac{\sqrt{1+\frac{dx^6}{c}}\;\text{AppellF1}\left[-\frac{1}{6},2,\frac{1}{2},\frac{5}{6},-\frac{bx^6}{a},-\frac{dx^6}{c}\right]}{a^2x\sqrt{c+dx^6}}$$

Result (type 6, 399 leaves):

$$\left(\frac{55\left(c+dx^6\right)\left(-6a^2d+7b^2cx^6+6ab\left(c-dx^6\right)\right)}{c}-\left(121a\left(7b^2c^2-24abcd+12a^2d^2\right)x^6\,\text{AppellF1}\left[\frac{5}{6},\frac{1}{2},1,\frac{11}{6},-\frac{dx^6}{c},-\frac{bx^6}{a}\right]\right)\Big/\left(-11ac\,\text{AppellF1}\left[\frac{5}{6},\frac{1}{2},1,\frac{11}{6},-\frac{dx^6}{c},-\frac{bx^6}{a}\right]+3x^6\left(2bc\,\text{AppellF1}\left[\frac{11}{6},\frac{1}{2},2,\frac{17}{6},-\frac{dx^6}{c},-\frac{bx^6}{a}\right]+ad\,\text{AppellF1}\left[\frac{11}{6},\frac{3}{2},1,\frac{17}{6},-\frac{dx^6}{c},-\frac{bx^6}{a}\right]\right)\right)+\left(170abd\left(7bc-6ad\right)x^{12}\,\text{AppellF1}\left[\frac{11}{6},\frac{1}{2},1,\frac{17}{6},-\frac{dx^6}{c},-\frac{bx^6}{a}\right]\right)\Big/\left(-17ac\,\text{AppellF1}\left[\frac{11}{6},\frac{1}{2},1,\frac{17}{6},-\frac{dx^6}{c},-\frac{bx^6}{a}\right]+3x^6\left(2bc\,\text{AppellF1}\left[\frac{17}{6},\frac{1}{2},2,\frac{23}{6},-\frac{dx^6}{c},-\frac{bx^6}{a}\right]+ad\,\text{AppellF1}\left[\frac{17}{6},\frac{3}{2},1,\frac{23}{6},-\frac{dx^6}{c},-\frac{bx^6}{a}\right]\right)\right)\right)\Big/$$

$$\left(330a^2\left(-bc+ad\right)x\left(a+bx^6\right)\sqrt{c+dx^6}\right)$$

Problem 717: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^3 \left(a+b x^6\right)^2 \sqrt{c+d x^6}} \, dx$$

Optimal (type 6, 64 leaves, 3 steps):

$$-\frac{\sqrt{1+\frac{d x^6}{c}} \text{ AppellF1}\left[-\frac{1}{3}, 2, \frac{1}{2}, \frac{2}{3}, -\frac{b x^6}{a}, -\frac{d x^6}{c}\right]}{2 a^2 x^2 \sqrt{c+d x^6}}$$

Result (type 6, 399 leaves):

$$\left(\frac{10 \left(c+d x^6\right) \left(-3 a^2 d+4 b^2 c x^6+3 a b \left(c-d x^6\right)\right)}{c} - \left(25 a \left(8 b^2 c^2-15 a b c d+3 a^2 d^2\right) x^6 \text{ AppellF1}\left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{d x^6}{c}, -\frac{b x^6}{a}\right]\right) \Big/ \left(-10 a c \text{ AppellF1}\left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{d x^6}{c}, -\frac{b x^6}{a}\right] + 3 x^6 \left(2 b c \text{ AppellF1}\left[\frac{5}{3}, \frac{1}{2}, 2, \frac{8}{3}, -\frac{d x^6}{c}, -\frac{b x^6}{a}\right] + a d \text{ AppellF1}\left[\frac{5}{3}, \frac{3}{2}, 1, \frac{8}{3}, -\frac{d x^6}{c}, -\frac{b x^6}{a}\right]\right)\right) + \left(16 a b d \left(4 b c-3 a d\right) x^{12} \text{ AppellF1}\left[\frac{5}{3}, \frac{1}{2}, 1, \frac{8}{3}, -\frac{d x^6}{c}, -\frac{b x^6}{a}\right]\right) \Big/ \left(-16 a c \text{ AppellF1}\left[\frac{5}{3}, \frac{1}{2}, 1, \frac{8}{3}, -\frac{d x^6}{c}, -\frac{b x^6}{a}\right] + 3 x^6 \left(2 b c \text{ AppellF1}\left[\frac{8}{3}, \frac{1}{2}, 2, \frac{11}{3}, -\frac{d x^6}{c}, -\frac{b x^6}{a}\right] + a d \text{ AppellF1}\left[\frac{8}{3}, \frac{3}{2}, 1, \frac{11}{3}, -\frac{d x^6}{c}, -\frac{b x^6}{a}\right]\right)\right)\right) \Big/ \left(60 a^2 \left(-b c+a d\right) x^2 \left(a+b x^6\right) \sqrt{c+d x^6}\right)$$

Problem 718: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^5 \left(a+b x^6\right)^2 \sqrt{c+d x^6}} \, dx$$

Optimal (type 6, 64 leaves, 3 steps):

$$-\frac{\sqrt{1+\frac{d x^6}{c}} \text{ AppellF1}\left[-\frac{2}{3}, 2, \frac{1}{2}, \frac{1}{3}, -\frac{b x^6}{a}, -\frac{d x^6}{c}\right]}{4 a^2 x^4 \sqrt{c+d x^6}}$$

Result (type 6, 399 leaves):

$$\left(\frac{4\left(c+d x^6\right)\left(-3 a^2 d+5 b^2 c x^6+3 a b\left(c-d x^6\right)\right)}{c}+\left(16 a\left(-20 b^2 c^2+21 a b c d+3 a^2 d^2\right) x^6 \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]\right)\Big/\left(-8 a c \text{AppellF1}\left[\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]+3 x^6\left(2 b c \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]+a d \text{AppellF1}\left[\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]\right)\right)+\left(7 a b d\left(-5 b c+3 a d\right) x^{12} \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]\right)\Big/\left(-14 a c \text{AppellF1}\left[\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]+3 x^6\left(2 b c \text{AppellF1}\left[\frac{7}{3},\frac{1}{2},2,\frac{10}{3},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]+a d \text{AppellF1}\left[\frac{7}{3},\frac{3}{2},1,\frac{10}{3},-\frac{d x^6}{c},-\frac{b x^6}{a}\right]\right)\right)\right)\Big/\left(48 a^2\left(-b c+a d\right) x^4\left(a+b x^6\right)\sqrt{c+d x^6}\right)$$

Problem 722: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x\left(a+b x^8\right)\sqrt{c+d x^8}}\,dx$$

Optimal (type 3, 85 leaves, 6 steps):

$$-\frac{\text{ArcTanh}\left[\frac{\sqrt{c+d x^8}}{\sqrt{c}}\right]}{4 a\sqrt{c}}+\frac{\sqrt{b}\,\text{ArcTanh}\left[\frac{\sqrt{b}\sqrt{c+d x^8}}{\sqrt{b c-a d}}\right]}{4 a\sqrt{b c-a d}}$$

Result (type 6, 162 leaves):

$$\left(5 b d x^8 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^8},-\frac{a}{b x^8}\right]\right)\Big/\left(12\left(a+b x^8\right)\sqrt{c+d x^8}\left(-5 b d x^8 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^8},-\frac{a}{b x^8}\right]+2 a d \text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^8},-\frac{a}{b x^8}\right]+b c \text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^8},-\frac{a}{b x^8}\right]\right)\right)$$

Problem 723: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{x^9\left(a+b x^8\right)\sqrt{c+d x^8}}\,dx$$

Optimal (type 3, 117 leaves, 7 steps):

$$-\frac{\sqrt{c+d x^8}}{8 a c x^8}+\frac{(2 b c+a d)\operatorname{ArcTanh}\left[\frac{\sqrt{c+d x^8}}{\sqrt{c}}\right]}{8 a^2 c^{3/2}}-\frac{b^{3/2}\operatorname{ArcTanh}\left[\frac{\sqrt{b}\sqrt{c+d x^8}}{\sqrt{b c-a d}}\right]}{4 a^2\sqrt{b c-a d}}$$

Result (type 6, 410 leaves):

$$\left(\left(6 b d x^{16}\operatorname{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\Big/\left(-4 a c\operatorname{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+x^8\left(2 b c\operatorname{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+a d\operatorname{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\right)+\left(5 b d x^8\left(a\left(3 c+2 d x^8\right)+b x^8\left(c+3 d x^8\right)\right)\operatorname{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^8},-\frac{a}{b x^8}\right]-3\left(a+b x^8\right)\left(c+d x^8\right)\left(2 a d\operatorname{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^8},-\frac{a}{b x^8}\right]+b c\operatorname{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^8},-\frac{a}{b x^8}\right]\right)\right)\Big/\left(a c\left(-5 b d x^8\operatorname{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^8},-\frac{a}{b x^8}\right]+2 a d\operatorname{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^8},-\frac{a}{b x^8}\right]+b c\operatorname{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^8},-\frac{a}{b x^8}\right]\right)\right)\right)\Big/\left(24 x^8\left(a+b x^8\right)\sqrt{c+d x^8}\right)$$

Problem 729: Result unnecessarily involves higher level functions.

$$\int\frac{x^9}{\left(a+b x^8\right)\sqrt{c+d x^8}}\,dx$$

Optimal (type 4, 851 leaves, 10 steps):

$$-\frac{(-a)^{1/4}\,\text{ArcTan}\left[\frac{\sqrt{b\,c-a\,d}\;x^2}{(-a)^{1/4}\,b^{1/4}\,\sqrt{c+d\,x^8}}\right]}{8\,b^{3/4}\,\sqrt{b\,c-a\,d}}-\frac{(-a)^{1/4}\,\text{ArcTan}\left[\frac{\sqrt{-b\,c+a\,d}\;x^2}{(-a)^{1/4}\,b^{1/4}\,\sqrt{c+d\,x^8}}\right]}{8\,b^{3/4}\,\sqrt{-b\,c+a\,d}}+$$

$$\frac{\left(\sqrt{c}+\sqrt{d}\;x^4\right)\sqrt{\frac{c+d\,x^8}{\left(\sqrt{c}+\sqrt{d}\;x^4\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}\,x^2}{c^{1/4}}\right],\frac{1}{2}\right]}{4\,b\,c^{1/4}\,d^{1/4}\,\sqrt{c+d\,x^8}}-$$

$$\left(a\left(\frac{\sqrt{b}\;\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)d^{1/4}\left(\sqrt{c}+\sqrt{d}\;x^4\right)\sqrt{\frac{c+d\,x^8}{\left(\sqrt{c}+\sqrt{d}\;x^4\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}\,x^2}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Bigg/$$

$$\left(8\,b\,c^{1/4}\,(b\,c+a\,d)\,\sqrt{c+d\,x^8}\right)-\left(\left(\sqrt{-a}\;\sqrt{b}\;\sqrt{c}+a\,\sqrt{d}\right)d^{1/4}\left(\sqrt{c}+\sqrt{d}\;x^4\right)\right.$$

$$\left.\sqrt{\frac{c+d\,x^8}{\left(\sqrt{c}+\sqrt{d}\;x^4\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}\,x^2}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Bigg/\left(8\,b\,c^{1/4}\,(b\,c+a\,d)\,\sqrt{c+d\,x^8}\right)-$$

$$\left(\left(\sqrt{b}\;\sqrt{c}+\sqrt{-a}\;\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}\;x^4\right)\sqrt{\frac{c+d\,x^8}{\left(\sqrt{c}+\sqrt{d}\;x^4\right)^2}}\;\text{EllipticPi}\left[\right.\right.$$

$$\left.\left.-\frac{\left(\sqrt{b}\;\sqrt{c}-\sqrt{-a}\;\sqrt{d}\right)^2}{4\,\sqrt{-a}\;\sqrt{b}\;\sqrt{c}\;\sqrt{d}},\,2\,\text{ArcTan}\left[\frac{d^{1/4}\,x^2}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Bigg/\left(16\,b\,c^{1/4}\,d^{1/4}\,(b\,c+a\,d)\,\sqrt{c+d\,x^8}\right)-$$

$$\left(\left(\sqrt{b}\;\sqrt{c}-\sqrt{-a}\;\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}\;x^4\right)\sqrt{\frac{c+d\,x^8}{\left(\sqrt{c}+\sqrt{d}\;x^4\right)^2}}\;\text{EllipticPi}\left[\right.\right.$$

$$\left.\left.\frac{\left(\sqrt{b}\;\sqrt{c}+\sqrt{-a}\;\sqrt{d}\right)^2}{4\,\sqrt{-a}\;\sqrt{b}\;\sqrt{c}\;\sqrt{d}},\,2\,\text{ArcTan}\left[\frac{d^{1/4}\,x^2}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Bigg/\left(16\,b\,c^{1/4}\,d^{1/4}\,(b\,c+a\,d)\,\sqrt{c+d\,x^8}\right)$$

Result (type 6, 165 leaves):

$$-\left(\left(9\,a\,c\,x^{10}\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\Bigg/\right.$$

$$\left(10\,(a+b\,x^8)\,\sqrt{c+d\,x^8}\left(-9\,a\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+2\,x^8\left(2\,b\,c\,\text{AppellF1}\left[\frac{9}{4},\right.\right.\right.\right.$$

$$\left.\left.\left.\left.\left.\frac{1}{2},2,\frac{13}{4},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{9}{4},\frac{3}{2},1,\frac{13}{4},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\right)\right)\right)$$

Problem 730: Result unnecessarily involves higher level functions.

$$\int \frac{x}{\left(a+b x^8\right) \sqrt{c+d x^8}} \, dx$$

Optimal (type 4, 754 leaves, 8 steps):

$$-\frac{b^{1/4} \operatorname{ArcTan}\left[\frac{\sqrt{b c-a d}\, x^2}{(-a)^{1/4} b^{1/4} \sqrt{c+d x^8}}\right]}{8 (-a)^{3/4} \sqrt{b c-a d}}-\frac{b^{1/4} \operatorname{ArcTan}\left[\frac{\sqrt{-b c+a d}\, x^2}{(-a)^{1/4} b^{1/4} \sqrt{c+d x^8}}\right]}{8 (-a)^{3/4} \sqrt{-b c+a d}}+\left(\left(\frac{\sqrt{b} \sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right) d^{1/4} \left(\sqrt{c}+\sqrt{d}\, x^4\right) \sqrt{\frac{c+d x^8}{\left(\sqrt{c}+\sqrt{d}\, x^4\right)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{d^{1/4} x^2}{c^{1/4}}\right], \frac{1}{2}\right]\right) \Big/ \left(8 c^{1/4} (b c+a d) \sqrt{c+d x^8}\right)+\left(\left(\sqrt{-a} \sqrt{b} \sqrt{c}+a \sqrt{d}\right) d^{1/4} \left(\sqrt{c}+\sqrt{d}\, x^4\right) \sqrt{\frac{c+d x^8}{\left(\sqrt{c}+\sqrt{d}\, x^4\right)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{d^{1/4} x^2}{c^{1/4}}\right], \frac{1}{2}\right]\right) \Big/ \left(8 a c^{1/4} (b c+a d) \sqrt{c+d x^8}\right)+\left(\left(\sqrt{b} \sqrt{c}+\sqrt{-a} \sqrt{d}\right)^2 \left(\sqrt{c}+\sqrt{d}\, x^4\right) \sqrt{\frac{c+d x^8}{\left(\sqrt{c}+\sqrt{d}\, x^4\right)^2}} \operatorname{EllipticPi}\left[-\frac{\left(\sqrt{b} \sqrt{c}-\sqrt{-a} \sqrt{d}\right)^2}{4 \sqrt{-a} \sqrt{b} \sqrt{c} \sqrt{d}}, 2 \operatorname{ArcTan}\left[\frac{d^{1/4} x^2}{c^{1/4}}\right], \frac{1}{2}\right]\right) \Big/ \left(16 a c^{1/4} d^{1/4} (b c+a d) \sqrt{c+d x^8}\right)+\left(\left(\sqrt{b} \sqrt{c}-\sqrt{-a} \sqrt{d}\right)^2 \left(\sqrt{c}+\sqrt{d}\, x^4\right) \sqrt{\frac{c+d x^8}{\left(\sqrt{c}+\sqrt{d}\, x^4\right)^2}} \operatorname{EllipticPi}\left[\frac{\left(\sqrt{b} \sqrt{c}+\sqrt{-a} \sqrt{d}\right)^2}{4 \sqrt{-a} \sqrt{b} \sqrt{c} \sqrt{d}}, 2 \operatorname{ArcTan}\left[\frac{d^{1/4} x^2}{c^{1/4}}\right], \frac{1}{2}\right]\right) \Big/ \left(16 a c^{1/4} d^{1/4} (b c+a d) \sqrt{c+d x^8}\right)$$

Result (type 6, 165 leaves):

$$-\left(\left(5 a c x^2 \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, -\frac{d x^8}{c}, -\frac{b x^8}{a}\right]\right) \Big/ \left(2 \left(a+b x^8\right) \sqrt{c+d x^8} \left(-5 a c \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, -\frac{d x^8}{c}, -\frac{b x^8}{a}\right]+2 x^8 \left(2 b c \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, -\frac{d x^8}{c}, -\frac{b x^8}{a}\right]+a d \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, -\frac{d x^8}{c}, -\frac{b x^8}{a}\right]\right)\right)\right)\right)$$

Problem 731: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^7 \left(a+b x^8\right) \sqrt{c+d x^8}} \, dx$$

Optimal (type 4, 878 leaves, 11 steps):

$$-\frac{\sqrt{c+d x^8}}{6 a c x^6}-\frac{b^{5/4} \text{ArcTan}\left[\frac{\sqrt{b c-a d}\ x^2}{(-a)^{1/4} b^{1/4} \sqrt{c+d x^8}}\right]}{8 (-a)^{7/4} \sqrt{b c-a d}}-\frac{b^{5/4} \text{ArcTan}\left[\frac{\sqrt{-b c+a d}\ x^2}{(-a)^{1/4} b^{1/4} \sqrt{c+d x^8}}\right]}{8 (-a)^{7/4} \sqrt{-b c+a d}}-$$
$$\frac{d^{3/4} \left(\sqrt{c}+\sqrt{d}\ x^4\right) \sqrt{\frac{c+d x^8}{\left(\sqrt{c}+\sqrt{d}\ x^4\right)^2}}\ \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{d^{1/4} x^2}{c^{1/4}}\right],\frac{1}{2}\right]}{12 a c^{5/4} \sqrt{c+d x^8}}-$$
$$\left(b \left(\frac{\sqrt{b}\ \sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right) d^{1/4} \left(\sqrt{c}+\sqrt{d}\ x^4\right) \sqrt{\frac{c+d x^8}{\left(\sqrt{c}+\sqrt{d}\ x^4\right)^2}}\ \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{d^{1/4} x^2}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$
$$\left(8 a c^{1/4} (b c+a d) \sqrt{c+d x^8}\right)-\left(b \left(\sqrt{-a}\ \sqrt{b}\ \sqrt{c}+a \sqrt{d}\right) d^{1/4} \left(\sqrt{c}+\sqrt{d}\ x^4\right)\right.$$
$$\left.\sqrt{\frac{c+d x^8}{\left(\sqrt{c}+\sqrt{d}\ x^4\right)^2}}\ \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{d^{1/4} x^2}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(8 a^2 c^{1/4} (b c+a d) \sqrt{c+d x^8}\right)-$$
$$\left(b \left(\sqrt{b}\ \sqrt{c}+\sqrt{-a}\ \sqrt{d}\right)^2 \left(\sqrt{c}+\sqrt{d}\ x^4\right) \sqrt{\frac{c+d x^8}{\left(\sqrt{c}+\sqrt{d}\ x^4\right)^2}}\ \text{EllipticPi}\left[\right.\right.$$
$$\left.\left.-\frac{\left(\sqrt{b}\ \sqrt{c}-\sqrt{-a}\ \sqrt{d}\right)^2}{4 \sqrt{-a}\ \sqrt{b}\ \sqrt{c}\ \sqrt{d}},2 \text{ArcTan}\left[\frac{d^{1/4} x^2}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(16 a^2 c^{1/4} d^{1/4} (b c+a d) \sqrt{c+d x^8}\right)-$$
$$\left(b \left(\sqrt{b}\ \sqrt{c}-\sqrt{-a}\ \sqrt{d}\right)^2 \left(\sqrt{c}+\sqrt{d}\ x^4\right) \sqrt{\frac{c+d x^8}{\left(\sqrt{c}+\sqrt{d}\ x^4\right)^2}}\ \text{EllipticPi}\left[\right.\right.$$
$$\left.\left.\frac{\left(\sqrt{b}\ \sqrt{c}+\sqrt{-a}\ \sqrt{d}\right)^2}{4 \sqrt{-a}\ \sqrt{b}\ \sqrt{c}\ \sqrt{d}},2 \text{ArcTan}\left[\frac{d^{1/4} x^2}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(16 a^2 c^{1/4} d^{1/4} (b c+a d) \sqrt{c+d x^8}\right)$$

Result (type 6, 344 leaves):

$$\frac{1}{30\,x^{6}\,\sqrt{c+d\,x^{8}}}\left(-\frac{5\,\left(c+d\,x^{8}\right)}{a\,c}+\left(25\,\left(3\,b\,c+a\,d\right)\,x^{8}\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{d\,x^{8}}{c},-\frac{b\,x^{8}}{a}\right]\right)\Big/\left(\left(a+b\,x^{8}\right)\left(-5\,a\,c\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{d\,x^{8}}{c},-\frac{b\,x^{8}}{a}\right]+2\,x^{8}\left(2\,b\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},2,\frac{9}{4},-\frac{d\,x^{8}}{c},-\frac{b\,x^{8}}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{2},1,\frac{9}{4},-\frac{d\,x^{8}}{c},-\frac{b\,x^{8}}{a}\right]\right)\right)\right)+\left(9\,b\,d\,x^{16}\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{d\,x^{8}}{c},-\frac{b\,x^{8}}{a}\right]\right)\Big/\left(\left(a+b\,x^{8}\right)\left(-9\,a\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{d\,x^{8}}{c},-\frac{b\,x^{8}}{a}\right]+2\,x^{8}\left(2\,b\,c\,\text{AppellF1}\left[\frac{9}{4},\frac{1}{2},2,\frac{13}{4},-\frac{d\,x^{8}}{c},-\frac{b\,x^{8}}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{9}{4},\frac{3}{2},1,\frac{13}{4},-\frac{d\,x^{8}}{c},-\frac{b\,x^{8}}{a}\right]\right)\right)\right)\right)$$

Problem 732: Result unnecessarily involves higher level functions.

$$\int\frac{x^{13}}{\left(a+b\,x^{8}\right)\,\sqrt{c+d\,x^{8}}}\,dx$$

Optimal (type 4, 1005 leaves, 12 steps):

$$\frac{x^2\sqrt{c+d\,x^8}}{2\,b\,\sqrt{d}\,\left(\sqrt{c}+\sqrt{d}\,x^4\right)}+\frac{(-a)^{3/4}\,\text{ArcTan}\left[\frac{\sqrt{b\,c-a\,d}\,x^2}{(-a)^{1/4}\,b^{1/4}\,\sqrt{c+d\,x^8}}\right]}{8\,b^{5/4}\,\sqrt{b\,c-a\,d}}-\frac{(-a)^{3/4}\,\text{ArcTan}\left[\frac{\sqrt{-b\,c+a\,d}\,x^2}{(-a)^{1/4}\,b^{1/4}\,\sqrt{c+d\,x^8}}\right]}{8\,b^{5/4}\,\sqrt{-b\,c+a\,d}}-$$

$$\frac{c^{1/4}\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+d\,x^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\;\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}\,x^2}{c^{1/4}}\right],\frac{1}{2}\right]}{2\,b\,d^{3/4}\,\sqrt{c+d\,x^8}}+$$

$$\frac{c^{1/4}\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+d\,x^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}\,x^2}{c^{1/4}}\right],\frac{1}{2}\right]}{4\,b\,d^{3/4}\,\sqrt{c+d\,x^8}}+$$

$$\left(a\left(\sqrt{c}-\frac{\sqrt{-a}\,\sqrt{d}}{\sqrt{b}}\right)d^{1/4}\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+d\,x^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}\,x^2}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(8\,b\,c^{1/4}\,(b\,c+a\,d)\,\sqrt{c+d\,x^8}\right)+$$

$$\left(a\left(\sqrt{c}+\frac{\sqrt{-a}\,\sqrt{d}}{\sqrt{b}}\right)d^{1/4}\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+d\,x^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}\,x^2}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(8\,b\,c^{1/4}\,(b\,c+a\,d)\,\sqrt{c+d\,x^8}\right)+$$

$$\left(\sqrt{-a}\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+d\,x^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\;\text{EllipticPi}\left[\right.\right.$$

$$\left.\left.-\frac{\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2}{4\,\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}},2\,\text{ArcTan}\left[\frac{d^{1/4}\,x^2}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(16\,b^{3/2}\,c^{1/4}\,d^{1/4}\,(b\,c+a\,d)\,\sqrt{c+d\,x^8}\right)-$$

$$\left(\sqrt{-a}\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+d\,x^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\;\text{EllipticPi}\left[\right.\right.$$

$$\left.\left.\frac{\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2}{4\,\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}},2\,\text{ArcTan}\left[\frac{d^{1/4}\,x^2}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(16\,b^{3/2}\,c^{1/4}\,d^{1/4}\,(b\,c+a\,d)\,\sqrt{c+d\,x^8}\right)$$

Result (type 6, 165 leaves):

$$-\left(\left(11\,a\,c\,x^{14}\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},1,\frac{11}{4},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\Big/\right.$$

$$\left(14\,(a+b\,x^8)\,\sqrt{c+d\,x^8}\,\left(-11\,a\,c\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},1,\frac{11}{4},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+2\,x^8\left(2\,b\,c\,\text{AppellF1}\left[\right.\right.\right.\right.$$

$$\left.\left.\left.\left.\left.\frac{11}{4},\frac{1}{2},2,\frac{15}{4},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{11}{4},\frac{3}{2},1,\frac{15}{4},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\right)\right)\right)$$

Problem 733: Result unnecessarily involves higher level functions.

$$\int \frac{x^5}{\left(a+b x^8\right) \sqrt{c+d x^8}} \, dx$$

Optimal (type 4, 768 leaves, 8 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{b c-a d}\, x^2}{(-a)^{1/4} b^{1/4} \sqrt{c+d x^8}}\right]}{8 (-a)^{1/4} b^{1/4} \sqrt{b c-a d}}-\frac{\text{ArcTan}\left[\frac{\sqrt{-b c+a d}\, x^2}{(-a)^{1/4} b^{1/4} \sqrt{c+d x^8}}\right]}{8 (-a)^{1/4} b^{1/4} \sqrt{-b c+a d}}-$$

$$\left(\left(\sqrt{c}-\frac{\sqrt{-a} \sqrt{d}}{\sqrt{b}}\right) d^{1/4} \left(\sqrt{c}+\sqrt{d} x^4\right) \sqrt{\frac{c+d x^8}{\left(\sqrt{c}+\sqrt{d} x^4\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{d^{1/4} x^2}{c^{1/4}}\right], \frac{1}{2}\right]\right) \Big/$$

$$\left(8 c^{1/4} (b c+a d) \sqrt{c+d x^8}\right) -$$

$$\left(\left(\sqrt{c}+\frac{\sqrt{-a} \sqrt{d}}{\sqrt{b}}\right) d^{1/4} \left(\sqrt{c}+\sqrt{d} x^4\right) \sqrt{\frac{c+d x^8}{\left(\sqrt{c}+\sqrt{d} x^4\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{d^{1/4} x^2}{c^{1/4}}\right], \frac{1}{2}\right]\right) \Big/$$

$$\left(8 c^{1/4} (b c+a d) \sqrt{c+d x^8}\right) + \left(\left(\sqrt{b} \sqrt{c}+\sqrt{-a} \sqrt{d}\right)^2 \left(\sqrt{c}+\sqrt{d} x^4\right)\right.$$

$$\left.\sqrt{\frac{c+d x^8}{\left(\sqrt{c}+\sqrt{d} x^4\right)^2}} \text{EllipticPi}\left[-\frac{\left(\sqrt{b} \sqrt{c}-\sqrt{-a} \sqrt{d}\right)^2}{4 \sqrt{-a} \sqrt{b} \sqrt{c} \sqrt{d}}, 2 \text{ArcTan}\left[\frac{d^{1/4} x^2}{c^{1/4}}\right], \frac{1}{2}\right]\right) \Big/$$

$$\left(16 \sqrt{-a} \sqrt{b} c^{1/4} d^{1/4} (b c+a d) \sqrt{c+d x^8}\right) - \left(\left(\sqrt{b} \sqrt{c}-\sqrt{-a} \sqrt{d}\right)^2 \left(\sqrt{c}+\sqrt{d} x^4\right)\right.$$

$$\left.\sqrt{\frac{c+d x^8}{\left(\sqrt{c}+\sqrt{d} x^4\right)^2}} \text{EllipticPi}\left[\frac{\left(\sqrt{b} \sqrt{c}+\sqrt{-a} \sqrt{d}\right)^2}{4 \sqrt{-a} \sqrt{b} \sqrt{c} \sqrt{d}}, 2 \text{ArcTan}\left[\frac{d^{1/4} x^2}{c^{1/4}}\right], \frac{1}{2}\right]\right) \Big/$$

$$\left(16 \sqrt{-a} \sqrt{b} c^{1/4} d^{1/4} (b c+a d) \sqrt{c+d x^8}\right)$$

Result (type 6, 165 leaves):

$$-\left(\left(7 a c x^6 \text{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\frac{d x^8}{c}, -\frac{b x^8}{a}\right]\right) \Big/\right.$$

$$\left(6 \left(a+b x^8\right) \sqrt{c+d x^8} \left(-7 a c \text{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\frac{d x^8}{c}, -\frac{b x^8}{a}\right]+2 x^8 \left(2 b c \text{AppellF1}\left[\frac{7}{4},\right.\right.\right.\right.$$

$$\left.\left.\left.\left.\left.\frac{1}{2}, 2, \frac{11}{4}, -\frac{d x^8}{c}, -\frac{b x^8}{a}\right]+a d \text{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, -\frac{d x^8}{c}, -\frac{b x^8}{a}\right]\right)\right)\right)\right)$$

Problem 734: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^3 \left(a+b x^8\right) \sqrt{c+d x^8}} \, dx$$

Optimal (type 4, 1032 leaves, 14 steps):

$$-\frac{\sqrt{c+d x^8}}{2 a c x^2}+\frac{\sqrt{d}\, x^2 \sqrt{c+d x^8}}{2 a c \left(\sqrt{c}+\sqrt{d}\, x^4\right)}+\frac{b^{3/4} \operatorname{ArcTan}\left[\frac{\sqrt{b c-a d}\, x^2}{(-a)^{1/4} b^{1/4} \sqrt{c+d x^8}}\right]}{8 (-a)^{5/4} \sqrt{b c-a d}}-\frac{b^{3/4} \operatorname{ArcTan}\left[\frac{\sqrt{-b c+a d}\, x^2}{(-a)^{1/4} b^{1/4} \sqrt{c+d x^8}}\right]}{8 (-a)^{5/4} \sqrt{-b c+a d}}-$$
$$\frac{d^{1/4} \left(\sqrt{c}+\sqrt{d}\, x^4\right) \sqrt{\frac{c+d x^8}{\left(\sqrt{c}+\sqrt{d}\, x^4\right)^2}}\, \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{d^{1/4} x^2}{c^{1/4}}\right], \frac{1}{2}\right]}{2 a c^{3/4} \sqrt{c+d x^8}}+$$
$$\frac{d^{1/4} \left(\sqrt{c}+\sqrt{d}\, x^4\right) \sqrt{\frac{c+d x^8}{\left(\sqrt{c}+\sqrt{d}\, x^4\right)^2}}\, \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{d^{1/4} x^2}{c^{1/4}}\right], \frac{1}{2}\right]}{4 a c^{3/4} \sqrt{c+d x^8}}+$$
$$\left(b \left(\sqrt{c}-\frac{\sqrt{-a}\, \sqrt{d}}{\sqrt{b}}\right) d^{1/4} \left(\sqrt{c}+\sqrt{d}\, x^4\right) \sqrt{\frac{c+d x^8}{\left(\sqrt{c}+\sqrt{d}\, x^4\right)^2}}\, \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{d^{1/4} x^2}{c^{1/4}}\right], \frac{1}{2}\right]\right) \Big/$$
$$\left(8 a c^{1/4} (b c+a d) \sqrt{c+d x^8}\right)+$$
$$\left(b \left(\sqrt{c}+\frac{\sqrt{-a}\, \sqrt{d}}{\sqrt{b}}\right) d^{1/4} \left(\sqrt{c}+\sqrt{d}\, x^4\right) \sqrt{\frac{c+d x^8}{\left(\sqrt{c}+\sqrt{d}\, x^4\right)^2}}\, \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{d^{1/4} x^2}{c^{1/4}}\right], \frac{1}{2}\right]\right) \Big/$$
$$\left(8 a c^{1/4} (b c+a d) \sqrt{c+d x^8}\right)+\left(\sqrt{b} \left(\sqrt{b}\, \sqrt{c}+\sqrt{-a}\, \sqrt{d}\right)^2 \left(\sqrt{c}+\sqrt{d}\, x^4\right)\right.$$
$$\left.\sqrt{\frac{c+d x^8}{\left(\sqrt{c}+\sqrt{d}\, x^4\right)^2}}\, \operatorname{EllipticPi}\left[-\frac{\left(\sqrt{b}\, \sqrt{c}-\sqrt{-a}\, \sqrt{d}\right)^2}{4 \sqrt{-a}\, \sqrt{b}\, \sqrt{c}\, \sqrt{d}}, 2 \operatorname{ArcTan}\left[\frac{d^{1/4} x^2}{c^{1/4}}\right], \frac{1}{2}\right]\right) \Big/$$
$$\left(16 (-a)^{3/2} c^{1/4} d^{1/4} (b c+a d) \sqrt{c+d x^8}\right)-\left(\sqrt{b} \left(\sqrt{b}\, \sqrt{c}-\sqrt{-a}\, \sqrt{d}\right)^2 \left(\sqrt{c}+\sqrt{d}\, x^4\right)\right.$$
$$\left.\sqrt{\frac{c+d x^8}{\left(\sqrt{c}+\sqrt{d}\, x^4\right)^2}}\, \operatorname{EllipticPi}\left[\frac{\left(\sqrt{b}\, \sqrt{c}+\sqrt{-a}\, \sqrt{d}\right)^2}{4 \sqrt{-a}\, \sqrt{b}\, \sqrt{c}\, \sqrt{d}}, 2 \operatorname{ArcTan}\left[\frac{d^{1/4} x^2}{c^{1/4}}\right], \frac{1}{2}\right]\right) \Big/$$
$$\left(16 (-a)^{3/2} c^{1/4} d^{1/4} (b c+a d) \sqrt{c+d x^8}\right)$$

Result (type 6, 344 leaves):

$$\frac{1}{42 x^2 \sqrt{c+d x^8}}\left(-\frac{21 \left(c+d x^8\right)}{a c}+\left(49 \left(b c-a d\right) x^8 \text{AppellF1}\left[\frac{3}{4},\frac{1}{2},1,\frac{7}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\Big/\left(\left(a+b x^8\right)\left(-7 a c \text{AppellF1}\left[\frac{3}{4},\frac{1}{2},1,\frac{7}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+2 x^8 \left(2 b c \text{AppellF1}\left[\frac{7}{4},\frac{1}{2},2,\frac{11}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+a d \text{AppellF1}\left[\frac{7}{4},\frac{3}{2},1,\frac{11}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\right)\right)-\left(33 b d x^{16} \text{AppellF1}\left[\frac{7}{4},\frac{1}{2},1,\frac{11}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\Big/\left(\left(a+b x^8\right)\left(-11 a c \text{AppellF1}\left[\frac{7}{4},\frac{1}{2},1,\frac{11}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+2 x^8 \left(2 b c \text{AppellF1}\left[\frac{11}{4},\frac{1}{2},2,\frac{15}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+a d \text{AppellF1}\left[\frac{11}{4},\frac{3}{2},1,\frac{15}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\right)\right)\right)$$

Problem 735: Result more than twice size of optimal antiderivative.

$$\int \frac{x^4}{\left(a+b x^8\right) \sqrt{c+d x^8}} \, dx$$

Optimal (type 6, 64 leaves, 2 steps):

$$\frac{x^5 \sqrt{1+\frac{d x^8}{c}} \text{AppellF1}\left[\frac{5}{8},1,\frac{1}{2},\frac{13}{8},-\frac{b x^8}{a},-\frac{d x^8}{c}\right]}{5 a \sqrt{c+d x^8}}$$

Result (type 6, 165 leaves):

$$-\left(\left(13 a c x^5 \text{AppellF1}\left[\frac{5}{8},\frac{1}{2},1,\frac{13}{8},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\Big/\left(5 \left(a+b x^8\right) \sqrt{c+d x^8} \left(-13 a c \text{AppellF1}\left[\frac{5}{8},\frac{1}{2},1,\frac{13}{8},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+4 x^8 \left(2 b c \text{AppellF1}\left[\frac{13}{8},\frac{1}{2},2,\frac{21}{8},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+a d \text{AppellF1}\left[\frac{13}{8},\frac{3}{2},1,\frac{21}{8},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\right)\right)\right)$$

Problem 736: Result more than twice size of optimal antiderivative.

$$\int \frac{x^2}{\left(a+b x^8\right) \sqrt{c+d x^8}} \, dx$$

Optimal (type 6, 64 leaves, 2 steps):

$$\frac{x^3 \sqrt{1+\frac{d x^8}{c}} \text{AppellF1}\left[\frac{3}{8},1,\frac{1}{2},\frac{11}{8},-\frac{b x^8}{a},-\frac{d x^8}{c}\right]}{3 a \sqrt{c+d x^8}}$$

Result (type 6, 165 leaves):

$$-\left(\left(11\,a\,c\,x^3\,\text{AppellF1}\left[\frac{3}{8},\frac{1}{2},1,\frac{11}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\Big/\left(3\left(a+b\,x^8\right)\sqrt{c+d\,x^8}\left(-11\,a\,c\,\text{AppellF1}\left[\frac{3}{8},\frac{1}{2},1,\frac{11}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+4\,x^8\left(2\,b\,c\,\text{AppellF1}\left[\frac{11}{8},\frac{1}{2},2,\frac{19}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{11}{8},\frac{3}{2},1,\frac{19}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\right)\right)\right)$$

Problem 737: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(a+b\,x^8\right)\sqrt{c+d\,x^8}}\,dx$$

Optimal (type 6, 59 leaves, 2 steps):

$$\frac{x\,\sqrt{1+\frac{d\,x^8}{c}}\,\text{AppellF1}\left[\frac{1}{8},1,\frac{1}{2},\frac{9}{8},-\frac{b\,x^8}{a},-\frac{d\,x^8}{c}\right]}{a\,\sqrt{c+d\,x^8}}$$

Result (type 6, 161 leaves):

$$-\left(\left(9\,a\,c\,x\,\text{AppellF1}\left[\frac{1}{8},\frac{1}{2},1,\frac{9}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\Big/\left(\left(a+b\,x^8\right)\sqrt{c+d\,x^8}\left(-9\,a\,c\,\text{AppellF1}\left[\frac{1}{8},\frac{1}{2},1,\frac{9}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+4\,x^8\left(2\,b\,c\,\text{AppellF1}\left[\frac{9}{8},\frac{1}{2},2,\frac{17}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{9}{8},\frac{3}{2},1,\frac{17}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\right)\right)\right)$$

Problem 738: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^2\left(a+b\,x^8\right)\sqrt{c+d\,x^8}}\,dx$$

Optimal (type 6, 62 leaves, 2 steps):

$$-\frac{\sqrt{1+\frac{d\,x^8}{c}}\,\text{AppellF1}\left[-\frac{1}{8},1,\frac{1}{2},\frac{7}{8},-\frac{b\,x^8}{a},-\frac{d\,x^8}{c}\right]}{a\,x\,\sqrt{c+d\,x^8}}$$

Result (type 6, 344 leaves):

$$\frac{1}{35\,x\,\sqrt{c+d\,x^8}}\left(-\frac{35\,(c+d\,x^8)}{a\,c}+\left(75\,(b\,c-3\,a\,d)\,x^8\,\text{AppellF1}\left[\frac{7}{8},\frac{1}{2},1,\frac{15}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\Big/\left((a+b\,x^8)\left(-15\,a\,c\,\text{AppellF1}\left[\frac{7}{8},\frac{1}{2},1,\frac{15}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+4\,x^8\left(2\,b\,c\,\text{AppellF1}\left[\frac{15}{8},\frac{1}{2},2,\frac{23}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{15}{8},\frac{3}{2},1,\frac{23}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\right)\right)-\left(161\,b\,d\,x^{16}\,\text{AppellF1}\left[\frac{15}{8},\frac{1}{2},1,\frac{23}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\Big/\left((a+b\,x^8)\left(-23\,a\,c\,\text{AppellF1}\left[\frac{15}{8},\frac{1}{2},1,\frac{23}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+4\,x^8\left(2\,b\,c\,\text{AppellF1}\left[\frac{23}{8},\frac{1}{2},2,\frac{31}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{23}{8},\frac{3}{2},1,\frac{31}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\right)\right)\right)$$

Problem 739: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^4\,(a+b\,x^8)\,\sqrt{c+d\,x^8}}\,dx$$

Optimal (type 6, 64 leaves, 2 steps):

$$-\frac{\sqrt{1+\frac{d\,x^8}{c}}\,\text{AppellF1}\left[-\frac{3}{8},1,\frac{1}{2},\frac{5}{8},-\frac{b\,x^8}{a},-\frac{d\,x^8}{c}\right]}{3\,a\,x^3\,\sqrt{c+d\,x^8}}$$

Result (type 6, 345 leaves):

$$\frac{1}{195\,x^3\,\sqrt{c+d\,x^8}}\left(-\frac{65\,(c+d\,x^8)}{a\,c}+\left(169\,(3\,b\,c-a\,d)\,x^8\,\text{AppellF1}\left[\frac{5}{8},\frac{1}{2},1,\frac{13}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\Big/\left((a+b\,x^8)\left(-13\,a\,c\,\text{AppellF1}\left[\frac{5}{8},\frac{1}{2},1,\frac{13}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+4\,x^8\left(2\,b\,c\,\text{AppellF1}\left[\frac{13}{8},\frac{1}{2},2,\frac{21}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{13}{8},\frac{3}{2},1,\frac{21}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\right)\right)-\left(105\,b\,d\,x^{16}\,\text{AppellF1}\left[\frac{13}{8},\frac{1}{2},1,\frac{21}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\Big/\left((a+b\,x^8)\left(-21\,a\,c\,\text{AppellF1}\left[\frac{13}{8},\frac{1}{2},1,\frac{21}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+4\,x^8\left(2\,b\,c\,\text{AppellF1}\left[\frac{21}{8},\frac{1}{2},2,\frac{29}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{21}{8},\frac{3}{2},1,\frac{29}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\right)\right)\right)$$

Problem 743: Result unnecessarily involves higher level functions and more

than twice size of optimal antiderivative.

$$\int \frac{1}{x \left(a+b x^8\right)^2 \sqrt{c+d x^8}} \, dx$$

Optimal (type 3, 132 leaves, 7 steps):

$$\frac{b \sqrt{c+d x^8}}{8 a (b c-a d) \left(a+b x^8\right)}-\frac{\text{ArcTanh}\left[\frac{\sqrt{c+d x^8}}{\sqrt{c}}\right]}{4 a^2 \sqrt{c}}+\frac{\sqrt{b} (2 b c-3 a d) \text{ArcTanh}\left[\frac{\sqrt{b} \sqrt{c+d x^8}}{\sqrt{b c-a d}}\right]}{8 a^2 (b c-a d)^{3/2}}$$

Result (type 6, 396 leaves):

$$\Bigg(b \Bigg(\bigg(6 c d x^8 \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\bigg)\Big/ \bigg(-4 a c \text{AppellF1}\left[1,\frac{1}{2},1,2,-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+x^8 \left(2 b c \text{AppellF1}\left[2,\frac{1}{2},2,3,-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+a d \text{AppellF1}\left[2,\frac{3}{2},1,3,-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\bigg)+\bigg(5 d x^8 \left(2 a d+b \left(c+3 d x^8\right)\right) \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^8},-\frac{a}{b x^8}\right]-3 \left(c+d x^8\right) \left(2 a d \text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^8},-\frac{a}{b x^8}\right]+b c \text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^8},-\frac{a}{b x^8}\right]\right)\bigg)\Big/ \bigg(a \bigg(-5 b d x^8 \text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},-\frac{c}{d x^8},-\frac{a}{b x^8}\right]+2 a d \text{AppellF1}\left[\frac{5}{2},\frac{1}{2},2,\frac{7}{2},-\frac{c}{d x^8},-\frac{a}{b x^8}\right]+b c \text{AppellF1}\left[\frac{5}{2},\frac{3}{2},1,\frac{7}{2},-\frac{c}{d x^8},-\frac{a}{b x^8}\right]\bigg)\bigg)\Bigg)\Bigg)\Big/ \left(24 (-b c+a d) \left(a+b x^8\right) \sqrt{c+d x^8}\right)$$

Problem 744: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{x^9 \left(a+b x^8\right)^2 \sqrt{c+d x^8}} \, dx$$

Optimal (type 3, 185 leaves, 8 steps):

$$-\frac{b (2 b c-a d) \sqrt{c+d x^8}}{8 a^2 c (b c-a d) \left(a+b x^8\right)}-\frac{\sqrt{c+d x^8}}{8 a c x^8 \left(a+b x^8\right)}+\frac{(4 b c+a d) \text{ArcTanh}\left[\frac{\sqrt{c+d x^8}}{\sqrt{c}}\right]}{8 a^3 c^{3/2}}-\frac{b^{3/2} (4 b c-5 a d) \text{ArcTanh}\left[\frac{\sqrt{b} \sqrt{c+d x^8}}{\sqrt{b c-a d}}\right]}{8 a^3 (b c-a d)^{3/2}}$$

Result (type 6, 489 leaves):

$$\left(\left(6\,a\,b\,d\,\left(-2\,b\,c+a\,d\right)\,x^{16}\,\text{AppellF1}\left[1,\,\frac{1}{2},\,1,\,2,\,-\frac{d\,x^8}{c},\,-\frac{b\,x^8}{a}\right]\right)\Big/\right.$$

$$\left(\left(-b\,c+a\,d\right)\,\left(-4\,a\,c\,\text{AppellF1}\left[1,\,\frac{1}{2},\,1,\,2,\,-\frac{d\,x^8}{c},\,-\frac{b\,x^8}{a}\right]+x^8\,\left(2\,b\,c\right.\right.\right.$$

$$\left.\left.\left.\text{AppellF1}\left[2,\,\frac{1}{2},\,2,\,3,\,-\frac{d\,x^8}{c},\,-\frac{b\,x^8}{a}\right]+a\,d\,\text{AppellF1}\left[2,\,\frac{3}{2},\,1,\,3,\,-\frac{d\,x^8}{c},\,-\frac{b\,x^8}{a}\right]\right)\right)\right)+$$

$$\left(5\,b\,d\,x^8\,\left(-a^2\,d\,\left(3\,c+2\,d\,x^8\right)+2\,b^2\,c\,x^8\,\left(c+3\,d\,x^8\right)+3\,a\,b\,\left(c^2+c\,d\,x^8-d^2\,x^{16}\right)\right)\right.$$

$$\text{AppellF1}\left[\frac{3}{2},\,\frac{1}{2},\,1,\,\frac{5}{2},\,-\frac{c}{d\,x^8},\,-\frac{a}{b\,x^8}\right]+$$

$$3\,\left(c+d\,x^8\right)\,\left(a^2\,d-2\,b^2\,c\,x^8+a\,b\,\left(-c+d\,x^8\right)\right)\,\left(2\,a\,d\,\text{AppellF1}\left[\frac{5}{2},\,\frac{1}{2},\,2,\,\frac{7}{2},\,-\frac{c}{d\,x^8},\,-\frac{a}{b\,x^8}\right]+\right.$$

$$\left.\left.b\,c\,\text{AppellF1}\left[\frac{5}{2},\,\frac{3}{2},\,1,\,\frac{7}{2},\,-\frac{c}{d\,x^8},\,-\frac{a}{b\,x^8}\right]\right)\right)\Big/$$

$$\left(c\,\left(b\,c-a\,d\right)\,\left(-5\,b\,d\,x^8\,\text{AppellF1}\left[\frac{3}{2},\,\frac{1}{2},\,1,\,\frac{5}{2},\,-\frac{c}{d\,x^8},\,-\frac{a}{b\,x^8}\right]+\right.\right.$$

$$2\,a\,d\,\text{AppellF1}\left[\frac{5}{2},\,\frac{1}{2},\,2,\,\frac{7}{2},\,-\frac{c}{d\,x^8},\,-\frac{a}{b\,x^8}\right]+$$

$$\left.\left.\left.b\,c\,\text{AppellF1}\left[\frac{5}{2},\,\frac{3}{2},\,1,\,\frac{7}{2},\,-\frac{c}{d\,x^8},\,-\frac{a}{b\,x^8}\right]\right)\right)\right)\Big/\left(24\,a^2\,x^8\,\left(a+b\,x^8\right)\,\sqrt{c+d\,x^8}\right)$$

Problem 750: Result unnecessarily involves higher level functions.

$$\int\frac{x^9}{\left(a+b\,x^8\right)^2\,\sqrt{c+d\,x^8}}\,dx$$

Optimal (type 4, 924 leaves, 11 steps):

$$-\frac{x^2\sqrt{c+d\,x^8}}{8\,(b\,c-a\,d)\,(a+b\,x^8)}-\frac{(b\,c+a\,d)\,\text{ArcTan}\left[\frac{\sqrt{b\,c-a\,d}\,x^2}{(-a)^{1/4}\,b^{1/4}\,\sqrt{c+d\,x^8}}\right]}{32\,(-a)^{3/4}\,b^{3/4}\,(b\,c-a\,d)^{3/2}}+\frac{(b\,c+a\,d)\,\text{ArcTan}\left[\frac{\sqrt{-b\,c+a\,d}\,x^2}{(-a)^{1/4}\,b^{1/4}\,\sqrt{c+d\,x^8}}\right]}{32\,(-a)^{3/4}\,b^{3/4}\,(-b\,c+a\,d)^{3/2}}+$$

$$\left(\left(\frac{\sqrt{b}\,\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)d^{1/4}\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+d\,x^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}\,x^2}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(32\,b\,c^{1/4}\,(b\,c-a\,d)\,\sqrt{c+d\,x^8}\right)+\left(\left(\sqrt{-a}\,\sqrt{b}\,\sqrt{c}+a\,\sqrt{d}\right)d^{1/4}\left(\sqrt{c}+\sqrt{d}\,x^4\right)\right.$$

$$\left.\sqrt{\frac{c+d\,x^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}\,x^2}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(32\,a\,b\,c^{1/4}\,(b\,c-a\,d)\,\sqrt{c+d\,x^8}\right)-$$

$$\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+d\,x^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}\,x^2}{c^{1/4}}\right],\frac{1}{2}\right]}{16\,b\,c^{1/4}\,(b\,c-a\,d)\,\sqrt{c+d\,x^8}}+$$

$$\left(\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+d\,x^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\,\text{EllipticPi}\left[\right.\right.$$

$$\left.\left.-\frac{\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2}{4\,\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}},2\,\text{ArcTan}\left[\frac{d^{1/4}\,x^2}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(64\,a\,b\,c^{1/4}\,d^{1/4}\,(b\,c-a\,d)\,\sqrt{c+d\,x^8}\right)+$$

$$\left(\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+d\,x^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\,\text{EllipticPi}\left[\right.\right.$$

$$\left.\left.\frac{\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2}{4\,\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}},2\,\text{ArcTan}\left[\frac{d^{1/4}\,x^2}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(64\,a\,b\,c^{1/4}\,d^{1/4}\,(b\,c-a\,d)\,\sqrt{c+d\,x^8}\right)$$

Result (type 6, 333 leaves) :

$$\left(x^2 \left(5 \left(c+d x^8\right)+\left(25 a c^2 \text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\Big/\left(-5 a c \text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+2 x^8 \left(2 b c\, \text{AppellF1}\left[\frac{5}{4},\frac{1}{2},2,\frac{9}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+a d \text{AppellF1}\left[\frac{5}{4},\frac{3}{2},1,\frac{9}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\right)-\left(9 a c d x^8 \text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\Big/\left(-9 a c \text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+2 x^8 \left(2 b c \text{AppellF1}\left[\frac{9}{4},\frac{1}{2},2,\frac{13}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+a d \text{AppellF1}\left[\frac{9}{4},\frac{3}{2},1,\frac{13}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\right)\right)\right)\Big/\left(40 \left(-b c+a d\right) \left(a+b x^8\right) \sqrt{c+d x^8}\right)$$

Problem 751: Result unnecessarily involves higher level functions.

$$\int \frac{x}{\left(a+b x^8\right)^2 \sqrt{c+d x^8}}\, dx$$

Optimal (type 4, 999 leaves, 11 steps):

$$\frac{b\,x^2\sqrt{c+d\,x^8}}{8\,a\,(b\,c-a\,d)\,(a+b\,x^8)}+\frac{b^{1/4}\,(3\,b\,c-5\,a\,d)\,\text{ArcTan}\left[\frac{\sqrt{b\,c-a\,d}\;x^2}{(-a)^{1/4}\,b^{1/4}\sqrt{c+d\,x^8}}\right]}{32\,(-a)^{7/4}\,(b\,c-a\,d)^{3/2}}-$$

$$\frac{b^{1/4}\,(3\,b\,c-5\,a\,d)\,\text{ArcTan}\left[\frac{\sqrt{-b\,c+a\,d}\;x^2}{(-a)^{1/4}\,b^{1/4}\sqrt{c+d\,x^8}}\right]}{32\,(-a)^{7/4}\,(-b\,c+a\,d)^{3/2}}+$$

$$\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}\;x^4\right)\sqrt{\frac{c+d\,x^8}{\left(\sqrt{c}+\sqrt{d}\;x^4\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}\,x^2}{c^{1/4}}\right],\frac{1}{2}\right]}{16\,a\,c^{1/4}\,(b\,c-a\,d)\sqrt{c+d\,x^8}}+$$

$$\left(\left(\frac{\sqrt{b}\;\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)d^{1/4}\,(3\,b\,c-5\,a\,d)\left(\sqrt{c}+\sqrt{d}\;x^4\right)\sqrt{\frac{c+d\,x^8}{\left(\sqrt{c}+\sqrt{d}\;x^4\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}\,x^2}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(32\,a\,c^{1/4}\,(b\,c-a\,d)\,(b\,c+a\,d)\sqrt{c+d\,x^8}\right)+$$

$$\left(\left(\sqrt{b}\;\sqrt{c}-\sqrt{-a}\;\sqrt{d}\right)d^{1/4}\,(3\,b\,c-5\,a\,d)\left(\sqrt{c}+\sqrt{d}\;x^4\right)\sqrt{\frac{c+d\,x^8}{\left(\sqrt{c}+\sqrt{d}\;x^4\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}\,x^2}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(32\,(-a)^{3/2}\,c^{1/4}\,(b\,c-a\,d)\,(b\,c+a\,d)\sqrt{c+d\,x^8}\right)+$$

$$\left(\left(\sqrt{b}\;\sqrt{c}+\sqrt{-a}\;\sqrt{d}\right)^2(3\,b\,c-5\,a\,d)\left(\sqrt{c}+\sqrt{d}\;x^4\right)\sqrt{\frac{c+d\,x^8}{\left(\sqrt{c}+\sqrt{d}\;x^4\right)^2}}\right.$$

$$\left.\text{EllipticPi}\left[-\frac{\left(\sqrt{b}\;\sqrt{c}-\sqrt{-a}\;\sqrt{d}\right)^2}{4\sqrt{-a}\;\sqrt{b}\;\sqrt{c}\;\sqrt{d}},\,2\,\text{ArcTan}\left[\frac{d^{1/4}\,x^2}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(64\,a^2\,c^{1/4}\,d^{1/4}\,(b\,c-a\,d)\,(b\,c+a\,d)\sqrt{c+d\,x^8}\right)+$$

$$\left(\left(\sqrt{b}\;\sqrt{c}-\sqrt{-a}\;\sqrt{d}\right)^2(3\,b\,c-5\,a\,d)\left(\sqrt{c}+\sqrt{d}\;x^4\right)\sqrt{\frac{c+d\,x^8}{\left(\sqrt{c}+\sqrt{d}\;x^4\right)^2}}\right.$$

$$\left.\text{EllipticPi}\left[\frac{\left(\sqrt{b}\;\sqrt{c}+\sqrt{-a}\;\sqrt{d}\right)^2}{4\sqrt{-a}\;\sqrt{b}\;\sqrt{c}\;\sqrt{d}},\,2\,\text{ArcTan}\left[\frac{d^{1/4}\,x^2}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(64\,a^2\,c^{1/4}\,d^{1/4}\,(b\,c-a\,d)\,(b\,c+a\,d)\sqrt{c+d\,x^8}\right)$$

Result (type 6, 343 leaves):

$$\left(x^2 \left(-\frac{5 b \left(c+d x^8\right)}{a}+\left(25 c \left(3 b c-4 a d\right) \text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\Big/\left(-5 a c \text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+2 x^8 \left(2 b c \text{AppellF1}\left[\frac{5}{4},\frac{1}{2},2,\frac{9}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+a d \text{AppellF1}\left[\frac{5}{4},\frac{3}{2},1,\frac{9}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\right)+\left(9 b c d x^8 \text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\Big/\left(-9 a c \text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+2 x^8 \left(2 b c \text{AppellF1}\left[\frac{9}{4},\frac{1}{2},2,\frac{13}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+a d \text{AppellF1}\left[\frac{9}{4},\frac{3}{2},1,\frac{13}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\right)\right)\right)\Big/\left(40 \left(-b c+a d\right) \left(a+b x^8\right) \sqrt{c+d x^8}\right)$$

Problem 752: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^7 \left(a+b x^8\right)^2 \sqrt{c+d x^8}} \, dx$$

Optimal (type 4, 1060 leaves, 12 steps):

$$-\frac{(7bc-4ad)\sqrt{c+dx^8}}{24a^2c\,(bc-ad)\,x^6}+\frac{b\sqrt{c+dx^8}}{8a\,(bc-ad)\,x^6\,(a+bx^8)}+$$

$$\frac{b^{5/4}(7bc-9ad)\,\text{ArcTan}\left[\frac{\sqrt{bc-ad}\,x^2}{(-a)^{1/4}b^{1/4}\sqrt{c+dx^8}}\right]}{32\,(-a)^{11/4}(bc-ad)^{3/2}}-\frac{b^{5/4}(7bc-9ad)\,\text{ArcTan}\left[\frac{\sqrt{-bc+ad}\,x^2}{(-a)^{1/4}b^{1/4}\sqrt{c+dx^8}}\right]}{32\,(-a)^{11/4}(-bc+ad)^{3/2}}-$$

$$\left(d^{3/4}(7bc-4ad)\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}x^2}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(48a^2c^{5/4}(bc-ad)\sqrt{c+dx^8}\right)+$$

$$\left(b\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)d^{1/4}(7bc-9ad)\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}x^2}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(32\,(-a)^{5/2}c^{1/4}(bc-ad)(bc+ad)\sqrt{c+dx^8}\right)-$$

$$\left(b\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)d^{1/4}(7bc-9ad)\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}x^2}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(32\,(-a)^{5/2}c^{1/4}(bc-ad)(bc+ad)\sqrt{c+dx^8}\right)-$$

$$\left(b\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2(7bc-9ad)\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\right.$$

$$\left.\text{EllipticPi}\left[-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\,\text{ArcTan}\left[\frac{d^{1/4}x^2}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(64a^3c^{1/4}d^{1/4}(bc-ad)(bc+ad)\sqrt{c+dx^8}\right)-$$

$$\left(b\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2(7bc-9ad)\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\right.$$

$$\left.\text{EllipticPi}\left[\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\,\text{ArcTan}\left[\frac{d^{1/4}x^2}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(64a^3c^{1/4}d^{1/4}(bc-ad)(bc+ad)\sqrt{c+dx^8}\right)$$

Result (type 6, 399 leaves):

$$\left(\frac{5\left(c+d x^8\right)\left(-4 a^2 d+7 b^2 c x^8+4 a b\left(c-d x^8\right)\right)}{c}+\left(25 a\left(-21 b^2 c^2+20 a b c d+4 a^2 d^2\right) x^8 \text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\Big/\left(-5 a c\, \text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+2 x^8\left(2 b c\, \text{AppellF1}\left[\frac{5}{4},\frac{1}{2},2,\frac{9}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+a d\, \text{AppellF1}\left[\frac{5}{4},\frac{3}{2},1,\frac{9}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\right)+\left(9 a b d\left(-7 b c+4 a d\right) x^{16} \text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\Big/\left(-9 a c\, \text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+2 x^8\left(2 b c\, \text{AppellF1}\left[\frac{9}{4},\frac{1}{2},2,\frac{13}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+a d\, \text{AppellF1}\left[\frac{9}{4},\frac{3}{2},1,\frac{13}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\right)\right)\Big/\left(120 a^2\left(-b c+a d\right) x^6\left(a+b x^8\right)\sqrt{c+d x^8}\right)$$

Problem 753: Result unnecessarily involves higher level functions.

$$\int \frac{x^{13}}{\left(a+b x^8\right)^2 \sqrt{c+d x^8}}\, dx$$

Optimal (type 4, 1164 leaves, 14 steps):

$$\frac{\sqrt{d}\, x^2 \sqrt{c+d x^8}}{8 b\left(b c-a d\right)\left(\sqrt{c}+\sqrt{d}\, x^4\right)}-\frac{x^6 \sqrt{c+d x^8}}{8\left(b c-a d\right)\left(a+b x^8\right)}+\frac{\left(3 b c-a d\right) \text{ArcTan}\left[\frac{\sqrt{b c-a d}\, x^2}{(-a)^{1/4} b^{1/4} \sqrt{c+d x^8}}\right]}{32\,(-a)^{1/4} b^{5/4}\left(b c-a d\right)^{3/2}}+\frac{\left(3 b c-a d\right) \text{ArcTan}\left[\frac{\sqrt{-b c+a d}\, x^2}{(-a)^{1/4} b^{1/4} \sqrt{c+d x^8}}\right]}{32\,(-a)^{1/4} b^{5/4}\left(-b c+a d\right)^{3/2}}-\left(c^{1/4} d^{1/4}\left(\sqrt{c}+\sqrt{d}\, x^4\right)\sqrt{\frac{c+d x^8}{\left(\sqrt{c}+\sqrt{d}\, x^4\right)^2}}\, \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{d^{1/4} x^2}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(8 b\left(b c-a d\right)\sqrt{c+d x^8}\right)+\left(c^{1/4} d^{1/4}\left(\sqrt{c}+\sqrt{d}\, x^4\right)\sqrt{\frac{c+d x^8}{\left(\sqrt{c}+\sqrt{d}\, x^4\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{d^{1/4} x^2}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(16 b\left(b c-a d\right)\sqrt{c+d x^8}\right)-$$

$$\left(\left(\sqrt{c}-\frac{\sqrt{-a}\,\sqrt{d}}{\sqrt{b}}\right) d^{1/4}\,(3\,b\,c-a\,d)\,\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+d\,x^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}\,x^2}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(32\,b\,c^{1/4}\,(b\,c-a\,d)\,(b\,c+a\,d)\,\sqrt{c+d\,x^8}\right)-$$

$$\left(\left(\sqrt{c}+\frac{\sqrt{-a}\,\sqrt{d}}{\sqrt{b}}\right) d^{1/4}\,(3\,b\,c-a\,d)\,\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+d\,x^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}\,x^2}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(32\,b\,c^{1/4}\,(b\,c-a\,d)\,(b\,c+a\,d)\,\sqrt{c+d\,x^8}\right)+$$

$$\left(\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2(3\,b\,c-a\,d)\,\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+d\,x^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\;\text{EllipticPi}\left[-\frac{\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2}{4\,\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}},2\,\text{ArcTan}\left[\frac{d^{1/4}\,x^2}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(64\,\sqrt{-a}\,b^{3/2}\,c^{1/4}\,d^{1/4}\,(b\,c-a\,d)\,(b\,c+a\,d)\,\sqrt{c+d\,x^8}\right)-$$

$$\left(\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2(3\,b\,c-a\,d)\,\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+d\,x^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\;\text{EllipticPi}\left[\frac{\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2}{4\,\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}},2\,\text{ArcTan}\left[\frac{d^{1/4}\,x^2}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(64\,\sqrt{-a}\,b^{3/2}\,c^{1/4}\,d^{1/4}\,(b\,c-a\,d)\,(b\,c+a\,d)\,\sqrt{c+d\,x^8}\right)$$

Result (type 6, 333 leaves) :

$$\left(x^6 \left(7 \left(c+d x^8\right)+\left(49 a c^2 \text{AppellF1}\left[\frac{3}{4},\frac{1}{2},1,\frac{7}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\Big/\left(-7 a c \text{AppellF1}\left[\frac{3}{4},\frac{1}{2},1,\frac{7}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+2 x^8 \left(2 b c \text{AppellF1}\left[\frac{7}{4},\frac{1}{2},2,\frac{11}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+a d \text{AppellF1}\left[\frac{7}{4},\frac{3}{2},1,\frac{11}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\right)+\left(11 a c d x^8 \text{AppellF1}\left[\frac{7}{4},\frac{1}{2},1,\frac{11}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\Big/\left(-11 a c \text{AppellF1}\left[\frac{7}{4},\frac{1}{2},1,\frac{11}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+2 x^8 \left(2 b c \text{AppellF1}\left[\frac{11}{4},\frac{1}{2},2,\frac{15}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+a d \text{AppellF1}\left[\frac{11}{4},\frac{3}{2},1,\frac{15}{4},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\right)\right)\right)\Big/\left(56 (-b c+a d) \left(a+b x^8\right) \sqrt{c+d x^8}\right)$$

Problem 754: Result unnecessarily involves higher level functions.

$$\int \frac{x^5}{\left(a+b x^8\right)^2 \sqrt{c+d x^8}} \, dx$$

Optimal (type 4, 1162 leaves, 14 steps):

$$-\frac{\sqrt{d}\, x^2 \sqrt{c+d x^8}}{8 a (b c-a d) \left(\sqrt{c}+\sqrt{d}\, x^4\right)}+\frac{b x^6 \sqrt{c+d x^8}}{8 a (b c-a d) \left(a+b x^8\right)}-\frac{(b c-3 a d) \text{ArcTan}\left[\frac{\sqrt{b c-a d}\, x^2}{(-a)^{1/4} b^{1/4} \sqrt{c+d x^8}}\right]}{32 (-a)^{5/4} b^{1/4} (b c-a d)^{3/2}}-\frac{(b c-3 a d) \text{ArcTan}\left[\frac{\sqrt{-b c+a d}\, x^2}{(-a)^{1/4} b^{1/4} \sqrt{c+d x^8}}\right]}{32 (-a)^{5/4} b^{1/4} (-b c+a d)^{3/2}}+$$

$$\left(c^{1/4} d^{1/4} \left(\sqrt{c}+\sqrt{d}\, x^4\right) \sqrt{\frac{c+d x^8}{\left(\sqrt{c}+\sqrt{d}\, x^4\right)^2}}\, \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{d^{1/4} x^2}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(8 a (b c-a d) \sqrt{c+d x^8}\right)-$$

$$\left(c^{1/4} d^{1/4} \left(\sqrt{c}+\sqrt{d}\, x^4\right) \sqrt{\frac{c+d x^8}{\left(\sqrt{c}+\sqrt{d}\, x^4\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{d^{1/4} x^2}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(16 a (b c-a d) \sqrt{c+d x^8}\right)-$$

$$\left(\left(\sqrt{c}-\frac{\sqrt{-a}\,\sqrt{d}}{\sqrt{b}}\right) d^{1/4} (b c-3 a d) \left(\sqrt{c}+\sqrt{d}\, x^4\right) \sqrt{\frac{c+d x^8}{\left(\sqrt{c}+\sqrt{d}\, x^4\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{d^{1/4} x^2}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(32 a c^{1/4} (b c-a d) (b c+a d) \sqrt{c+d x^8}\right)-$$

$$\left(\left(\sqrt{c}+\frac{\sqrt{-a}\,\sqrt{d}}{\sqrt{b}}\right) d^{1/4}\,(b\,c-3\,a\,d)\,\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+d\,x^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}\,x^2}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(32\,a\,c^{1/4}\,(b\,c-a\,d)\,(b\,c+a\,d)\,\sqrt{c+d\,x^8}\right)-$$

$$\left(\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2 (b\,c-3\,a\,d)\,\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+d\,x^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\right.$$

$$\left.\text{EllipticPi}\left[-\frac{\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2}{4\,\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}},2\,\text{ArcTan}\left[\frac{d^{1/4}\,x^2}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(64\,(-a)^{3/2}\sqrt{b}\,c^{1/4}\,d^{1/4}\,(b\,c-a\,d)\,(b\,c+a\,d)\,\sqrt{c+d\,x^8}\right)+$$

$$\left(\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2 (b\,c-3\,a\,d)\,\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+d\,x^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\right.$$

$$\left.\text{EllipticPi}\left[\frac{\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2}{4\,\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}},2\,\text{ArcTan}\left[\frac{d^{1/4}\,x^2}{c^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(64\,(-a)^{3/2}\sqrt{b}\,c^{1/4}\,d^{1/4}\,(b\,c-a\,d)\,(b\,c+a\,d)\,\sqrt{c+d\,x^8}\right)$$

Result (type 6, 342 leaves):

$$\left(x^6\left(-\frac{21\,b\,(c+d\,x^8)}{a}+\left(49\,c\,(b\,c-4\,a\,d)\,\text{AppellF1}\left[\frac{3}{4},\frac{1}{2},1,\frac{7}{4},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\Big/\right.\right.$$

$$\left(-7\,a\,c\,\text{AppellF1}\left[\frac{3}{4},\frac{1}{2},1,\frac{7}{4},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+2\,x^8\left(2\,b\,c\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},2,\right.\right.\right.$$

$$\left.\left.\left.\frac{11}{4},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{7}{4},\frac{3}{2},1,\frac{11}{4},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\right)-$$

$$\left(33\,b\,c\,d\,x^8\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},1,\frac{11}{4},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\Big/$$

$$\left(-11\,a\,c\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},1,\frac{11}{4},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+\right.$$

$$2\,x^8\left(2\,b\,c\,\text{AppellF1}\left[\frac{11}{4},\frac{1}{2},2,\frac{15}{4},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{11}{4},\frac{3}{2},\right.\right.$$

$$\left.\left.\left.\left.\left.1,\frac{15}{4},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\right)\right)\right)\Big/\left(168\,(-b\,c+a\,d)\,(a+b\,x^8)\,\sqrt{c+d\,x^8}\right)$$

Problem 755: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^3 \left(a+b x^8\right)^2 \sqrt{c+d x^8}} \, dx$$

Optimal (type 4, 1243 leaves, 15 steps):

$$-\frac{(5 b c-4 a d) \sqrt{c+d x^8}}{8 a^2 c (b c-a d) x^2}+\frac{\sqrt{d} (5 b c-4 a d) x^2 \sqrt{c+d x^8}}{8 a^2 c (b c-a d) \left(\sqrt{c}+\sqrt{d} x^4\right)}+\frac{b \sqrt{c+d x^8}}{8 a (b c-a d) x^2 \left(a+b x^8\right)}-$$

$$\frac{b^{3/4} (5 b c-7 a d) \text{ArcTan}\left[\frac{\sqrt{b c-a d}\, x^2}{(-a)^{1/4} b^{1/4} \sqrt{c+d x^8}}\right]}{32 (-a)^{9/4} (b c-a d)^{3/2}}-\frac{b^{3/4} (5 b c-7 a d) \text{ArcTan}\left[\frac{\sqrt{-b c+a d}\, x^2}{(-a)^{1/4} b^{1/4} \sqrt{c+d x^8}}\right]}{32 (-a)^{9/4} (-b c+a d)^{3/2}}-$$

$$\left(d^{1/4} (5 b c-4 a d) \left(\sqrt{c}+\sqrt{d} x^4\right) \sqrt{\frac{c+d x^8}{\left(\sqrt{c}+\sqrt{d} x^4\right)^2}} \,\text{EllipticE}\left[2 \text{ArcTan}\left[\frac{d^{1/4} x^2}{c^{1/4}}\right], \frac{1}{2}\right]\right) \Big/$$

$$\left(8 a^2 c^{3/4} (b c-a d) \sqrt{c+d x^8}\right)+$$

$$\left(d^{1/4} (5 b c-4 a d) \left(\sqrt{c}+\sqrt{d} x^4\right) \sqrt{\frac{c+d x^8}{\left(\sqrt{c}+\sqrt{d} x^4\right)^2}} \,\text{EllipticF}\left[2 \text{ArcTan}\left[\frac{d^{1/4} x^2}{c^{1/4}}\right], \frac{1}{2}\right]\right) \Big/$$

$$\left(16 a^2 c^{3/4} (b c-a d) \sqrt{c+d x^8}\right)+$$

$$\left(b \left(\sqrt{c}-\frac{\sqrt{-a} \sqrt{d}}{\sqrt{b}}\right) d^{1/4} (5 b c-7 a d) \left(\sqrt{c}+\sqrt{d} x^4\right) \sqrt{\frac{c+d x^8}{\left(\sqrt{c}+\sqrt{d} x^4\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[2 \text{ArcTan}\left[\frac{d^{1/4} x^2}{c^{1/4}}\right], \frac{1}{2}\right]\right) \Big/ \left(32 a^2 c^{1/4} (b c-a d) (b c+a d) \sqrt{c+d x^8}\right)+$$

$$\left(b \left(\sqrt{c}+\frac{\sqrt{-a} \sqrt{d}}{\sqrt{b}}\right) d^{1/4} (5 b c-7 a d) \left(\sqrt{c}+\sqrt{d} x^4\right) \sqrt{\frac{c+d x^8}{\left(\sqrt{c}+\sqrt{d} x^4\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[2 \text{ArcTan}\left[\frac{d^{1/4} x^2}{c^{1/4}}\right], \frac{1}{2}\right]\right) \Big/ \left(32 a^2 c^{1/4} (b c-a d) (b c+a d) \sqrt{c+d x^8}\right)-$$

$$\left(\sqrt{b} \left(\sqrt{b} \sqrt{c}+\sqrt{-a} \sqrt{d}\right)^2 (5 b c-7 a d) \left(\sqrt{c}+\sqrt{d} x^4\right) \sqrt{\frac{c+d x^8}{\left(\sqrt{c}+\sqrt{d} x^4\right)^2}}\right.$$

$$\text{EllipticPi}\left[-\frac{\left(\sqrt{b}\ \sqrt{c}-\sqrt{-a}\ \sqrt{d}\right)^2}{4\sqrt{-a}\ \sqrt{b}\ \sqrt{c}\ \sqrt{d}},\ 2\,\text{ArcTan}\left[\frac{d^{1/4}\,x^2}{c^{1/4}}\right],\ \frac{1}{2}\right]\Bigg/$$
$$\left(64\,(-a)^{5/2}\,c^{1/4}\,d^{1/4}\,(b\,c-a\,d)\,(b\,c+a\,d)\,\sqrt{c+d\,x^8}\right)+$$
$$\left(\sqrt{b}\,\left(\sqrt{b}\ \sqrt{c}-\sqrt{-a}\ \sqrt{d}\right)^2\,(5\,b\,c-7\,a\,d)\,\left(\sqrt{c}+\sqrt{d}\ x^4\right)\sqrt{\frac{c+d\,x^8}{\left(\sqrt{c}+\sqrt{d}\ x^4\right)^2}}\right.$$
$$\left.\text{EllipticPi}\left[\frac{\left(\sqrt{b}\ \sqrt{c}+\sqrt{-a}\ \sqrt{d}\right)^2}{4\sqrt{-a}\ \sqrt{b}\ \sqrt{c}\ \sqrt{d}},\ 2\,\text{ArcTan}\left[\frac{d^{1/4}\,x^2}{c^{1/4}}\right],\ \frac{1}{2}\right]\right)\Bigg/$$
$$\left(64\,(-a)^{5/2}\,c^{1/4}\,d^{1/4}\,(b\,c-a\,d)\,(b\,c+a\,d)\,\sqrt{c+d\,x^8}\right)$$

Result (type 6, 399 leaves) :

$$\left(\frac{21\,(c+d\,x^8)\,\left(-4\,a^2\,d+5\,b^2\,c\,x^8+4\,a\,b\,(c-d\,x^8)\right)}{c}-\right.$$
$$\left(49\,a\,(5\,b^2\,c^2-12\,a\,b\,c\,d+4\,a^2\,d^2)\,x^8\,\text{AppellF1}\left[\frac{3}{4},\frac{1}{2},1,\frac{7}{4},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\Bigg/$$
$$\left(-7\,a\,c\,\text{AppellF1}\left[\frac{3}{4},\frac{1}{2},1,\frac{7}{4},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+2\,x^8\,\left(2\,b\,c\right.\right.$$
$$\left.\left.\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},2,\frac{11}{4},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{7}{4},\frac{3}{2},1,\frac{11}{4},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\right)+$$
$$\left(33\,a\,b\,d\,(5\,b\,c-4\,a\,d)\,x^{16}\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},1,\frac{11}{4},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\Bigg/$$
$$\left(-11\,a\,c\,\text{AppellF1}\left[\frac{7}{4},\frac{1}{2},1,\frac{11}{4},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+\right.$$
$$2\,x^8\,\left(2\,b\,c\,\text{AppellF1}\left[\frac{11}{4},\frac{1}{2},2,\frac{15}{4},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+\right.$$
$$\left.\left.\left.a\,d\,\text{AppellF1}\left[\frac{11}{4},\frac{3}{2},1,\frac{15}{4},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\right)\right)\Bigg/$$
$$\left(168\,a^2\,(-b\,c+a\,d)\,x^2\,(a+b\,x^8)\,\sqrt{c+d\,x^8}\right)$$

Problem 756: Result more than twice size of optimal antiderivative.

$$\int\frac{x^4}{(a+b\,x^8)^2\,\sqrt{c+d\,x^8}}\,dx$$

Optimal (type 6, 64 leaves, 2 steps) :

$$\frac{x^5 \sqrt{1+\frac{d x^8}{c}}\ \text{AppellF1}\left[\frac{5}{8},2,\frac{1}{2},\frac{13}{8},-\frac{b x^8}{a},-\frac{d x^8}{c}\right]}{5 a^2 \sqrt{c+d x^8}}$$

Result (type 6, 343 leaves):

$$\left(x^5 \left(-\frac{65 b \left(c+d x^8\right)}{a}+\left(169 c \left(3 b c-8 a d\right) \text{AppellF1}\left[\frac{5}{8},\frac{1}{2},1,\frac{13}{8},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\Big/\left(-13 a c\, \text{AppellF1}\left[\frac{5}{8},\frac{1}{2},1,\frac{13}{8},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+4 x^8 \left(2 b c\, \text{AppellF1}\left[\frac{13}{8},\frac{1}{2},2,\frac{21}{8},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+a d\, \text{AppellF1}\left[\frac{13}{8},\frac{3}{2},1,\frac{21}{8},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\right)-\left(105 b c d x^8 \text{AppellF1}\left[\frac{13}{8},\frac{1}{2},1,\frac{21}{8},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\Big/\left(-21 a c\, \text{AppellF1}\left[\frac{13}{8},\frac{1}{2},1,\frac{21}{8},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+4 x^8 \left(2 b c\, \text{AppellF1}\left[\frac{21}{8},\frac{1}{2},2,\frac{29}{8},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+a d\, \text{AppellF1}\left[\frac{21}{8},\frac{3}{2},1,\frac{29}{8},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\right)\right)\right)\Big/\left(520 \left(-b c+a d\right) \left(a+b x^8\right) \sqrt{c+d x^8}\right)$$

Problem 757: Result more than twice size of optimal antiderivative.

$$\int \frac{x^2}{\left(a+b x^8\right)^2 \sqrt{c+d x^8}}\, dx$$

Optimal (type 6, 64 leaves, 2 steps):

$$\frac{x^3 \sqrt{1+\frac{d x^8}{c}}\ \text{AppellF1}\left[\frac{3}{8},2,\frac{1}{2},\frac{11}{8},-\frac{b x^8}{a},-\frac{d x^8}{c}\right]}{3 a^2 \sqrt{c+d x^8}}$$

Result (type 6, 343 leaves):

$$\left(x^3\left(-\frac{33\,b\,\left(c+d\,x^8\right)}{a}+\left(121\,c\,\left(5\,b\,c-8\,a\,d\right)\,\text{AppellF1}\left[\frac{3}{8},\frac{1}{2},1,\frac{11}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\Big/\left(-11\,a\,c\,\text{AppellF1}\left[\frac{3}{8},\frac{1}{2},1,\frac{11}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+4\,x^8\left(2\,b\,c\,\text{AppellF1}\left[\frac{11}{8},\frac{1}{2},2,\frac{19}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{11}{8},\frac{3}{2},1,\frac{19}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\right)+\left(57\,b\,c\,d\,x^8\,\text{AppellF1}\left[\frac{11}{8},\frac{1}{2},1,\frac{19}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\Big/\left(-19\,a\,c\,\text{AppellF1}\left[\frac{11}{8},\frac{1}{2},1,\frac{19}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+4\,x^8\left(2\,b\,c\,\text{AppellF1}\left[\frac{19}{8},\frac{1}{2},2,\frac{27}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{19}{8},\frac{3}{2},1,\frac{27}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\right)\right)\right)\Big/\left(264\,\left(-b\,c+a\,d\right)\,\left(a+b\,x^8\right)\,\sqrt{c+d\,x^8}\right)$$

Problem 758: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(a+b\,x^8\right)^2\,\sqrt{c+d\,x^8}}\,dx$$

Optimal (type 6, 59 leaves, 2 steps):

$$\frac{x\,\sqrt{1+\frac{d\,x^8}{c}}\,\text{AppellF1}\left[\frac{1}{8},2,\frac{1}{2},\frac{9}{8},-\frac{b\,x^8}{a},-\frac{d\,x^8}{c}\right]}{a^2\,\sqrt{c+d\,x^8}}$$

Result (type 6, 341 leaves):

$$\left(x\left(-\frac{3\,b\,\left(c+d\,x^8\right)}{a}+\left(27\,c\,\left(7\,b\,c-8\,a\,d\right)\,\text{AppellF1}\left[\frac{1}{8},\frac{1}{2},1,\frac{9}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\Big/\left(-9\,a\,c\,\text{AppellF1}\left[\frac{1}{8},\frac{1}{2},1,\frac{9}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+4\,x^8\left(2\,b\,c\,\text{AppellF1}\left[\frac{9}{8},\frac{1}{2},2,\frac{17}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{9}{8},\frac{3}{2},1,\frac{17}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\right)+\left(17\,b\,c\,d\,x^8\,\text{AppellF1}\left[\frac{9}{8},\frac{1}{2},1,\frac{17}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\Big/\left(-17\,a\,c\,\text{AppellF1}\left[\frac{9}{8},\frac{1}{2},1,\frac{17}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+4\,x^8\left(2\,b\,c\,\text{AppellF1}\left[\frac{17}{8},\frac{1}{2},2,\frac{25}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]+a\,d\,\text{AppellF1}\left[\frac{17}{8},\frac{3}{2},1,\frac{25}{8},-\frac{d\,x^8}{c},-\frac{b\,x^8}{a}\right]\right)\right)\right)\right)\Big/\left(24\,\left(-b\,c+a\,d\right)\,\left(a+b\,x^8\right)\,\sqrt{c+d\,x^8}\right)$$

Problem 759: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^2 \left(a+b x^8\right)^2 \sqrt{c+d x^8}} \, dx$$

Optimal (type 6, 62 leaves, 2 steps):

$$-\frac{\sqrt{1+\frac{d x^8}{c}} \text{ AppellF1}\left[-\frac{1}{8},2,\frac{1}{2},\frac{7}{8},-\frac{b x^8}{a},-\frac{d x^8}{c}\right]}{a^2 x \sqrt{c+d x^8}}$$

Result (type 6, 399 leaves):

$$\left(\frac{35 \left(c+d x^8\right) \left(-8 a^2 d+9 b^2 c x^8+8 a b \left(c-d x^8\right)\right)}{c}-\left(75 a \left(9 b^2 c^2-40 a b c d+24 a^2 d^2\right) x^8 \text{ AppellF1}\left[\frac{7}{8},\frac{1}{2},1,\frac{15}{8},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right) \Big/ \left(-15 a c \text{ AppellF1}\left[\frac{7}{8},\frac{1}{2},1,\frac{15}{8},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+4 x^8 \left(2 b c \text{ AppellF1}\left[\frac{15}{8},\frac{1}{2},2,\frac{23}{8},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+a d \text{ AppellF1}\left[\frac{15}{8},\frac{3}{2},1,\frac{23}{8},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\right)+\left(161 a b d (9 b c-8 a d) x^{16} \text{ AppellF1}\left[\frac{15}{8},\frac{1}{2},1,\frac{23}{8},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right) \Big/ \left(-23 a c \text{ AppellF1}\left[\frac{15}{8},\frac{1}{2},1,\frac{23}{8},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+4 x^8 \left(2 b c \text{ AppellF1}\left[\frac{23}{8},\frac{1}{2},2,\frac{31}{8},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+a d \text{ AppellF1}\left[\frac{23}{8},\frac{3}{2},1,\frac{31}{8},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\right)\right) \Big/ \left(280 a^2 (-b c+a d) x \left(a+b x^8\right) \sqrt{c+d x^8}\right)$$

Problem 760: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{x^4 \left(a+b x^8\right)^2 \sqrt{c+d x^8}} \, dx$$

Optimal (type 6, 64 leaves, 2 steps):

$$-\frac{\sqrt{1+\frac{d x^8}{c}} \text{ AppellF1}\left[-\frac{3}{8},2,\frac{1}{2},\frac{5}{8},-\frac{b x^8}{a},-\frac{d x^8}{c}\right]}{3 a^2 x^3 \sqrt{c+d x^8}}$$

Result (type 6, 399 leaves):

$$\left(\frac{65\left(c+d x^8\right)\left(-8 a^2 d+11 b^2 c x^8+8 a b\left(c-d x^8\right)\right)}{c}-\right.$$
$$\left(169 a\left(33 b^2 c^2-56 a b c d+8 a^2 d^2\right) x^8 \text{AppellF1}\left[\frac{5}{8},\frac{1}{2},1,\frac{13}{8},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\Big/$$
$$\left(-13 a c \text{AppellF1}\left[\frac{5}{8},\frac{1}{2},1,\frac{13}{8},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+4 x^8\left(2 b c \text{AppellF1}\left[\frac{13}{8},\frac{1}{2},\right.\right.\right.$$
$$\left.\left.\left.2,\frac{21}{8},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+a d \text{AppellF1}\left[\frac{13}{8},\frac{3}{2},1,\frac{21}{8},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\right)+$$
$$\left(105 a b d\left(11 b c-8 a d\right) x^{16} \text{AppellF1}\left[\frac{13}{8},\frac{1}{2},1,\frac{21}{8},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\Big/$$
$$\left(-21 a c \text{AppellF1}\left[\frac{13}{8},\frac{1}{2},1,\frac{21}{8},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+\right.$$
$$4 x^8\left(2 b c \text{AppellF1}\left[\frac{21}{8},\frac{1}{2},2,\frac{29}{8},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]+\right.$$
$$\left.\left.\left.a d \text{AppellF1}\left[\frac{21}{8},\frac{3}{2},1,\frac{29}{8},-\frac{d x^8}{c},-\frac{b x^8}{a}\right]\right)\right)\right)\Big/$$
$$\left(1560 a^2\left(-b c+a d\right) x^3\left(a+b x^8\right)\sqrt{c+d x^8}\right)$$

Problem 818: Result more than twice size of optimal antiderivative.

$$\int\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q (e x)^m \, dx$$

Optimal (type 6, 105 leaves, 4 steps):

$$\frac{1}{e\left(1+m\right)}\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{a x^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{c x^2}\right)^{-q}$$
$$(e x)^{1+m} \text{AppellF1}\left[\frac{1}{2}\left(-1-m\right),-p,-q,\frac{1-m}{2},-\frac{b}{a x^2},-\frac{d}{c x^2}\right]$$

Result (type 6, 284 leaves):

$$\left(b d\left(3+m-2 p-2 q\right)\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q x\,(e x)^m\right.$$
$$\left.\text{AppellF1}\left[\frac{1}{2}\left(1+m-2 p-2 q\right),-p,-q,\frac{1}{2}\left(3+m-2 p-2 q\right),-\frac{a x^2}{b},-\frac{c x^2}{d}\right]\right)\Big/$$
$$\left(\left(1+m-2 p-2 q\right)\left(b d\left(3+m-2 p-2 q\right)\right.\right.$$
$$\text{AppellF1}\left[\frac{1}{2}\left(1+m-2 p-2 q\right),-p,-q,\frac{1}{2}\left(3+m-2 p-2 q\right),-\frac{a x^2}{b},-\frac{c x^2}{d}\right]+$$
$$2 x^2\left(a d p \text{AppellF1}\left[\frac{1}{2}\left(3+m-2 p-2 q\right),1-p,-q,\frac{1}{2}\left(5+m-2 p-2 q\right),-\frac{a x^2}{b},-\frac{c x^2}{d}\right]+\right.$$
$$\left.\left.\left.b c q \text{AppellF1}\left[\frac{1}{2}\left(3+m-2 p-2 q\right),-p,1-q,\frac{1}{2}\left(5+m-2 p-2 q\right),-\frac{a x^2}{b},-\frac{c x^2}{d}\right]\right)\right)\right)$$

Problem 819: Result more than twice size of optimal antiderivative.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^4 \, dx$$

Optimal (type 6, 84 leaves, 4 steps):

$$\frac{1}{5} \left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{a x^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{c x^2}\right)^{-q} x^5 \, \text{AppellF1}\left[-\frac{5}{2}, -p, -q, -\frac{3}{2}, -\frac{b}{a x^2}, -\frac{d}{c x^2}\right]$$

Result (type 6, 254 leaves):

$$\left(b d (-7+2 p+2 q) \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^5 \, \text{AppellF1}\left[\frac{5}{2}-p-q, -p, -q, \frac{7}{2}-p-q, -\frac{a x^2}{b}, -\frac{c x^2}{d}\right]\right) \Big/ \left((-5+2 p+2 q) \left(b d (7-2 p-2 q) \, \text{AppellF1}\left[\frac{5}{2}-p-q, -p, -q, \frac{7}{2}-p-q, -\frac{a x^2}{b}, -\frac{c x^2}{d}\right] + 2 x^2 \left(a d p \, \text{AppellF1}\left[\frac{7}{2}-p-q, 1-p, -q, \frac{9}{2}-p-q, -\frac{a x^2}{b}, -\frac{c x^2}{d}\right] + b c q \, \text{AppellF1}\left[\frac{7}{2}-p-q, -p, 1-q, \frac{9}{2}-p-q, -\frac{a x^2}{b}, -\frac{c x^2}{d}\right]\right)\right)\right)$$

Problem 820: Result more than twice size of optimal antiderivative.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^3 \, dx$$

Optimal (type 6, 100 leaves, 3 steps):

$$\frac{1}{2 a^3 (1+p)} b^2 \left(a+\frac{b}{x^2}\right)^{1+p} \left(c+\frac{d}{x^2}\right)^q \left(\frac{b \left(c+\frac{d}{x^2}\right)}{b c-a d}\right)^{-q} \text{AppellF1}\left[1+p, -q, 3, 2+p, -\frac{d \left(a+\frac{b}{x^2}\right)}{b c-a d}, \frac{a+\frac{b}{x^2}}{a}\right]$$

Result (type 6, 229 leaves):

$$\left(b d (-3+p+q) \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^4 \, \text{AppellF1}\left[2-p-q, -p, -q, 3-p-q, -\frac{a x^2}{b}, -\frac{c x^2}{d}\right]\right) \Big/ \left(2 (-2+p+q) \left(-b d (-3+p+q) \, \text{AppellF1}\left[2-p-q, -p, -q, 3-p-q, -\frac{a x^2}{b}, -\frac{c x^2}{d}\right] + x^2 \left(a d p \, \text{AppellF1}\left[3-p-q, 1-p, -q, 4-p-q, -\frac{a x^2}{b}, -\frac{c x^2}{d}\right] + b c q \, \text{AppellF1}\left[3-p-q, -p, 1-q, 4-p-q, -\frac{a x^2}{b}, -\frac{c x^2}{d}\right]\right)\right)\right)$$

Problem 821: Result more than twice size of optimal antiderivative.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^2 \, dx$$

Optimal (type 6, 84 leaves, 4 steps):

$$\frac{1}{3}\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{a x^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{c x^2}\right)^{-q} x^3\,\text{AppellF1}\left[-\frac{3}{2},-p,-q,-\frac{1}{2},-\frac{b}{a x^2},-\frac{d}{c x^2}\right]$$

Result (type 6, 254 leaves):

$$\left(b\,d\,(-5+2p+2q)\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q x^3\,\text{AppellF1}\left[\frac{3}{2}-p-q,-p,-q,\frac{5}{2}-p-q,-\frac{a x^2}{b},-\frac{c x^2}{d}\right]\right)\Big/$$
$$\left((-3+2p+2q)\left(b\,d\,(5-2p-2q)\,\text{AppellF1}\left[\frac{3}{2}-p-q,-p,-q,\frac{5}{2}-p-q,-\frac{a x^2}{b},-\frac{c x^2}{d}\right]+\right.\right.$$
$$2x^2\left(a\,d\,p\,\text{AppellF1}\left[\frac{5}{2}-p-q,1-p,-q,\frac{7}{2}-p-q,-\frac{a x^2}{b},-\frac{c x^2}{d}\right]+\right.$$
$$\left.\left.\left.b\,c\,q\,\text{AppellF1}\left[\frac{5}{2}-p-q,-p,1-q,\frac{7}{2}-p-q,-\frac{a x^2}{b},-\frac{c x^2}{d}\right]\right)\right)\right)$$

Problem 822: Result more than twice size of optimal antiderivative.

$$\int\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q x\,dx$$

Optimal (type 6, 98 leaves, 3 steps):

$$-\frac{1}{2a^2(1+p)}b\left(a+\frac{b}{x^2}\right)^{1+p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{b\left(c+\frac{d}{x^2}\right)}{b c-a d}\right)^{-q}\text{AppellF1}\left[1+p,-q,2,2+p,-\frac{d\left(a+\frac{b}{x^2}\right)}{b c-a d},\frac{a+\frac{b}{x^2}}{a}\right]$$

Result (type 6, 229 leaves):

$$\left(b\,d\,(-2+p+q)\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q x^2\,\text{AppellF1}\left[1-p-q,-p,-q,2-p-q,-\frac{a x^2}{b},-\frac{c x^2}{d}\right]\right)\Big/$$
$$\left(2(-1+p+q)\left(-b\,d\,(-2+p+q)\,\text{AppellF1}\left[1-p-q,-p,-q,2-p-q,-\frac{a x^2}{b},-\frac{c x^2}{d}\right]+\right.\right.$$
$$x^2\left(a\,d\,p\,\text{AppellF1}\left[2-p-q,1-p,-q,3-p-q,-\frac{a x^2}{b},-\frac{c x^2}{d}\right]+\right.$$
$$\left.\left.\left.b\,c\,q\,\text{AppellF1}\left[2-p-q,-p,1-q,3-p-q,-\frac{a x^2}{b},-\frac{c x^2}{d}\right]\right)\right)\right)$$

Problem 823: Result more than twice size of optimal antiderivative.

$$\int\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q dx$$

Optimal (type 6, 79 leaves, 4 steps):

$$\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{a x^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{c x^2}\right)^{-q} x\,\text{AppellF1}\left[-\frac{1}{2},-p,-q,\frac{1}{2},-\frac{b}{a x^2},-\frac{d}{c x^2}\right]$$

Result (type 6, 252 leaves):

$$\left(b\,d\,(-3+2\,p+2\,q)\,\left(a+\frac{b}{x^2}\right)^p\,\left(c+\frac{d}{x^2}\right)^q\,x\,\text{AppellF1}\left[\frac{1}{2}-p-q,\,-p,\,-q,\,\frac{3}{2}-p-q,\,-\frac{a\,x^2}{b},\,-\frac{c\,x^2}{d}\right]\right)\Big/$$
$$\left((-1+2\,p+2\,q)\,\left(b\,d\,(3-2\,p-2\,q)\,\text{AppellF1}\left[\frac{1}{2}-p-q,\,-p,\,-q,\,\frac{3}{2}-p-q,\,-\frac{a\,x^2}{b},\,-\frac{c\,x^2}{d}\right]+\right.\right.$$
$$2\,x^2\,\left(a\,d\,p\,\text{AppellF1}\left[\frac{3}{2}-p-q,\,1-p,\,-q,\,\frac{5}{2}-p-q,\,-\frac{a\,x^2}{b},\,-\frac{c\,x^2}{d}\right]+\right.$$
$$\left.\left.\left.b\,c\,q\,\text{AppellF1}\left[\frac{3}{2}-p-q,\,-p,\,1-q,\,\frac{5}{2}-p-q,\,-\frac{a\,x^2}{b},\,-\frac{c\,x^2}{d}\right]\right)\right)\right)$$

Problem 824: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\,\left(c+\frac{d}{x^2}\right)^q}{x}\,dx$$

Optimal (type 6, 97 leaves, 3 steps):

$$\frac{1}{2\,a\,(1+p)}\left(a+\frac{b}{x^2}\right)^{1+p}\,\left(c+\frac{d}{x^2}\right)^q\,\left(\frac{b\,\left(c+\frac{d}{x^2}\right)}{b\,c-a\,d}\right)^{-q}\,\text{AppellF1}\left[1+p,\,-q,\,1,\,2+p,\,-\frac{d\,\left(a+\frac{b}{x^2}\right)}{b\,c-a\,d},\,\frac{a+\frac{b}{x^2}}{a}\right]$$

Result (type 6, 223 leaves):

$$-\left(\left(b\,d\,(-1+p+q)\,\left(a+\frac{b}{x^2}\right)^p\,\left(c+\frac{d}{x^2}\right)^q\,\text{AppellF1}\left[-p-q,\,-p,\,-q,\,1-p-q,\,-\frac{a\,x^2}{b},\,-\frac{c\,x^2}{d}\right]\right)\Big/\right.$$
$$\left(2\,(p+q)\,\left(b\,d\,(-1+p+q)\,\text{AppellF1}\left[-p-q,\,-p,\,-q,\,1-p-q,\,-\frac{a\,x^2}{b},\,-\frac{c\,x^2}{d}\right]-\right.\right.$$
$$x^2\,\left(a\,d\,p\,\text{AppellF1}\left[1-p-q,\,1-p,\,-q,\,2-p-q,\,-\frac{a\,x^2}{b},\,-\frac{c\,x^2}{d}\right]+\right.$$
$$\left.\left.\left.\left.b\,c\,q\,\text{AppellF1}\left[1-p-q,\,-p,\,1-q,\,2-p-q,\,-\frac{a\,x^2}{b},\,-\frac{c\,x^2}{d}\right]\right)\right)\right)\right)$$

Problem 825: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\,\left(c+\frac{d}{x^2}\right)^q}{x^2}\,dx$$

Optimal (type 6, 82 leaves, 4 steps):

$$-\frac{1}{x}\left(a+\frac{b}{x^2}\right)^p\,\left(1+\frac{b}{a\,x^2}\right)^{-p}\,\left(c+\frac{d}{x^2}\right)^q\,\left(1+\frac{d}{c\,x^2}\right)^{-q}\,\text{AppellF1}\left[\frac{1}{2},\,-p,\,-q,\,\frac{3}{2},\,-\frac{b}{a\,x^2},\,-\frac{d}{c\,x^2}\right]$$

Result (type 6, 254 leaves):

$$\left(b\,d\,(-1+2p+2q)\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q \text{AppellF1}\left[-\frac{1}{2}-p-q,-p,-q,\frac{1}{2}-p-q,-\frac{a\,x^2}{b},-\frac{c\,x^2}{d}\right]\right)\Bigg/\left((1+2p+2q)\,x\left(b\,d\,(1-2p-2q)\,\text{AppellF1}\left[-\frac{1}{2}-p-q,-p,-q,\frac{1}{2}-p-q,-\frac{a\,x^2}{b},-\frac{c\,x^2}{d}\right]+2x^2\left(a\,d\,p\,\text{AppellF1}\left[\frac{1}{2}-p-q,1-p,-q,\frac{3}{2}-p-q,-\frac{a\,x^2}{b},-\frac{c\,x^2}{d}\right]+b\,c\,q\,\text{AppellF1}\left[\frac{1}{2}-p-q,-p,1-q,\frac{3}{2}-p-q,-\frac{a\,x^2}{b},-\frac{c\,x^2}{d}\right]\right)\right)\right)$$

Problem 827: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^4}\,dx$$

Optimal (type 6, 84 leaves, 4 steps):

$$-\frac{1}{3x^3}\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{a\,x^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{c\,x^2}\right)^{-q}\text{AppellF1}\left[\frac{3}{2},-p,-q,\frac{5}{2},-\frac{b}{a\,x^2},-\frac{d}{c\,x^2}\right]$$

Result (type 6, 255 leaves):

$$\left(b\,d\,(1+2p+2q)\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q \text{AppellF1}\left[-\frac{3}{2}-p-q,-p,-q,-\frac{1}{2}-p-q,-\frac{a\,x^2}{b},-\frac{c\,x^2}{d}\right]\right)\Bigg/\left((3+2p+2q)\,x^3\left(-b\,d\,(1+2p+2q)\,\text{AppellF1}\left[-\frac{3}{2}-p-q,-p,-q,-\frac{1}{2}-p-q,-\frac{a\,x^2}{b},-\frac{c\,x^2}{d}\right]+2x^2\left(a\,d\,p\,\text{AppellF1}\left[-\frac{1}{2}-p-q,1-p,-q,\frac{1}{2}-p-q,-\frac{a\,x^2}{b},-\frac{c\,x^2}{d}\right]+b\,c\,q\,\text{AppellF1}\left[-\frac{1}{2}-p-q,-p,1-q,\frac{1}{2}-p-q,-\frac{a\,x^2}{b},-\frac{c\,x^2}{d}\right]\right)\right)\right)$$

Problem 828: Result more than twice size of optimal antiderivative.

$$\int\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q(e\,x)^{5/2}\,dx$$

Optimal (type 6, 91 leaves, 4 steps):

$$\frac{1}{7e}2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{a\,x^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{c\,x^2}\right)^{-q}(e\,x)^{7/2}\,\text{AppellF1}\left[-\frac{7}{4},-p,-q,-\frac{3}{4},-\frac{b}{a\,x^2},-\frac{d}{c\,x^2}\right]$$

Result (type 6, 260 leaves):

$$\left(2\,b\,d\,(-11+4\,p+4\,q)\,\left(a+\frac{b}{x^2}\right)^p\,\left(c+\frac{d}{x^2}\right)^q\,x\,(e\,x)^{5/2}\,\text{AppellF1}\left[\frac{7}{4}-p-q,\,-p,\,-q,\,\frac{11}{4}-p-q,\,-\frac{a\,x^2}{b},\,-\frac{c\,x^2}{d}\right]\right)\Big/\left((-7+4\,p+4\,q)\,\left(b\,d\,(11-4\,p-4\,q)\,\text{AppellF1}\left[\frac{7}{4}-p-q,\,-p,\,-q,\,\frac{11}{4}-p-q,\,-\frac{a\,x^2}{b},\,-\frac{c\,x^2}{d}\right]+4\,x^2\,\left(a\,d\,p\,\text{AppellF1}\left[\frac{11}{4}-p-q,\,1-p,\,-q,\,\frac{15}{4}-p-q,\,-\frac{a\,x^2}{b},\,-\frac{c\,x^2}{d}\right]+b\,c\,q\,\text{AppellF1}\left[\frac{11}{4}-p-q,\,-p,\,1-q,\,\frac{15}{4}-p-q,\,-\frac{a\,x^2}{b},\,-\frac{c\,x^2}{d}\right]\right)\right)\right)$$

Problem 829: Result more than twice size of optimal antiderivative.

$$\int\left(a+\frac{b}{x^2}\right)^p\,\left(c+\frac{d}{x^2}\right)^q\,(e\,x)^{3/2}\,dx$$

Optimal (type 6, 91 leaves, 4 steps):

$$\frac{1}{5\,e}2\,\left(a+\frac{b}{x^2}\right)^p\,\left(1+\frac{b}{a\,x^2}\right)^{-p}\,\left(c+\frac{d}{x^2}\right)^q\,\left(1+\frac{d}{c\,x^2}\right)^{-q}\,(e\,x)^{5/2}\,\text{AppellF1}\left[-\frac{5}{4},\,-p,\,-q,\,-\frac{1}{4},\,-\frac{b}{a\,x^2},\,-\frac{d}{c\,x^2}\right]$$

Result (type 6, 260 leaves):

$$\left(2\,b\,d\,(-9+4\,p+4\,q)\,\left(a+\frac{b}{x^2}\right)^p\,\left(c+\frac{d}{x^2}\right)^q\,x\,(e\,x)^{3/2}\,\text{AppellF1}\left[\frac{5}{4}-p-q,\,-p,\,-q,\,\frac{9}{4}-p-q,\,-\frac{a\,x^2}{b},\,-\frac{c\,x^2}{d}\right]\right)\Big/\left((-5+4\,p+4\,q)\,\left(b\,d\,(9-4\,p-4\,q)\,\text{AppellF1}\left[\frac{5}{4}-p-q,\,-p,\,-q,\,\frac{9}{4}-p-q,\,-\frac{a\,x^2}{b},\,-\frac{c\,x^2}{d}\right]+4\,x^2\,\left(a\,d\,p\,\text{AppellF1}\left[\frac{9}{4}-p-q,\,1-p,\,-q,\,\frac{13}{4}-p-q,\,-\frac{a\,x^2}{b},\,-\frac{c\,x^2}{d}\right]+b\,c\,q\,\text{AppellF1}\left[\frac{9}{4}-p-q,\,-p,\,1-q,\,\frac{13}{4}-p-q,\,-\frac{a\,x^2}{b},\,-\frac{c\,x^2}{d}\right]\right)\right)\right)$$

Problem 830: Result more than twice size of optimal antiderivative.

$$\int\left(a+\frac{b}{x^2}\right)^p\,\left(c+\frac{d}{x^2}\right)^q\,\sqrt{e\,x}\,dx$$

Optimal (type 6, 91 leaves, 4 steps):

$$\frac{1}{3\,e}2\,\left(a+\frac{b}{x^2}\right)^p\,\left(1+\frac{b}{a\,x^2}\right)^{-p}\,\left(c+\frac{d}{x^2}\right)^q\,\left(1+\frac{d}{c\,x^2}\right)^{-q}\,(e\,x)^{3/2}\,\text{AppellF1}\left[-\frac{3}{4},\,-p,\,-q,\,\frac{1}{4},\,-\frac{b}{a\,x^2},\,-\frac{d}{c\,x^2}\right]$$

Result (type 6, 260 leaves):

$$\left(2\,b\,d\,(-7+4\,p+4\,q)\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q x\,\sqrt{e\,x}\;\text{AppellF1}\left[\frac{3}{4}-p-q,-p,-q,\frac{7}{4}-p-q,-\frac{a\,x^2}{b},-\frac{c\,x^2}{d}\right]\right)\Big/\left(\left(-3+4\,p+4\,q\right)\left(b\,d\,(7-4\,p-4\,q)\,\text{AppellF1}\left[\frac{3}{4}-p-q,-p,-q,\frac{7}{4}-p-q,-\frac{a\,x^2}{b},-\frac{c\,x^2}{d}\right]+4\,x^2\left(a\,d\,p\,\text{AppellF1}\left[\frac{7}{4}-p-q,1-p,-q,\frac{11}{4}-p-q,-\frac{a\,x^2}{b},-\frac{c\,x^2}{d}\right]+b\,c\,q\,\text{AppellF1}\left[\frac{7}{4}-p-q,-p,1-q,\frac{11}{4}-p-q,-\frac{a\,x^2}{b},-\frac{c\,x^2}{d}\right]\right)\right)\right)$$

Problem 831: Result more than twice size of optimal antiderivative.

$$\int\frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{\sqrt{e\,x}}\,dx$$

Optimal (type 6, 89 leaves, 4 steps):

$$\frac{1}{e}2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{a\,x^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{c\,x^2}\right)^{-q}\sqrt{e\,x}\;\text{AppellF1}\left[-\frac{1}{4},-p,-q,\frac{3}{4},-\frac{b}{a\,x^2},-\frac{d}{c\,x^2}\right]$$

Result (type 6, 260 leaves):

$$\left(2\,b\,d\,(-5+4\,p+4\,q)\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q x\,\text{AppellF1}\left[\frac{1}{4}-p-q,-p,-q,\frac{5}{4}-p-q,-\frac{a\,x^2}{b},-\frac{c\,x^2}{d}\right]\right)\Big/\left(\left(-1+4\,p+4\,q\right)\sqrt{e\,x}\left(b\,d\,(5-4\,p-4\,q)\,\text{AppellF1}\left[\frac{1}{4}-p-q,-p,-q,\frac{5}{4}-p-q,-\frac{a\,x^2}{b},-\frac{c\,x^2}{d}\right]+4\,x^2\left(a\,d\,p\,\text{AppellF1}\left[\frac{5}{4}-p-q,1-p,-q,\frac{9}{4}-p-q,-\frac{a\,x^2}{b},-\frac{c\,x^2}{d}\right]+b\,c\,q\,\text{AppellF1}\left[\frac{5}{4}-p-q,-p,1-q,\frac{9}{4}-p-q,-\frac{a\,x^2}{b},-\frac{c\,x^2}{d}\right]\right)\right)\right)$$

Problem 832: Result more than twice size of optimal antiderivative.

$$\int\frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{(e\,x)^{3/2}}\,dx$$

Optimal (type 6, 89 leaves, 4 steps):

$$-\frac{1}{e\,\sqrt{e\,x}}2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{a\,x^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{c\,x^2}\right)^{-q}\text{AppellF1}\left[\frac{1}{4},-p,-q,\frac{5}{4},-\frac{b}{a\,x^2},-\frac{d}{c\,x^2}\right]$$

Result (type 6, 260 leaves):

$$\left(2\,b\,d\,(-3+4\,p+4\,q)\,\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q x\,\text{AppellF1}\left[-\frac{1}{4}-p-q,\,-p,\,-q,\,\frac{3}{4}-p-q,\,-\frac{a\,x^2}{b},\,-\frac{c\,x^2}{d}\right]\right)\Big/\left((1+4\,p+4\,q)\,(e\,x)^{3/2}\left(b\,d\,(3-4\,p-4\,q)\,\text{AppellF1}\left[-\frac{1}{4}-p-q,\,-p,\,-q,\,\frac{3}{4}-p-q,\,-\frac{a\,x^2}{b},\,-\frac{c\,x^2}{d}\right]+4\,x^2\left(a\,d\,p\,\text{AppellF1}\left[\frac{3}{4}-p-q,\,1-p,\,-q,\,\frac{7}{4}-p-q,\,-\frac{a\,x^2}{b},\,-\frac{c\,x^2}{d}\right]+b\,c\,q\,\text{AppellF1}\left[\frac{3}{4}-p-q,\,-p,\,1-q,\,\frac{7}{4}-p-q,\,-\frac{a\,x^2}{b},\,-\frac{c\,x^2}{d}\right]\right)\right)\right)$$

Problem 833: Result more than twice size of optimal antiderivative.

$$\int\frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{(e\,x)^{5/2}}\,dx$$

Optimal (type 6, 91 leaves, 4 steps):

$$-\frac{1}{3\,e\,(e\,x)^{3/2}}2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{a\,x^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{c\,x^2}\right)^{-q}\text{AppellF1}\left[\frac{3}{4},\,-p,\,-q,\,\frac{7}{4},\,-\frac{b}{a\,x^2},\,-\frac{d}{c\,x^2}\right]$$

Result (type 6, 260 leaves):

$$\left(2\,b\,d\,(-1+4\,p+4\,q)\,\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q x\,\text{AppellF1}\left[-\frac{3}{4}-p-q,\,-p,\,-q,\,\frac{1}{4}-p-q,\,-\frac{a\,x^2}{b},\,-\frac{c\,x^2}{d}\right]\right)\Big/\left((3+4\,p+4\,q)\,(e\,x)^{5/2}\left(b\,d\,(1-4\,p-4\,q)\,\text{AppellF1}\left[-\frac{3}{4}-p-q,\,-p,\,-q,\,\frac{1}{4}-p-q,\,-\frac{a\,x^2}{b},\,-\frac{c\,x^2}{d}\right]+4\,x^2\left(a\,d\,p\,\text{AppellF1}\left[\frac{1}{4}-p-q,\,1-p,\,-q,\,\frac{5}{4}-p-q,\,-\frac{a\,x^2}{b},\,-\frac{c\,x^2}{d}\right]+b\,c\,q\,\text{AppellF1}\left[\frac{1}{4}-p-q,\,-p,\,1-q,\,\frac{5}{4}-p-q,\,-\frac{a\,x^2}{b},\,-\frac{c\,x^2}{d}\right]\right)\right)\right)$$

Problem 846: Result more than twice size of optimal antiderivative.

$$\int\frac{1}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x}}\,dx$$

Optimal (type 3, 8 leaves, 2 steps):

$$2\,\text{ArcCosh}\left[\sqrt{x}\right]$$

Result (type 3, 20 leaves):

$$4\,\text{ArcSinh}\left[\frac{\sqrt{-1+\sqrt{x}}}{\sqrt{2}}\right]$$

Problem 883: Result more than twice size of optimal antiderivative.

$$\int x^{13} (b+c x)^{13} (b+2 c x) \, dx$$

Optimal (type 1, 14 leaves, 1 step):

$$\frac{1}{14} x^{14} (b+c x)^{14}$$

Result (type 1, 172 leaves):

$$\frac{b^{14} x^{14}}{14}+b^{13} c x^{15}+\frac{13}{2} b^{12} c^2 x^{16}+26 b^{11} c^3 x^{17}+\frac{143}{2} b^{10} c^4 x^{18}+143 b^9 c^5 x^{19}+\frac{429}{2} b^8 c^6 x^{20}+\frac{1716}{7} b^7 c^7 x^{21}+\frac{429}{2} b^6 c^8 x^{22}+143 b^5 c^9 x^{23}+\frac{143}{2} b^4 c^{10} x^{24}+26 b^3 c^{11} x^{25}+\frac{13}{2} b^2 c^{12} x^{26}+b c^{13} x^{27}+\frac{c^{14} x^{28}}{14}$$

Problem 884: Result more than twice size of optimal antiderivative.

$$\int x^{27} (b+c x^2)^{13} (b+2 c x^2) \, dx$$

Optimal (type 1, 16 leaves, 2 steps):

$$\frac{1}{28} x^{28} (b+c x^2)^{14}$$

Result (type 1, 182 leaves):

$$\frac{b^{14} x^{28}}{28}+\frac{1}{2} b^{13} c x^{30}+\frac{13}{4} b^{12} c^2 x^{32}+13 b^{11} c^3 x^{34}+\frac{143}{4} b^{10} c^4 x^{36}+\frac{143}{2} b^9 c^5 x^{38}+\frac{429}{4} b^8 c^6 x^{40}+\frac{858}{7} b^7 c^7 x^{42}+\frac{429}{4} b^6 c^8 x^{44}+\frac{143}{2} b^5 c^9 x^{46}+\frac{143}{4} b^4 c^{10} x^{48}+13 b^3 c^{11} x^{50}+\frac{13}{4} b^2 c^{12} x^{52}+\frac{1}{2} b c^{13} x^{54}+\frac{c^{14} x^{56}}{28}$$

Problem 885: Result more than twice size of optimal antiderivative.

$$\int x^{41} (b+c x^3)^{13} (b+2 c x^3) \, dx$$

Optimal (type 1, 16 leaves, 2 steps):

$$\frac{1}{42} x^{42} (b+c x^3)^{14}$$

Result (type 1, 186 leaves):

$$\frac{b^{14}x^{42}}{42}+\frac{1}{3}b^{13}c\,x^{45}+\frac{13}{6}b^{12}c^2x^{48}+\frac{26}{3}b^{11}c^3x^{51}+\frac{143}{6}b^{10}c^4x^{54}+$$
$$\frac{143}{3}b^9c^5x^{57}+\frac{143}{2}b^8c^6x^{60}+\frac{572}{7}b^7c^7x^{63}+\frac{143}{2}b^6c^8x^{66}+\frac{143}{3}b^5c^9x^{69}+$$
$$\frac{143}{6}b^4c^{10}x^{72}+\frac{26}{3}b^3c^{11}x^{75}+\frac{13}{6}b^2c^{12}x^{78}+\frac{1}{3}b\,c^{13}x^{81}+\frac{c^{14}x^{84}}{42}$$

Problem 895: Result more than twice size of optimal antiderivative.

$$\int\frac{x^{-1-7n}\left(b+2c\,x^n\right)}{\left(b+c\,x^n\right)^8}\,dx$$

Optimal (type 3, 21 leaves, 2 steps):

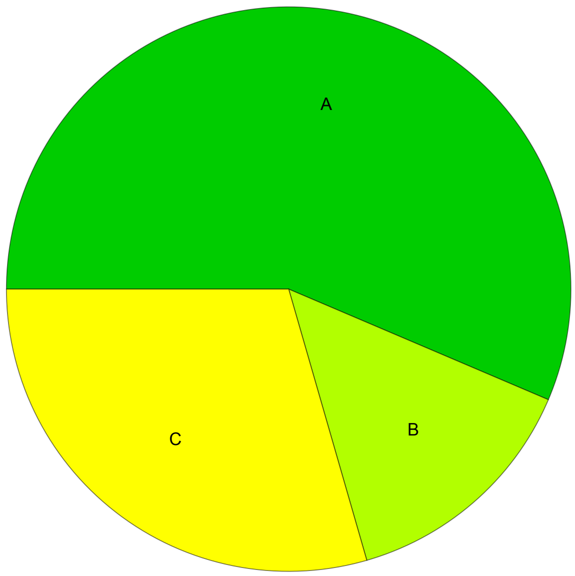
$$-\frac{x^{-7n}}{7n\left(b+c\,x^n\right)^7}$$

Result (type 3, 127 leaves):

$$-\frac{1}{7b^{14}n\left(b+c\,x^n\right)^7}$$
$$x^{-7n}\left(b^{14}+1716b^7c^7x^{7n}+12012b^6c^8x^{8n}+36036b^5c^9x^{9n}+60060b^4c^{10}x^{10n}+60060b^3c^{11}x^{11n}+\right.$$
$$\left.36036b^2c^{12}x^{12n}+12012b\,c^{13}x^{13n}+1716c^{14}x^{14n}\right)$$

Summary of Integration Test Results

913 integration problems



A - 515 optimal antiderivatives

B - 129 more than twice size of optimal antiderivatives

C - 269 unnecessarily complex antiderivatives

D - 0 unable to integrate problems

E - 0 integration timeouts